Abstracts of lectures in order of the meeting program

Scientific Meeting in Memory of Professor Józef Joachim Telega

IPPT (Institute of Fundamental Technological Research), Warsaw, Poland,

March 5, 2025, meeting time from 03:00 p.m. – 07:00 p.m. of Warsaw time.

SCHEDULE OF LECTURES

03:00 p.m.	Opening Barbara Gambin and Tomasz Lewiński	Short opening. Some words about the life and outcome of Joachim Telega
03:10 p.m.	Dieter Weichert	Joachim Telega- in memoriam
03:30 p.m.	Meir Shillor	The new damped normal compliance contact condition and applications
03:50 p.m.	Andrej Cherkaev	Optimal multimaterial composites: the structure of fields, bounds, laminates, and rank-one envelope
04:10 p.m.	Elena Cherkaeva	Stieltjes analytic representation of effective properties of composites: Forward and inverse homogenization.
04:30 p.m.	Graeme Milton	Bounds on the complex permittivity and quality factors of two-phase composites
04:50 p.m.	BREAK	
05:10 p.m.	Gennady Mishuris	Upscaling the toughness due to its periodic variation. Case study: hydraulic fracture
05:30 p.m.	Mircea Sofonea	Well-posedness concepts in nonlinear analysis and contact mechanics
05:50 p.m.	Grzegorz Dzierżanowski and Andrej Cherkaev	Optimum structures made of two isotropic materials and void
06:10 p.m.	StanisławJemioło	Two families fiber reinforced hyperelastic material model
06:30 p.m.	Tomasz Lewiński	Topology optimization of structures. Overview of results found by the team: Karol Bołbotowski, Sławomir Czarnecki, Radosław Czubacki, Grzegorz Dzierżanowski, Cezary Graczykowski, Krzysztof Kolanek, Michał Kursa, Tomasz Lewiński, Tomasz Łukasiak, Przemysław Sobczak, Tomasz Sokół.
06:50 p.m.	Barbara Gambin	Γ- convergence, Padé approximants, asymptotic expansions in various homogenization issues. Overview of results found by the team: Włodzimierz. Bielski, Andrzej Gałka, Barbara Gambin, Eleonora Kruglenko, Stanisław Tokarzewski, Ryszard Wojnar

The new Damped Normal Compliance contact condition and applications

Meir Shillor

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The new Damped Normal Compliance contact condition and applications The talk provides a short description of the new Damped Normal Compliance (DNC) contact condition and its applications to the impact of an elastic rod with a reactive obstacle and the vibrations of a Bernoulli beam when an obstacle is positioned below its right end. It is seen how the condition takes into account energy dissipation.

Optimal multimaterial composites: the structure of fields, bounds, laminates, and rank-one envelope.

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Optimal multimaterial composites: the structure of fields, bounds, laminates, and rank-one envelope. The problem of the best periodic composite structure is formulated as a problem of the quasiconvex envelope for a multiwell Lagrangian. Since we compare all structures, the minimizers - gradient fields or stresses in an optimal structure - are independent of the geometry and are determined by a few parameters: moduli of the mixed materials, their volume fractions, and the applied field; therefore the problem can be reduced to finite-dimensional optimization problem [1]. The problem describes the sets of variations of fields in materials in the optimal structures. The suggested pointwise inequalities constraints on the fields complement the translation bounds for multicomponent composites. The description of exact bounds is accompanied by the optimal laminates that realize them. The paper defines these laminates as paths of the Rank-One-Convex envelope that join the found sets of the optimal fields in materials. The Rank-One-Convex envelope's structure mimics a composite cell's optimal geometry. The optimal bounds and matching structures for isotropic three-material composites (one material is ideal) were found in [2]. Cherkaev and Zhang described the whole collection of anisotropic bounds and optimal structures for this problem [3]. The entire set of optimal elastic structures from two materials and void was found by Cherkaev and Dzierżanowski [4]; the optimal design of multimaterial composites discussed by Pedregal and Zhang [5], Briggs, Cherkaev, and Dzierżanowski [6], and others.

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Stieltjes analytic representation of effective properties of composites: Forward and inverse homogenization

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Stieltjes integral representations for the effective transport coefficients of composites investigated by Professor Telega and his colleagues, play a central role in the homogenization of microstructured media, linking composite microgeometry to the spectral properties of a related self-adjoint operator. The talk will discuss matrix Pade approximation and extensions of the Stieltjes representation to the case of effective viscoelastic shear modulus for two- and three-dimensional composites. I will show that the matrix spectral measure in the integral representation of the effective properties can be uniquely reconstructed; this uniqueness provides a basis for the inverse homogenization problem of recovering information about the microgeometry of the composite. Pade approximations of the spectral measure allow constructing bounds on the effective properties of composites in forward homogenization, lead to efficient computational methods for wave propagation in composite materials, and result in spectrally matched geometries in inverse homogenization, linking it to the inverse spectral problem.

Bounds on the complex permittivity and quality factors of two-phase composites

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Back in 1980 I derived bounds on the complex dielectric constant of a two phase composite and on the complex polarizability of particles in a medium. Some of these bounds were also independently obtained by David Bergman. In the case of isotropic composites the two-dimensional bounds were sharp. In three-dimensions the bounds confined the dielectric constant to lie within a lens shaped region of the complex plane. Five points on one circular arc of the lens were known to be optimal while the other arc of the lens was shown to be not optimal. While various generalizations of the bounds were made, including those of Joachim Telega and collaborators, our knowledge of the three dimensional isotropic bounds on the complex dielectric constant remained fairly stagnant. Then, in 2020, Christian Kern, Owen Miller and myself found that the non-optimal arc could be replaced by an optimal arc, attained by assemblages of doubly coated spheres. Moreover, and to my amazement, additional points on the other side of the lens were found by Christian that are optimal, being achieved by hierarchical laminates. It is not known if more points on the arc are attained. All these results were at a single frequency. Near resonance it is more important to bound the Q-factor, typically defined in terms of the width and height of the resonant response as a function of frequency. In joint work last year with Kshiteej Deshmukh we have obtained bounds on an appropriately defined Q-factor. The talk will review this progress.

Upscaling the toughness due to its periodic variation. Case study: Hydraulic Fracture.

Gennady Mishuris

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This talk will discuss whether and how an averaging-based approach to material toughness can be confidently utilized. Usually, various upscaling procedures are applied to achieve the goal. Recently, we have proposed a new averaging-based approach dependent on the material and process-dependent parameters. The respective measures come from temporal averaging (in contrast to the spatial one). They require a knowledge of the instantaneous crack tip velocity during each specific process. The temporal average approach is general in its nature, and can be used in the analysis of any stable fracture propagation process. Numerous simulations have been performed to verify the measure proposed. We used hydraulic fracture as it always produces a stable crack propagation. We utilize our extremely accurate and effective in-house built time-space adaptive solver, which can obtain solutions for all classic HF models (PKN, KGD, Radial) with arbitrary fluid rheology, leak off and pumping regime. The solver uses the crack opening and the fluid velocity as the basic unknowns in contrast to the conventional crack opening and fluid pressure pair [1]. We analyse the KGD and Radial HF models in an elastic homogeneous material characterised by periodic toughness distributions [**?**]. In particular, we show how local energy redistribution affects the process, resulting in local (in time and space) changes in the propagation regime. For example, even if both the maximum and minimum values of the toughness distribution correspond solely to the high toughness regime (under a given fluid rate), local regions exhibiting viscosity-dominated behaviour are apparent. Another interesting feature of the proposed measures: even though the toughness and energy release rate fracture criteria are equivalent in the problem under consideration (homogeneous elastic material), temporal averaging based on the energy argument appears more accurate. Finally, we show an interesting effect of the fluid reversal within the fracture for a small-time fraction and question the quasi-static approach commonly utilised in modelling propagation of HF fracture in inhomogeneous material.

References

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Well-posedness Concepts in Nonlinear Analysis and Contact Mechanics

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Well-posedness concepts for nonlinear problems represent an important topic in Functional Analysis which knew a significant development in the last decades. Originating in the papers of Tykhonov [1] and Levitin-Polyak [2], well-posedness concepts have been extended to a large number of problems, including nonlinear equations, optimization, inequalities, inclusions, fixed point, and optimal control problems. They describe the convergence to the solution of a nonlinear problem, unifies different convergence results and provides a framework in which the link between different problems can been established. Details can be found in [3]. The well-posedness concepts depend on the problem considered, vary from author to author, and even from paper to paper. Nevertheless, most of these concepts are based on two main ingredients: the existence and uniqueness of the solution and the convergence to it of a special class of sequences, the so-called approximating sequences.

We start this lecture by recalling the concepts of Tykhonov and Levitin-Polyak well-posedness for minimization problems. Then, we consider a variational-hemivariational inequality in a reflexive Banach space X, governed by a history-dependent operator. Besides the unique solvability of the inequality, we provide necessary and sufficient conditions which guarantee the uniform convergence of a sequence of functions to the solution. This allows us to introduce the best well-posedness concept in the study of this inequality. We then apply these abstract results in the study of a mathematical model which describes the frictionless contact of a viscoelastic body with a foundation made of a rigid body covered by a layer of deformable material. We also present numerical simulations which validate our theoretical results. In this way we illustrate the cross fertilization between the abstract mathematical concepts, on one hand, and their applications in Contact Mechanics, on the other hand.

References

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Optimum structures made of two isotropic materials and voids

Andrej Cherkaev and Grzegorz Dzierżanowski* with the collaboration of Nathan Briggs

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In this talk, we discuss several exact results in the optimum design of three-phase elastic structures. The problem is formulated as follows: Two isotropic materials, the "strong" and the "weak" one, are laid out with void in a given two-dimensional domain so that the functional of combined compliance and weight of the structure is minimized. The optimal distribution of three phases is determined on two levels, macro- and microscopic, similar to the classical two-phase problem. On the macro level, the design domain is divided into several subdomains. Some are filled with pure phases, while others are occupied by their mixtures (composite material). On the micro level, the study regards the optimality of multiphase microstructures. They are limited to high-rank orthogonal laminates, but other choices are also possible; e.g., the three-phase wheel assemblages are proven to be optimal if the homogenized stress tensor is isotropic. The main aim of the talk is to discuss the case when the optimal distribution of three phases is macroscopically non-unique. It occurs when the moduli of "strong" and "weak" isotropic materials are in a particular relation. We emphasize that such a non-uniqueness is relevant only in the three-phase problem; it does not occur in the two-material design.

Two families fiber reinforced hyperelastic material models

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We examine the constitutive relations of hyperelasticity for orthotropic and transversely isotropic materials. As specific instances of materials exhibiting orthotropic and transverse isotropic symmetry in the reference configuration, we consider isotropic materials reinforced with two families of fibers [8]. The theory of representation of anisotropic scalar tensor functions is employed [3]. General constitutive relations are formulated within the Lagrangian framework, and their counterparts in the Eulerian framework are derived, which is essential for their numerical implementation in the ABAQUS software via UMAT user subroutines [1].
Fundamental models of hyperelasticity for orthotropic and transversely isotropic materials are analyzed, as referenced in [2], [4] - [7]. Examples of strain energy functions and constitutive relations are provided. The objective is to identify constitutive relations for which the strain energy potential is a polyconvex function.
It is important to note that the assumption of elastic properties of the material is always an idealization. Consequently, generalizations of the proposed models within the frameworks of pseudo-hyperelasticity and visco-hyperelasticity theories are also included.
The potential applications of the proposed anisotropic hyperelastic constitutive relations are primarily in the fields of biomechanics and composite mechanics, such as elastomers reinforced with various types of continuous fibers [2]. In biomechanics, this includes constitutive modeling of skeletal muscles, cardiac muscles, arteries, and tendons [4], [7] - [9]. This work continues the research topics investigated and developed by the scientific team led by Prof. J.J. Telega, as referenced in [4],[9].

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Topology optimization of structures. Overview of results found by the team: Karol Bołbotowski, Sławomir Czarnecki, Radosław Czubacki, Grzegorz Dzierżanowski, Cezary Graczykowski, Krzysztof Kolanek, Michał Kursa, Tomasz Lewiński, Tomasz Łukasiak, Przemysław Sobczak, Tomasz Sokół

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The paper provides an overview on the main results published by the research group of the Department of Structural Mechanics and Computer Aided Engineering, Faculty of Civil Engineering, Warsaw University of Technology in years 1985-2025. The results have been achieved jointly with Professor Józef Joachim Telega or have been inspired by the concepts he promoted and creatively developed. The presentation refers to: -theory of homogenization of periodic composites, laminates, shells and plates, -theory of optimum design of multi-material composites -theory of optimal layout of elastic moduli -theory of optimal trusses and grillages -theory of optimal archgrids and vaults -theory of optimal design of plates, shells and membranes.

Γ- convergence, Padé approximants, asymptotic expansions in various homogenization issues. Overview of results found by the team: Włodzimierz Bielski, Andrzej Gałka, Barbara Gambin, Eleonora Kruglenko, Stanisław Tokarzewski, Ryszard Wojnar

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The Γ- convergence was applied in the relations between models of incompressible and slightly compressible magnetostrictive materials. The magnetostrictive material models were introduced by the form of the free energy functionals which depend on magnetization and elastic deformation as well as on their gradients. We demonstrate the existence of a minimum of an energy functional for a slightly compressible material. We also prove a theorem on the convergence of a sequence of minimizers of less and less compressible material energy functionals to a minimizer of energy of incompressible material. Besides the existence of a solution of the incompressible magnetostrictive problem is obtained. In addition, in the field of micromagnetism, E. Kruglenko wrote a doctoral thesis, starting with Joachim Telega's as a primary supervisor in which the methods of nonconvex functionals and Young measures were applied. The strength of the Γ- convergence concept was proved also by the demonstration of how the Γ limit of the fissured linear elastic matrix enables to obtain the homogenized nonlinear elastic body. The classical Pade approximants to determine bounds on transport coefficients were generalized by Stanisław Tokarzewski for matrix approximants which enabled calculation bounds of anisotropic effective transport coefficients for a two-phase medium. The asymptotic expansion method of homogenization has been extensively studied to calculate the form of homogenized various porous media properties. The first results come from years of cooperation with Joachim Telega with Włodzimierz Bielski, Andrzej Gałka, and Ryszard Wojnar. This resulted in many practical homogenization formulas. As an example, the effective nonlinear heat transport properties calculated for the periodic structure of a tissue liver were used in the application for controlling temperature change in hyperthermia as a cancer therapy. Another interesting example of porous media application was modeling the water flow water distribution by a canal with an overgrown bottom, and with a corrugated bottom important in geophysical experimental studies.