Numerical methods for solving ordinary and partial differential equations

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The aim of this cycle of lectures and laboratory exercises is to acquaint the students with the basic types of numerical methods used to obtain approximate solutions in various physical problems described by ordinary and partial differential equations. During the course will be discussed main one- and multi-step methods for solving ODEs (e.g., Euler and modified Euler methods, second-, fourth-, and higher-order Runge-Kutta methods, leapfrog integration, exponential integrators, etc.) as well as numerical techniques for solving all kinds of PDEs (e.g., finite difference, finite element, and finite volume methods, spectral methods, meshfree methods, domain decomposition methods, multigrid methods, etc.). The full understanding of the presented material demands from the students at least the basic knowledge of main types of ordinary and partial differential equations.

Main topics:

1. Introduction and examples of numerical integration of differential equations.
2. Numerical methods for solving first-order initial value problems: linear multistep, Runge-Kutta, and alternative methods, explicit and implicit methods, analysis of convergence, order, and stability of numerical methods.
3. Numerical methods for solving second-order 1D boundary value problems: finite difference methods, Dirichlet type and mixed boundary conditions, shooting methods, linear and nonlinear boundary value problems.
4. Forward differences for parabolic PDEs: heat conduction with two-point boundary value problem and given initial temperature distribution, implicit and backward difference methods, Crank-Nicolson method, stability analysis.
5. Finite differences for elliptic PDEs: Laplace's equation in the unit square, Galerkin and Ritz methods for multi-dimensional Poisson equation with homogeneous boundary conditions, finite element methods in structural mechanics.
6. Finite differences for hyperbolic PDEs: 1D wave equation, flux conservative form, hyperbolic conservation laws, Lax-Friedrichs and Lax-Wendroff methods, Leapfrog method, upwind differencing method, nonlinear model problem – inviscid Burgers' equation.
7. Other methods for time-dependent PDEs: method of lines, spectral methods, etc.

The total number of lecture hours: 30, laboratory exercises: 30 hours, self-teaching: 45 hours, direct tutoring and consultations: 15 hours.

ECTS Points: 4.