THE DOCTORAL SCHOOL OF IPPT PAN

COURSE OFFERED IN THE DOCTORAL SCHOOL OF IPPT PAN

Name of the course	Polish	**Podstawy Rachunku Tensorowego**			
	English	**Fundamentals of Tensor Calculus**			
Type of the course	**Introductory Lecture**				
Course coordinator	**prof. dr hab. inż Katarzyna Kowalczyk-Gajewska**	Course teacher	**prof. dr hab. inż Katarzyna Kowalczyk-Gajewska**		
Implementing unit	**ZMM**	Scientific discipline / disciplines	**Mechanical Engineering / Materials Science**		
Level of education	**Doctoral studies**	Semester	**Winter**		
Language of the course	**English or Polish**				
Type of assessment	**exam**	Number of hours in a semester	**45**	ECTS credits	**4**

Type of classes		Lecture	Auditory classes	Project classes	Laboratory	Seminar
Number of hours	in a week	**2**	**1**	**0**	**0**	**0**
	in a semester	**30**	**15**	**0**	**0**	**0**

1. Prerequisites

Basic knowledge in mathematics related to vector and matrix algebra, differential and integral calculus at the level of master technical studies, including the ability to solve ordinary and selected partial differential equations.

2. Course objectives

The goal of the course is to present and deepen the knowledge of candidates on fundamentals of tensor calculus necessary for the description of problems within continuum mechanics including basic notion of linear algebra, definition of a tensor product and other tensor operations, invariant decompositions of tensors of a second and fourth order, symmetry groups of tensors and tensor functions and elements of tensor analysis.

3. Course content (separate for each type of classes)

Lecture

1. Basic notions of linear algebra (e.g.: a group, a linear space, the Euclidean space). Basis of the Euclidean space. Transformation rules.
2. Tensor product and the tensor space of an arbitrary order. Basis of the tensor space and transformation rules for tensors. Tensor operations. Tensor as a linear operator. Automorphisms of a tensor space.
3. Spectral decomposition of a second order tensor. Orthogonal invariants of tensors. The Cayley-Hamilton theorem. Polar decomposition of the second order tensor.
4. Symmetry groups of tensors and tensor functions. Invariant decomposition of the fourth order tensors. Elements of the theory of tensor function representation. Derivatives of tensor functions.
5. Derivative and integral operations for a tensor fields in an affine space. Tensor fields in curvilinear coordinate systems.

Auditory Classes

Solving specific problems related to the theory presented in the course of the lecture.

6. Learning outcomes			
Number of the learning outcome	Learning outcomes description	Reference to the learning outcomes according to the 8th level of PRK	Learning outcomes verification methods*
Knowledge			
1	The graduate acquires basic knowledge of the theory of tensor calculus in terms of tensor operations and properties	P8S_WG	Homeworks/examination
2	The graduate acquires basic knowledge of the applicability of tensor calculus in the field of mechanics (i.e. spectral decomposition, symmetry groups)	P8S_WG	Homeworks/examination
3	The graduate knows how to transfer the acquired knowledge to his/her research field and proper dissemination of results	P8S_WK	Assessment of activity during classes
Skills			
1	The graduate is able to solve problems of finding tensor representation of different order in arbitrary basis and perform operations on tensors.	P8S_UW	Homeworks/examination
2	The graduate is able to solve the eigenvalue problem for a tensor of second order and give interpretation of the results.	P8S_UW	Homeworks/examination
3	The graduate is able to analyze the symmetry group of the tensor or tensor function and perform differentiation and integration of tensor field in any curvilinear system	P8S_UW	Homeworks/examination/ Assessment of activity during classes
4	The graduate is able to apply the learned techniques of tensor calculus in his research field and use in the industrial environment	P8S_UW	Assessment of activity during classes
Communication			
1	The graduate is able to communicate his/her results in a clear and rigorous way in the international scientific community	P8S_UK	Examination/Assessment of activity during classes
Social competences			
1	The graduate is ready to think and act in a creative and entrepreneurial way.	P8S_KO	Assessment of activity during classes
2	The graduate is ready to critically evaluate the achievements of the represented scientific discipline, including the existing literature and his or her own contribution to the development of this discipline	P8S_KK	Assessment of activity during classes

*Allowed learning outcomes verification methods: exam; oral exam; written test; oral test; project evaluation; report evaluation; presentation evaluation; active participation during classes; homework; tests

7. Assessment criteria
Activity during classes, written assignments (homeworks), results of oral examination

8. Literature

Primary references:

[1] M.A. Akivis, V.V. Goldberg Tensor Calculus with Applications. World Scientific, 2003

[2] M. Itskov, Tensor Algebra and Tensor Analysis for Engineers. II ed., Springer, 2010.

Secondary references:

[1] J. Ostrowska-Maciejewska, K. Kowalczyk-Gajewska. Rachunek Tensorowy w Mechanice Ośrodków Ciągłych. Biblioteka Mechaniki Stosowanej, IPPT PAN, Warszawa, 2013. [In Polish]

[2] S. Sahraee, P. Wriggers Tensor Calculus and Differential Geometry for Engineers, Springer, 2023

9. PhD student's workload necessary to achieve the learning outcomes**

No.	Description	Number of hours
1	Hours of scheduled instruction given by the lecturer in the classroom	**45**
2	Hours of consultations with the lecturer, exams, tests, etc.	**15**
3	Amount of time devoted to the preparation for classes, preparation of presentations, reports, projects, homework	**25**
4	Amount of time devoted to the preparation for exams, test, assessments	**15**
	Total number of hours	**100**
	ECTS credits	**4**

** 1 ECTS = 25–30 hours of the PhD students work (2 ECTS ≈ 60 hours; 4 ECTS ≈ 110 hours, etc.)