THE DOCTORAL SCHOOL OF IPPT PAN

COURSE OFFERED IN THE DOCTORAL SCHOOL OF IPPT PAN

Name of the course	Polish	**Numeryczne metody rozwiązywania równań różniczkowych zwyczajnych i cząstkowych**
	English	**Numerical methods for solving ordinary and partial differential equations**

Type of the course	**Specialized course**		
Course coordinator	**Wasyl Kowalczuk, PhD, D.Sc.**	Course teachers	**Wasyl Kowalczuk, PhD, D.Sc.** **Barbara Gołubowska, PhD**
Implementing unit	**ZTOCiN**	Scientific discipline / disciplines	**Mechanical engineering**

Level of education	**Doctoral studies**	Semester	**Winter or summer**		
Language of the course	**English**				
Type of assessment	**Examination**	Number of hours in a semester	**60**	ECTS credits	**4**

Type of classes		Lecture	Auditory classes	Project classes	Laboratory	Seminar
Number of hours	in a week	**2**	**2**	**0**	**0**	**0**
	in a semester	**30**	**30**	**0**	**0**	**0**

1. Prerequisites

The full understanding of the material presented in this course demands from the students at least the basic knowledge of main types of ordinary and partial differential equations.

2. Course objectives

The aim of this course is to acquaint the students with the basic types of numerical methods used to obtain approximate solutions in various physical problems described by ordinary and partial differential equations. During the course will be discussed main one- and multi-step methods for solving ODEs (e.g., Euler and modified Euler methods, second-, fourth-, and higher-order Runge-Kutta methods, leapfrog integration, exponential integrators, etc.) as well as numerical techniques for solving all kinds of PDEs (e.g., finite difference, finite element, and finite volume methods, spectral methods, meshfree methods, domain decomposition methods, multigrid methods, etc.).

3. Course content (separate for each type of classes)

Lecture

Main topics:

1. Introduction with examples of numerical integration of differential equations.
2. Numerical methods for solving first-order initial value problems: linear multistep, Runge-Kutta, alternative methods, explicit and implicit methods, analysis of convergence, order, and stability of numerical methods.
3. Numerical methods for solving second-order 1D boundary value problems: finite difference methods, Dirichlet-type and mixed boundary conditions, shooting methods, linear and nonlinear boundary value problems.
4. Forward differences for parabolic PDEs: heat conduction with two-point boundary value problem and given initial temperature distribution, implicit and backward difference methods, Crank-Nicolson method, stability analysis.
5. Finite differences for elliptic PDEs: Laplace's equation in the unit square, Galerkin and Ritz methods for multi-dimensional Poisson equation with homogeneous boundary conditions, finite element methods in structural mechanics.

6. Finite differences for hyperbolic PDEs: 1D wave equation, flux conservative form, hyperbolic conservation laws, Lax-Friedrichs and Lax-Wendroff methods, Leapfrog method, upwind differencing method, nonlinear model problem – inviscid Burgers' equation. 7. Other methods for time-dependent PDEs: method of lines, spectral methods, etc.
Laboratory
- does not apply

4. Learning outcomes			
Number of the learning outcome	Learning outcomes description	Reference to the learning outcomes according to the 8th level of PRK	Learning outcomes verification methods*
Knowledge			
1	The graduate acquires basic knowledge of main types of numerical methods for solving ordinary and partial differential equations.	P8S_WG	examination
2	The graduate knows how to apply the acquired knowledge to solution of practical problems, especially in the field of mechanical engineering.	P8S_WK	assessment of activity during auditory classes
Skills			
1	The graduate is able to solve practical problems in the field of mechanical engineering that demand to find a numerical solution of various ordinary and partial differential equations.	P8S_UW	assessment of activity during auditory classes and examination
2	The graduate is able to apply the acquired knowledge directly to the field of his/her scientific research as well as disseminate the obtained results to the scientific community.	P8S_UW	assessment of activity during auditory classes
Communication			
1	The graduate is able to initiate a debate and participate in scientific discourse, as well as to provide appropriate arguments in scientific discussions and public debates on various topics.	P8S_UK	assessment of activity during auditory classes
Social competences			
1	The graduate is ready to think and act in a creative and entrepreneurial way.	P8S_KO	assessment of activity during auditory classes
2	The graduate is ready to critically evaluate the achievements of the represented scientific discipline, including his or her own contribution to the development of this discipline.	P8S_KK	assessment of activity during auditory classes

*Allowed learning outcomes verification methods: exam; oral exam; written test; oral test; project evaluation; report evaluation; presentation evaluation; active participation during classes; homework; tests

5. Assessment criteria
Assessment of activity during auditory classes, result of final examination.

6. Literature
Primary references:
[1] J. C. Butcher. *Numerical methods for ordinary differential equations*. John Wiley & Sons Ltd., Chichester, 2008.
[2] R. J. LeVeque. *Finite difference methods for ordinary and partial differential equations*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2007.
Secondary references:
[1] S. C. Brenner, L. R. Scott. *The mathematical theory of finite element methods*, Vol. 15, *Texts in Applied Mathematics*. Springer, New York, 2008.
[2] C. Johnson. *Numerical solution of partial differential equations by the finite element method*. Dover Publications Inc., Mineola, NY, 2009.

7. PhD student's workload necessary to achieve the learning outcomes**		
No.	Description	Number of hours
1	Hours of scheduled instruction given by the lecturer in the classroom	**60**
2	Hours of consultations with the lecturer, exams, tests, etc.	**15**
3	Amount of time devoted to the preparation for classes, preparation of presentations, reports, projects, homework	**30**
4	Amount of time devoted to the preparation for exams, test, assessments	**15**
	Total number of hours	**120**
	ECTS credits	**4**

** 1 ECTS = 25–30 hours of the PhD students work (2 ECTS ≈ 60 hours; 4 ECTS ≈ 110 hours, etc.)