cemat
center for computational and stochastic mathematics

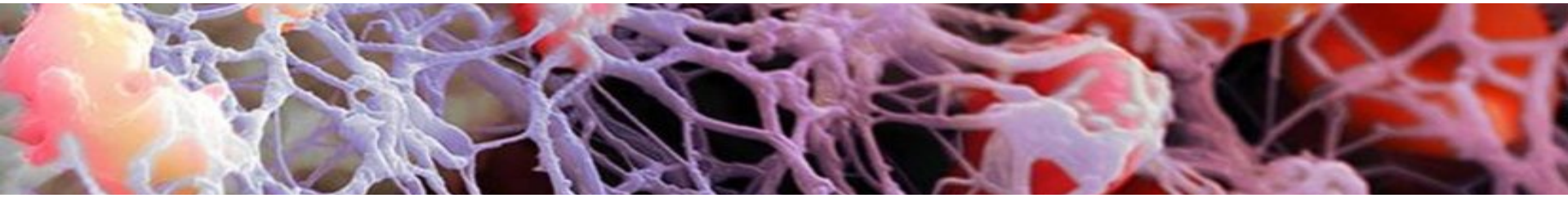
TÉCNICO LISBOA

ACADEMIA DAS CIÊNCIAS LISBOA

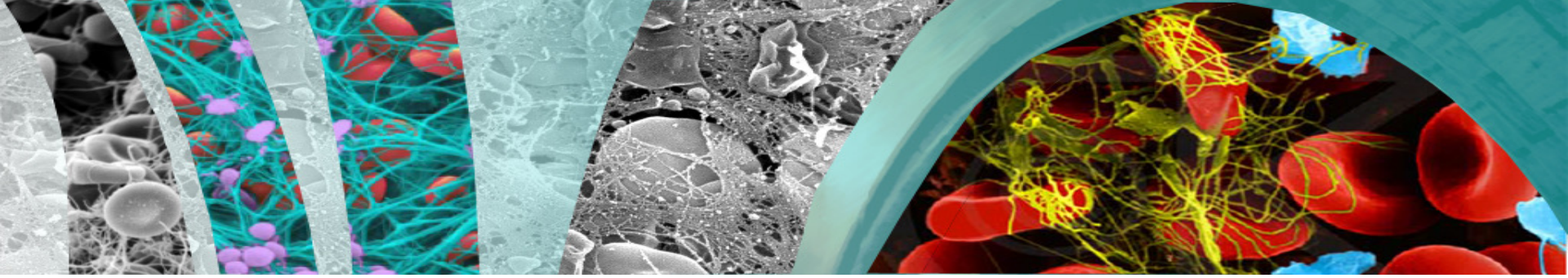
Cardiovascular Modeling and Simulations – A Mathematical Challenge

Adélia SEQUEIRA

Online Seminar



Polish Academy of Sciences, Warsaw, Poland, April 3, 2023



Main Topics (FCT & UT Austin - Portugal CoLab Projects)

Project: EXCL/MAT-NAN/0114/2012
Projects: UIDB/04621/2020, UIDP/04621/2020

PHYSIOMATH

Blood Rheology

Blood Coagulation Modeling and Simulations

Image-Based Modeling of Blood Flows in Cerebral Aneurysms

Mathematical Modeling and Simulations of Inflammation and Atherosclerosis

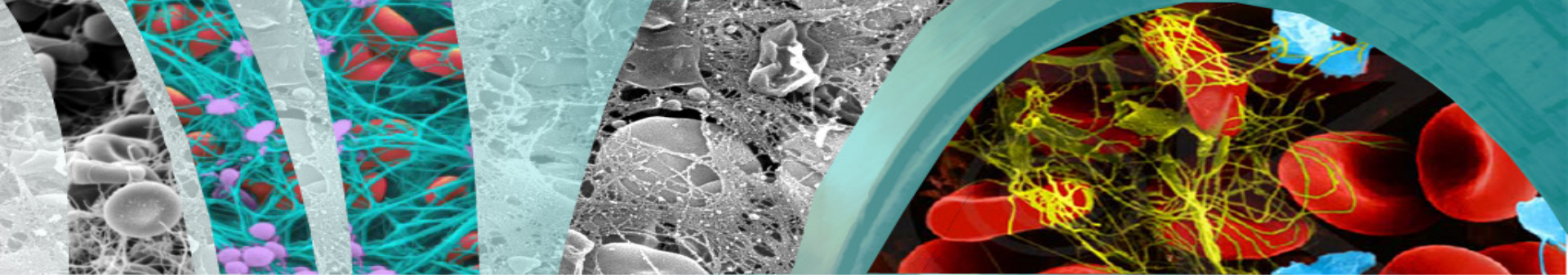
Aortopathies in BAV Patients

Mathematical and Computational Modeling of Drug Dissolution Applied to Coronary Stents

Collaborators: Tom Hughes, Shaolie Hossain, K. R. Rajagopal, Alfio Quarteroni, Antonio Fasano, Willi Jäger, Maria Neuss-Radu, Šárka Nečasová, Tomáš Bodnár, Nader El Khatib, Anne Robertson, Cassio Oishi, Vitaly Volpert, Yuri Vassilewski, …

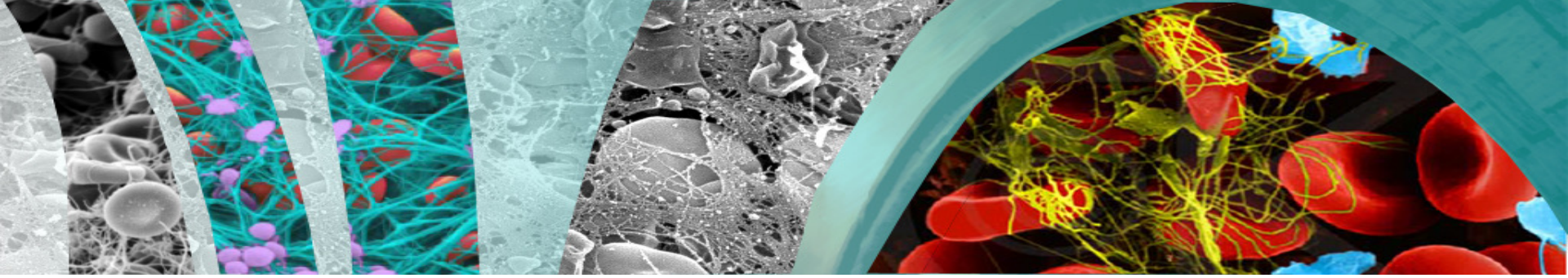
Present & Former Postdocs: Sonia Seyed Allaei, Oualid Kafi, Alberto Gambaruto, Elias Gudiño, Jorge Tiago, Alexandra Moura, Euripides Sellountos, Keith Smith, Lionel Nadau, Nadir Arada,…

Former PhD Students: Iolanda Velho, Telma Silva, Ana Jantarada, Jevgenija Pavlova, Paolo Tricerri, Simone Rossi, Telma Guerra, Luis Borges, Marília Pires, Fernando Carapau, Paulo Correia, João Janela, Juha Videman,….



Why do we need math in medicine???

- Complex biological processes and relations between them, usually involving many parameters
- After building hypotheses, validation through clinical results is needed as well as a quantitative description
- Mathematical modeling gives a tool to reduce the number of animal experiments by *in silico* modeling, or to make these tests (partially) obsolete
- Mathematical modeling gives a predictive tool to the clinicians to quantify the impact of treatment
- Major issue is that most parameters are patient-specific, which requires the involvement of uncertainty assessment

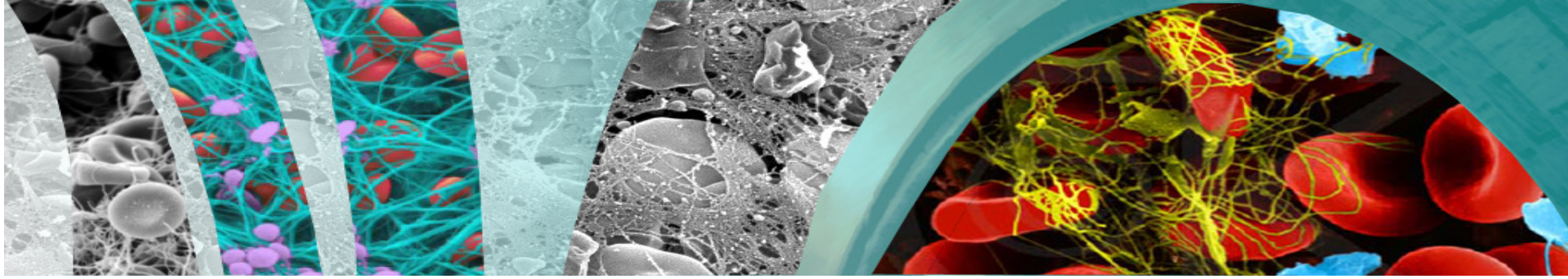


CARDIOVASCULAR MATHEMATICS IN CLINICS

- Cardiovascular diseases are the leading causes of death in developed countries
- Modeling and simulations of blood flow behavior and the applied stresses help to:
  - Understand several diseases – prediction (diagnosis & treatment)
  - Optimize surgical procedures
  - Design medical devices

New challenge:

Combining mechanism-driven models (e.g. based on physics and physiology) and data-driven models (e.g. based on machine learning and artificial intelligence) to analyze large and diverse datasets while attaining cause-to-effect interpretability



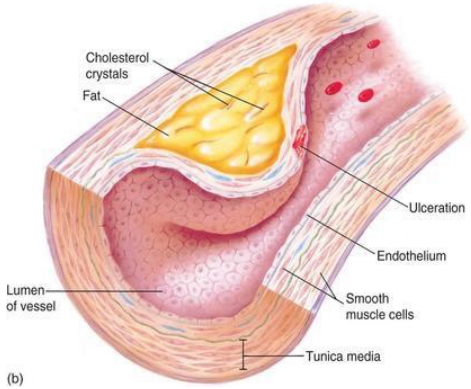
CARDIOVASCULAR DISEASES

Atherosclerosis

• Accumulation of fatty materials, fibrous elements and calcium in the intima of the arteries

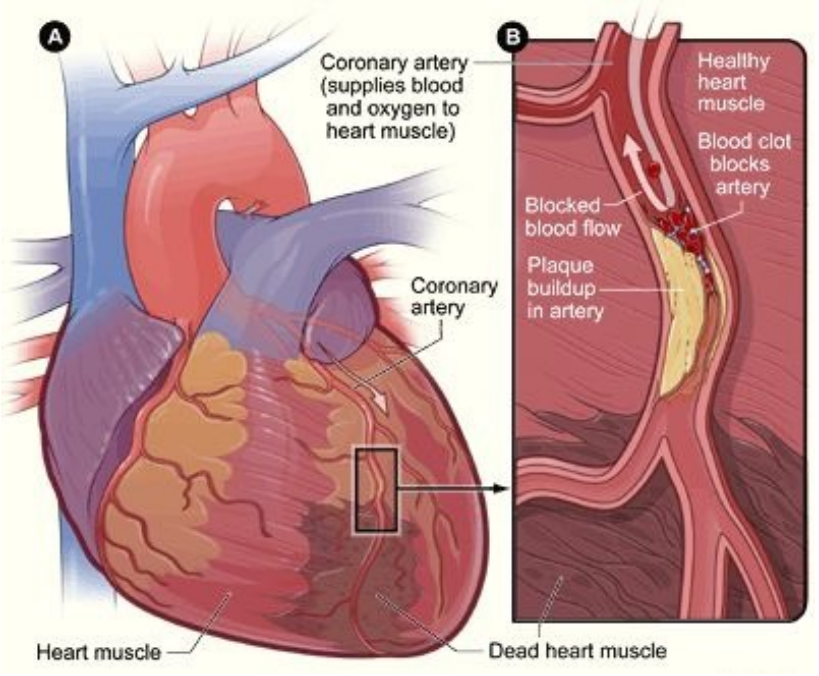
Causes:

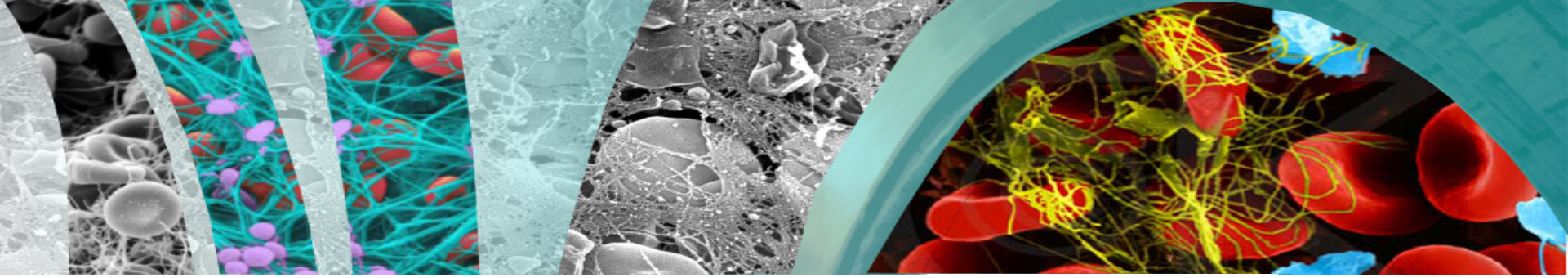
- LDL - Cholesterol
- High blood pressure
- Smoking



Consequences:

- Vessel narrowing
- Heart attack
- Stroke

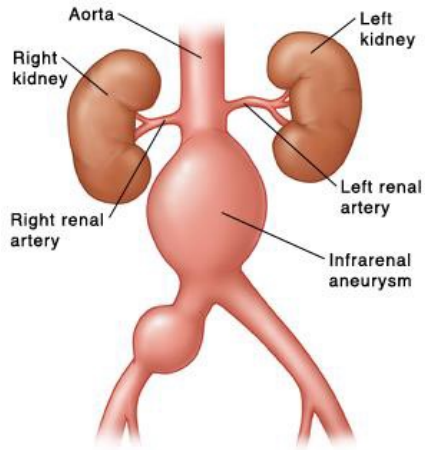


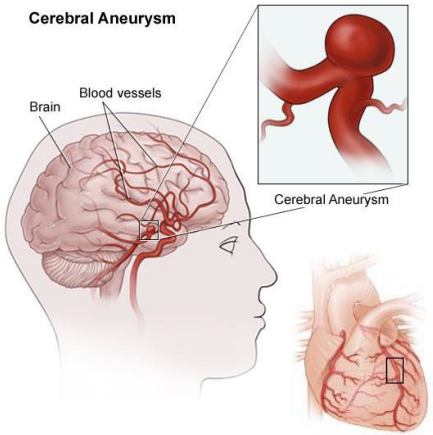


CARDIOVASCULAR DISEASES

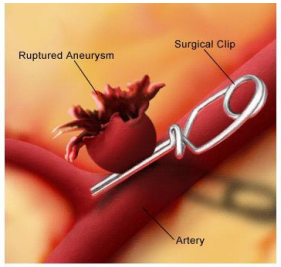
Aneurysms

- Gradual dilation of arterial segments
- <u>Consequences:</u>
  - Vessel stretches and becomes thinner
  - They can rupture causing hemorrhage

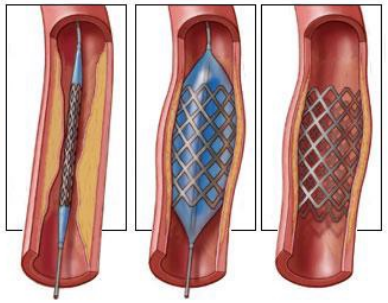


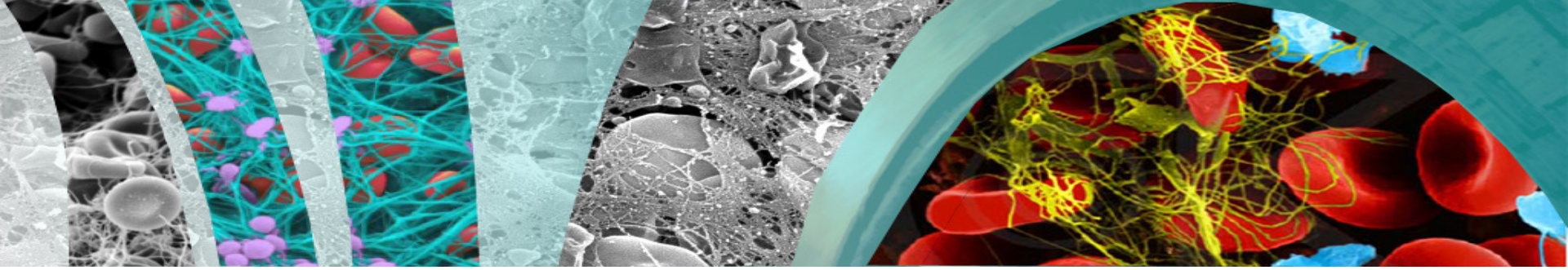


Aneurysm Clipping

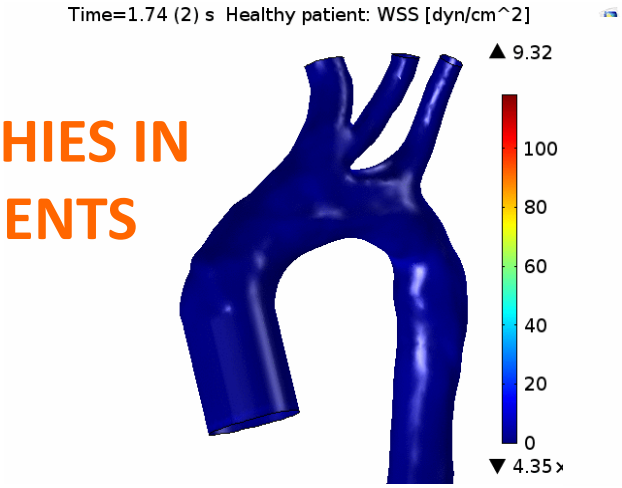


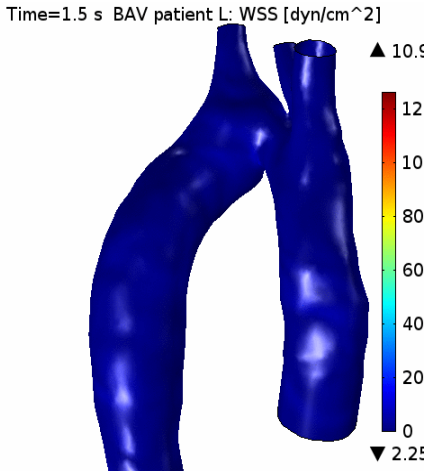
Stents



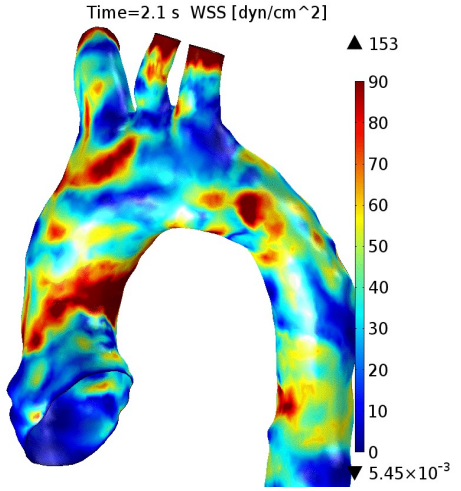


AORTOPATHIES IN BAV PATIENTS

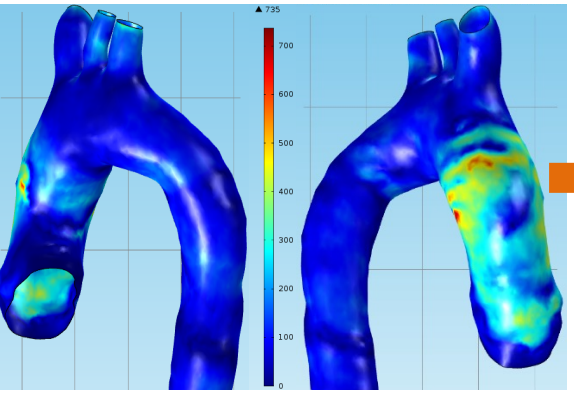


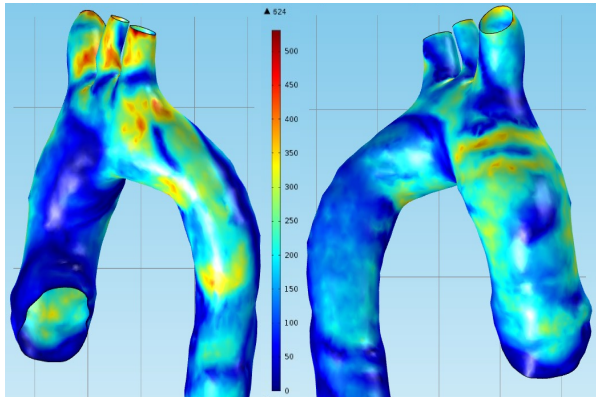


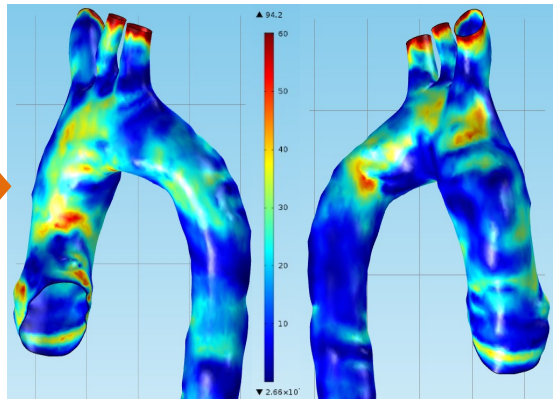
BAV patient : mild AR



BAV patient : severe AR





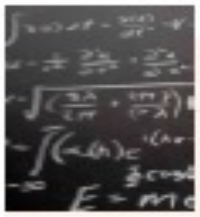


Peak systole

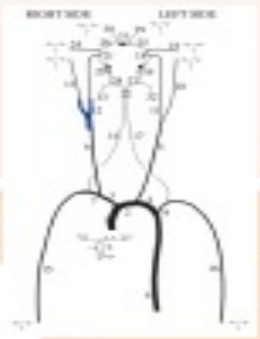
Mid-deceleration phase

Late diastole

Personalized numerical approach to disease diagnosis

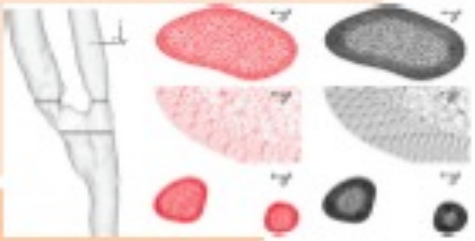


Mathematical models



Problem setting

Mesh generation



Geometry reconstruction

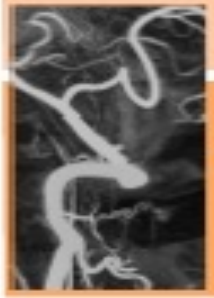
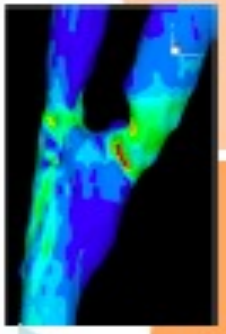
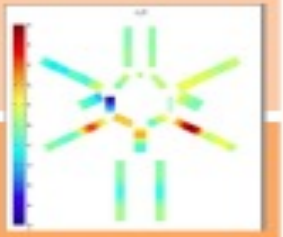


Image and medical data acquisition

Numerical simulations



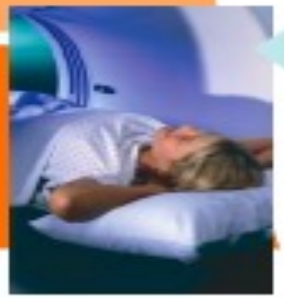


Post-processing visualization

FEEDBACK

Lab, animal and clinical studies

Patient's real data

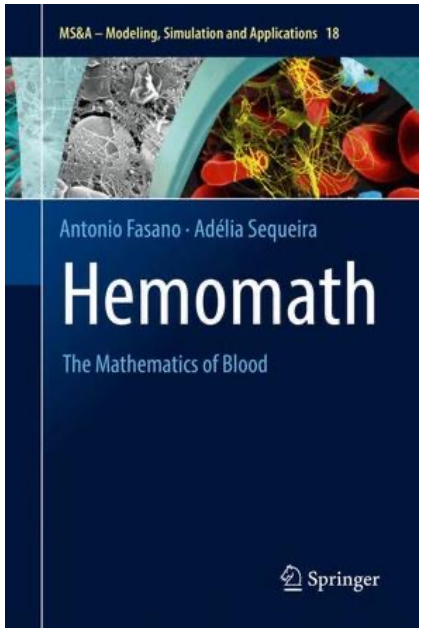


Clinical interpretation

Historical Remarks

Hemodynamics – study of blood flow in the circulatory system

- The importance of blood for life has been very clear since the old times with many implications at religious level
- For instance, Egyptians had great familiarity with the inside of the human body through the practice of mummification
- Egyptians and Mesopotamians certainly practiced bloodletting as a therapy for numerous illnesses
- The modern understanding of the circulatory system starts with the work of **William Harvey** (1578-1657) – publication of his seminal work in 1628
- Giovanni **Borelli (1608-1679) studied the contraction of the heart** and its interaction with the arteries and is seen by many as the *"father of Bioengineering"*
- In 1742, **Leonhard Euler** (1707-1783) presented the *"Principles for determining the motion of the blood through arteries".* This is the first known work on the mechanics of flows in elastic tubes, in which Euler applied his equations to analyze the flow of blood through arteries, driven by a piston pump simulating the heart. Euler is considered the *"father of Hemodynamics"*



Historical Remarks

$$\frac{\partial}{\partial t}(\rho s) + \frac{\partial}{\partial z}(\rho s v) = 0,$$

$$\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z}\right) = -\frac{\partial H}{\partial z},$$

XXXIII.

Principia pro motu sanguinis per arterias determinando.

§ 43. In motu igitur sanguinis explicando easdem offendimus insuperabiles difficultates, quae nos impediunt omnia plane opera Creatoris accuratius perscrutari; ubi perpetuo multo magis summam sapientiam cum omnipotentia conjunctam admirari ac venerari debemus, cum ne summum quidem ingenium humanum vel levissimae vibrillae veram structuram percipere atque explicare valeat.

. . . . . .

§ 15. Quoniam igitur relatio inter p et s constat, conveniet inde valorem formulae $\left(\frac{dp}{dz}\right)$ elicere, qui cum sola z variabilis hic occurrat reperietur:

$$\left(\frac{dp}{dz}\right) = \frac{c}{(\Sigma - s)^2}\left(\Sigma\left(\frac{ds}{dz}\right) - \frac{s\,d\Sigma}{dz}\right),$$

hic valor succinctius ita exhiberi potest:

$$\left(\frac{dp}{dz}\right) = \frac{c\Sigma^2}{(\Sigma - s)^2} \cdot \left(\frac{d.(s:\Sigma)}{dz}\right).$$

Sicque posterior aequatio induet hanc formam:

$$\frac{2gc\Sigma^2}{(\Sigma - s)^2} \cdot \left(\frac{d.(s:\Sigma)}{dz}\right) + \nu\left(\frac{dv}{dz}\right) + \left(\frac{dv}{dt}\right) = 0,$$

ita ut nunc duae tantum supersint functiones s et ν, per ambas variabiles principales

§ 16. Quo has duas aequationes magis evolvamus, eas ita repraesentemus:

I. $\nu\left(\frac{ds}{dz}\right) + s\left(\frac{dv}{dz}\right) + \left(\frac{ds}{dt}\right) = 0$ et II. $\left(\frac{dv}{dt}\right) + \nu\left(\frac{dv}{dz}\right) + \frac{2gc\Sigma^2}{(\Sigma - s)^2}$

Jam a posteriore in s ducta auferamus priorem in ν ductam et obtinebimus hanc aequationem:

$$s\left(\frac{dv}{dt}\right) - \nu\nu\left(\frac{ds}{dz}\right) - \nu\left(\frac{ds}{dt}\right) + \frac{2gcs\Sigma^2}{(\Sigma - s)^2}\left(\frac{d.(s:\Sigma)}{dz}\right) = 0,$$

43) "Thus in explaining the motion of the blood, we come up against the same insuperable difficulties which clearly prevent us from more accurately investigating all the works of the Creator; wherein we ought constantly to admire and to venerate much more the highest wisdom conjoined with omnipotence since truly not even the greatest human ingenuity avails to understand and explain the true structure of the slightest micro-organism".

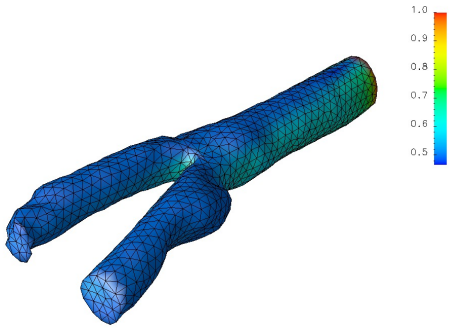
Acknowledgment:
L. Euler 1775

Mathematical Modeling and Simulation of the Human Cardiovascular System

Motivation:

Hemodynamics vs cardiovascular diseases: local fluid patterns and **wall shear stress** are strictly related to the development of cardiovascular diseases **(indicator of atherosclerosis)**

- **Difficulties in modeling blood flow**
  - **Blood Rheology**
  - **Blood flow interaction with the vessel walls**
  - **Complex Geometry**
  - **Closed System**



$$WSS = \mu\left(\frac{\partial \mathbf{u}}{\partial \mathbf{n}} \cdot \boldsymbol{\tau}\right)\Big|_{wall}$$

Local flow dynamics has an important role in the systemic circulation (and vice-versa)

3D flow simulations are restricted to specific regions of interest

Blood Flow Dynamics

BLOOD RHEOLOGY

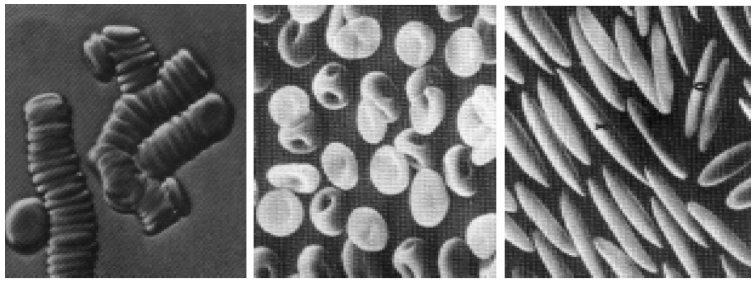
➢ **Why is blood a non-Newtonian fluid ?**

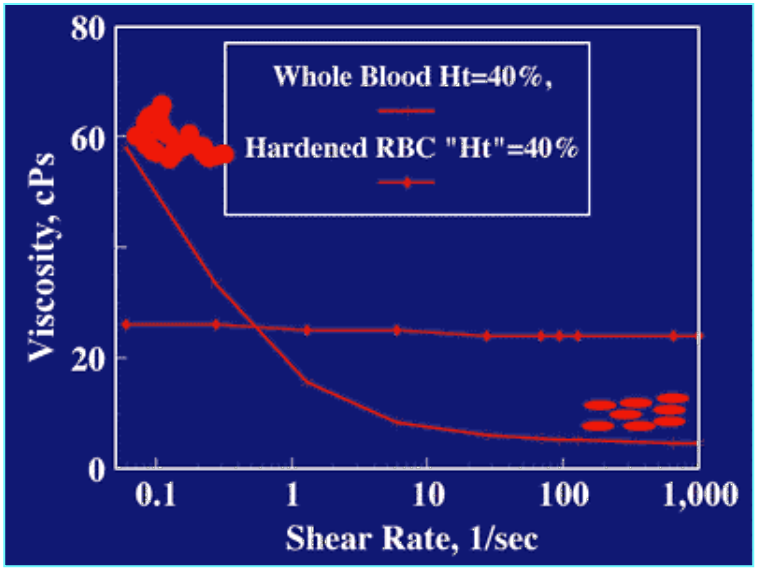
RBC aggregation and deformability

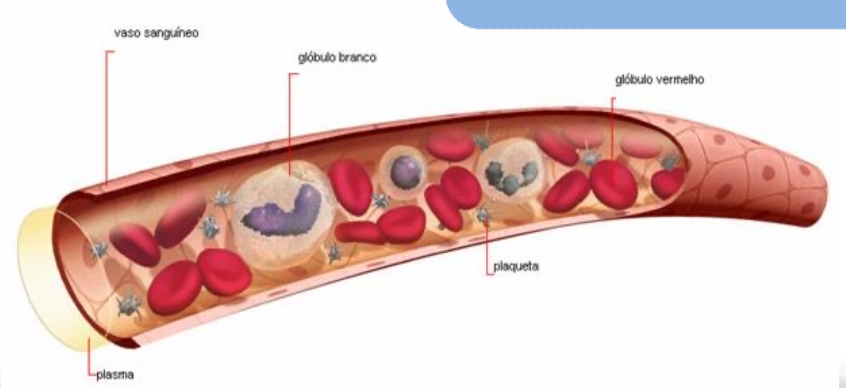
Shear –Thinning viscosity

Other factors

Haematocrit
Osmotic pressure
Plasma composition





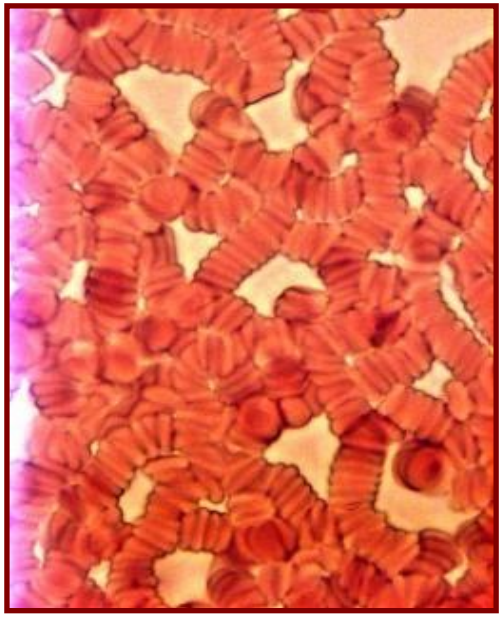


Blood Flow Dynamics

BLOOD RHEOLOGY

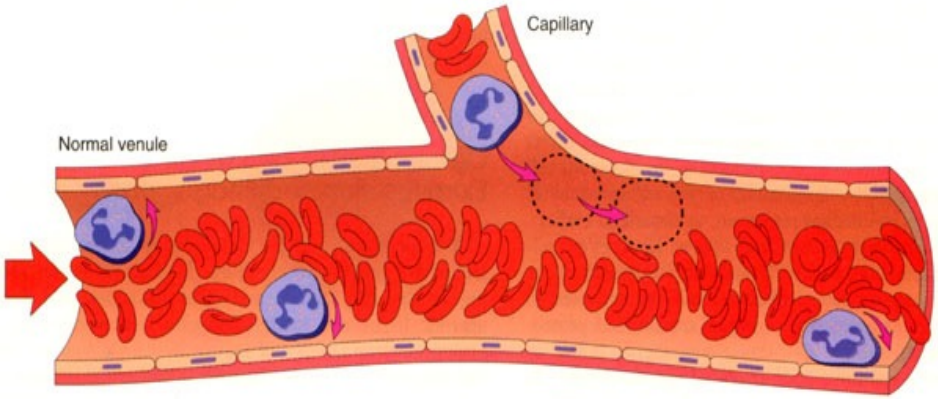
Viscosity depends on shear rate and vessel radius

Rouleaux aggregation



Red blood cells aggregate as in stack of coins

Fåhraeus-Lindquist effect



In small vessels (below 1mm radii) red blood cells move toward the central part of the vessel, and blood viscosity shifts toward plasma viscosity (much lower)

Blood Flow Dynamics

BLOOD RHEOLOGY

➢ **Why is blood a non-Newtonian fluid ?**

► **Non-Linear Viscoelasticity** ◄

- Elastic behavior of RBC (elongation and distortion)
- Formation and distortion of the rouleaux

Haematocrit
Temperature
Time (Thixotropy)
Experimental factors
Plasma viscosity
……..

Non-Linear
Creeping
Stress Relaxation
Normal Stress
Effects

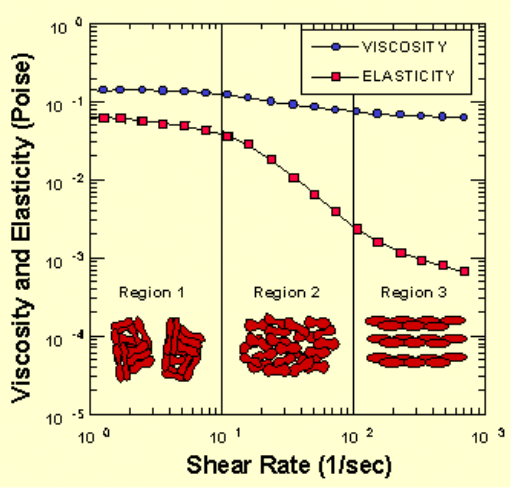


Fig. The shear rate dependence of normal human blood at 2Hz and 22ºC [Vilastic Sc. Inc]

Viscoelastic parameters *experimentally measured* *e.g.* with unsteady flow in capillary tube viscometers

Oscillatory and pulsatile flow analysis

Viscoelastic effects are only substantial at low shear rates

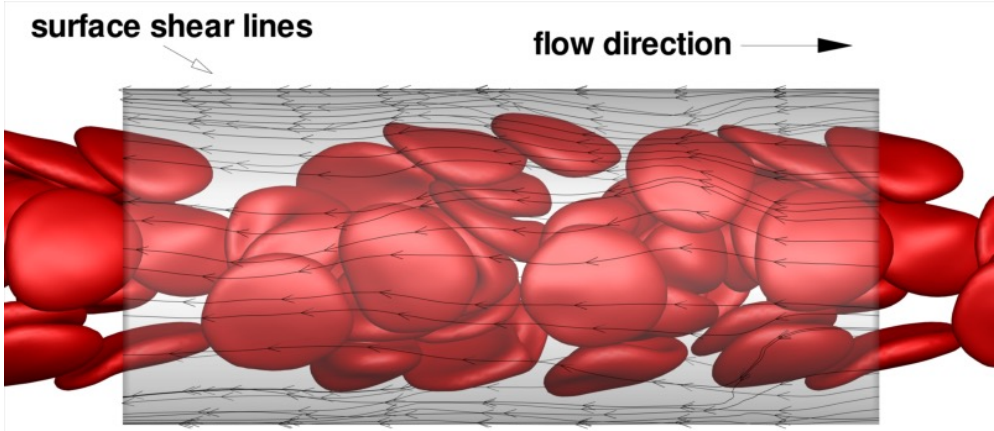
Blood Flow Dynamics

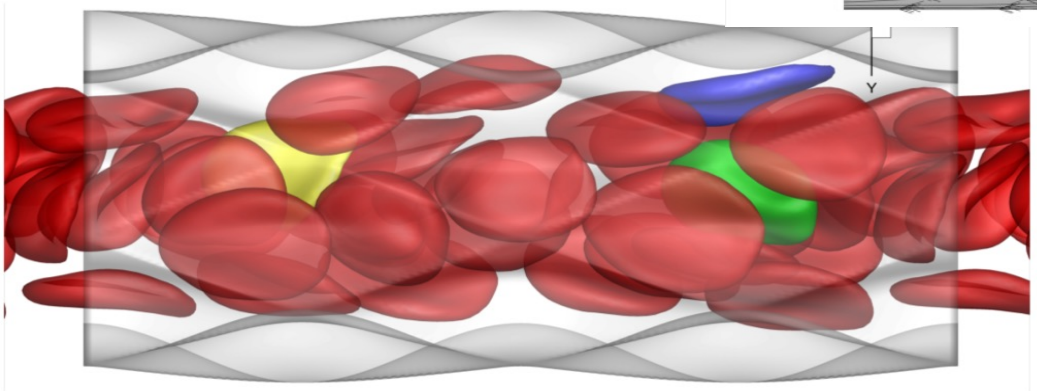
BLOOD RHEOLOGY

Particles Simulation

- **Dissipative Particle Dynamics (DPD)**

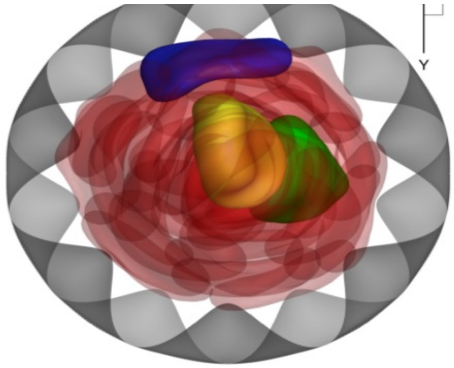
  stochastic mesoscopic simulation technique





A. Gambaruto

lateral view

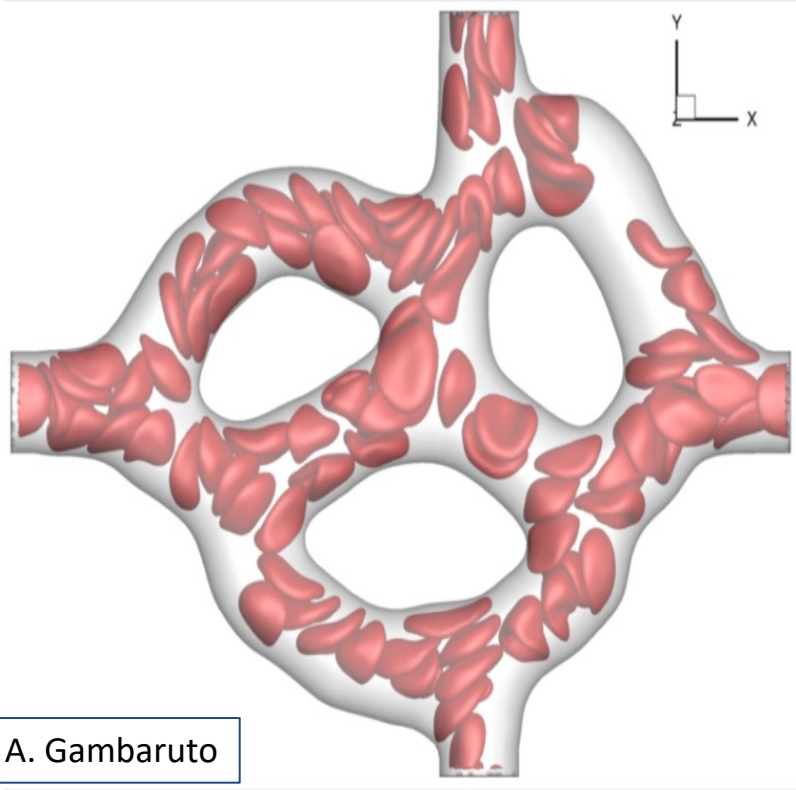


longitudinal view

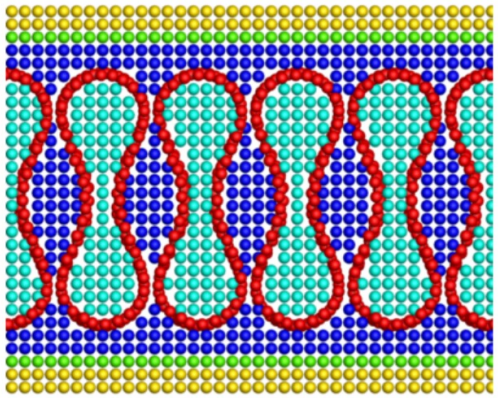
Blood Flow Dynamics

BLOOD RHEOLOGY

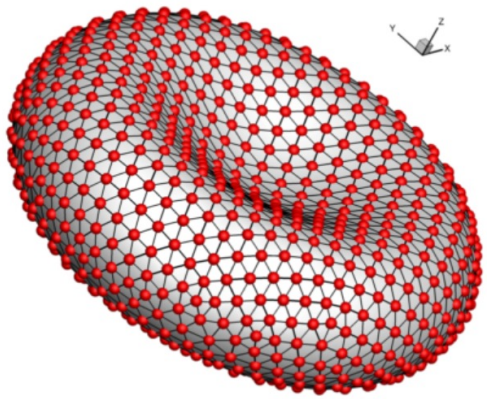
Particles Simulation



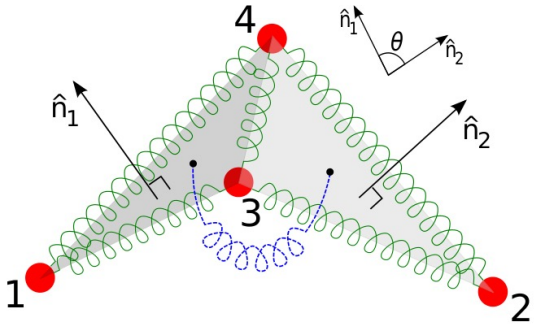
A. Gambaruto



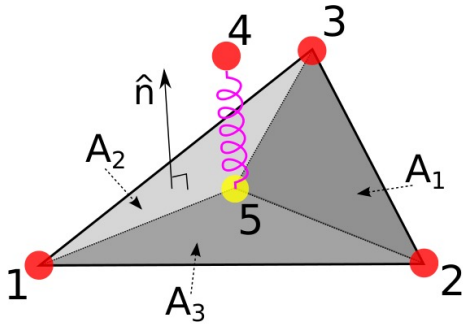
cross-section of a domain



detail of red blood cell



tension and bending spring elements



repulsion spring

BLOOD RHEOLOGY

Blood can also exhibit other non-Newtonian characteristics

- **Thixotropy:** Due to the finite time required for the formation and breakdown of the rouleaux. It is a function of shear rate.

- **Yield-Stress:** Some experiments show that blood can resist shear, behaving rigidly, until a critical level of stress is reached (the yield stress). Above this value blood appears to flow like a fluid.

Constitutive Models

SHEAR-THINNING BLOOD FLOW MODELS:

EXPERIMENTAL PARAMETERS

Model	non Newtonian viscosity	model constants for blood
Power-Law	$\eta(\dot{\gamma}) = k\dot{\gamma}^{n-1}$	$n = 0.61, k = 0.42$
Powell-Eyring	$\frac{\eta(\dot{\gamma}) - \eta_\infty}{\eta_0 - \eta_\infty} = \frac{sinh^{-1}(\lambda\dot{\gamma})}{\lambda\dot{\gamma}}$	$\eta_0 = 0.056Pas, \eta_\infty = 0.00345Pas$ $\lambda = 5.383s$
Cross	$\eta(\dot{\gamma}) = \eta_\infty + \frac{\eta_0 - \eta_\infty}{1 + (\lambda\dot{\gamma})^m}$	$\eta_0 = 0.056Pas, \eta_\infty = 0.00345Pas$ $\lambda = 1.007s, m = 1.028$
Modified Cross	$\eta(\dot{\gamma}) = \eta_\infty + \frac{\eta_0 - \eta_\infty}{(1 + (\lambda\dot{\gamma})^m)^a}$	$\eta_0 = 0.056Pas, \eta_\infty = 0.00345Pas$ $\lambda = 3.736s, m = 2.406, a = 0.254$
Carreau	$\frac{\eta(\dot{\gamma}) - \eta_\infty}{\eta_0 - \eta_\infty} = (1 + (\lambda\dot{\gamma})^2)^{(n-1)/2}$	$\eta_0 = 0.056Pas, \eta_\infty = 0.00345Pas$ $\lambda = 3.313s, n = 0.3568$
Carreau-Yasuda	$\frac{\eta(\dot{\gamma}) - \eta_\infty}{\eta_0 - \eta_\infty} = (1 + (\lambda\dot{\gamma})^a)^{(n-1)/a}$	$\eta_0 = 0.056Pas, \eta_\infty = 0.00345Pas$ $\lambda = 1.902s, n = 0.22, a = 1.25$

(Y.I.Cho and K.R.Kensey, *Biorheology*, 1991)

More about … BLOOD RHEOLOGY

- A. Fasano, A. Sequeira. **Hemomath – The Mathematics of Blood**. MS&A -Modeling, Simulation and Applications Series, Springer Verlag, ISBN: 978-3-319-60512-8, 2017.

- Anne M. Robertson, Adélia Sequeira and Marina V. Kameneva. **Hemorheology.** In: *Hemodynamical Flows: Modeling, Analysis and Simulation*, G. P. Galdi, R. Rannacher, A. M. Robertson, S. Turek, Oberwolfach Seminars, Vol. 37, pp.63-120, 2008.

- Anne M. Robertson, Adélia Sequeira and Robert Owens. **Rheological models for blood.** In: *Cardiovascular Mathematics,* A. Quarteroni, L. Formaggia and A. Veneziani (eds.), Springer-Verlag, 2009.

Blood Flow Dynamics

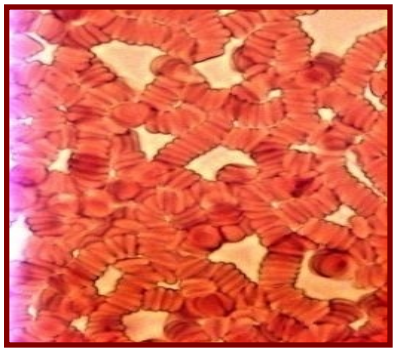
Blood Flow: **Generalized Newtonian fluid equations**

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}\right)+\nabla p-\nabla\cdot\boldsymbol{\tau}(\boldsymbol{u})=0 \quad \text{in } \Omega$$

$$\nabla\cdot\boldsymbol{u}=0 \quad \text{in } \Omega$$

$$\boldsymbol{\tau}(\boldsymbol{u})=2\mu(\dot{\gamma})\boldsymbol{D}(\boldsymbol{u}) \quad \text{in } \Omega$$

$$\boldsymbol{D}(\boldsymbol{u})=\frac{1}{2}(\nabla\boldsymbol{u}+(\nabla\boldsymbol{u})^T), \quad \dot{\gamma}=\sqrt{2\boldsymbol{D}(\boldsymbol{u}):\boldsymbol{D}(\boldsymbol{u})}$$



rouleaux aggregation

shear-thinning viscosity Carreau model

$$\frac{\mu(\dot{\gamma})-\mu_\infty}{\mu_0-\mu_\infty}=\left[1+(\lambda\dot{\gamma})^2\right]^{(n-1)/2}, \quad n\leq 1$$

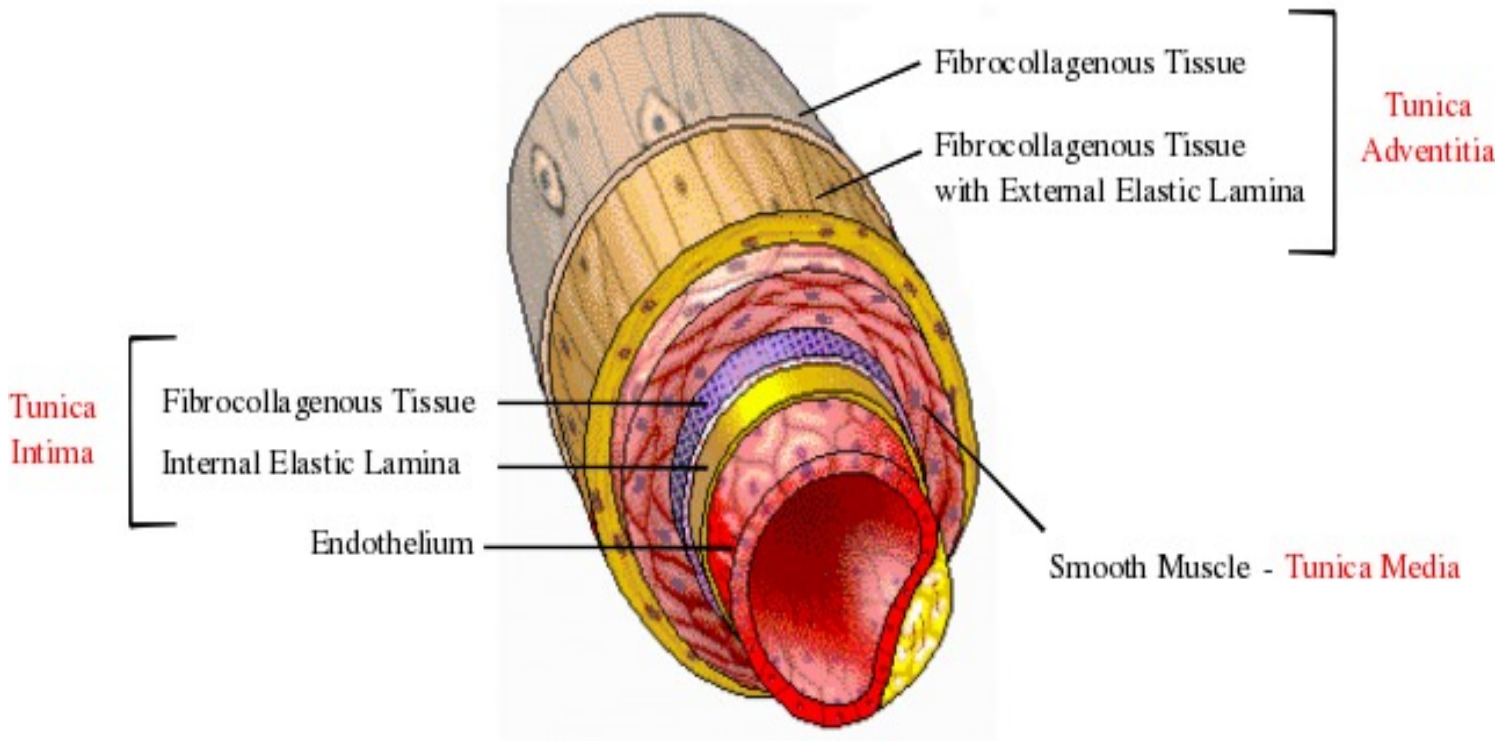
$$\mu_0=\lim_{\dot{\gamma}\to 0}\mu(\dot{\gamma})=0.056 Pa\,s$$

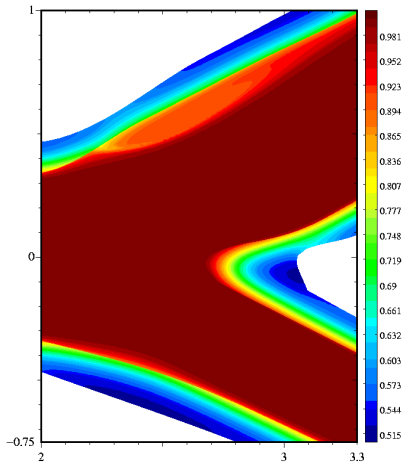
$$\mu_\infty=\lim_{\dot{\gamma}\to\infty}\mu(\dot{\gamma})=0.00345 Pa\,s$$

$$\lambda=3.313\,s$$

$$n=0.3568$$

Morphology of the Blood Vessels





Mechanical model of the arterial vessel: linear or non-linear elasticity in Lagrangian formulation

Mechanical interaction
(Fluid-wall coupling)

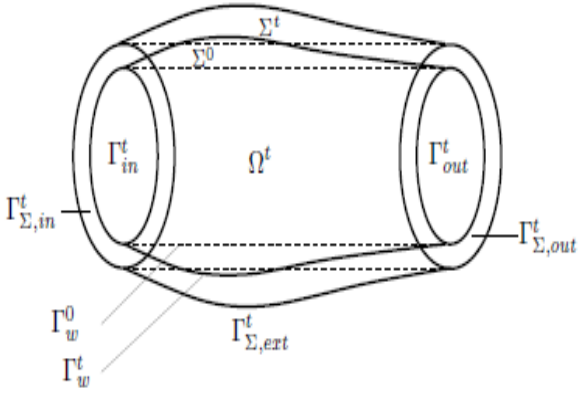
Biochemical interactions

Equations for the deformation of the vessel wall

3D nonlinear hyperelasticity (Lagrangian formulation)

$\Sigma^t \subset \mathbf{R}^3$ ⇒ structure domain

Σ^0 ⇒ reference configuration



$$\partial\Sigma^t = \Gamma^t_\omega \cup \Gamma^t_{\Sigma,ext} \cup \Gamma^t_{\Sigma,in} \cup \Gamma^t_{\Sigma,out}$$

$$\partial\Sigma^0 = \Gamma^0_\omega \cup \Gamma^0_{\Sigma,ext} \cup \Gamma^0_{\Sigma,in} \cup \Gamma^0_{\Sigma,out}$$ ⇒ boundary of the reference domain

Mechanical Interaction - FSI

3D nonlinear hyperelasticity (Lagrangian formulation)

$$\rho_w \frac{\partial^2 \eta}{\partial t^2} - \nabla_0 . \sigma(\eta) = 0 \quad \text{in } \Sigma^0, \forall t \in I$$

η ⟹ displacement vector

ρ_w ⟹ wall density

$$\sigma(\eta) = F(\eta) S(\eta) = (I + \nabla_0 \eta) S(\eta)$$ ⟹ first Piola-Kirchhoff tensor

gradient of deformation tensor

second Piola-Kirchhoff tensor

Mechanical Interaction - FSI

3D nonlinear hyperelasticity (Lagrangian formulation)

We consider a St Venant – Kirchhoff material for which
S is a linear function of ***E***

$$S(\eta) = \lambda tr(E) I + 2\nu E$$

Green-St Venant Stress tensor

$$E = E(\eta) = \frac{1}{2}(F^T F - I) = \frac{1}{2}\left((\nabla_0 \eta)^T + \nabla_0 \eta + (\nabla_0 \eta)^T \nabla_0 \eta\right)$$

$\lambda(\bar{E}, \xi), \nu(\bar{E}, \xi)$ ⟹ Lamé constants
(functions of the Young modulus and of the Poisson ratio)

$$\lambda = \frac{\bar{E}\xi}{(1+\xi)(1-2\xi)} \qquad \upsilon = \frac{\bar{E}\xi}{2(1+\xi)}$$

Mechanical Interaction - FSI

Equations for the deformation of the vessel wall

(Lagrangian formulation)

$$\rho_w \frac{\partial^2 \eta}{\partial t^2} - \nabla_0 . \sigma(\eta) = 0 \quad \text{in } \Sigma^0$$

initial & boundary conditions

$$\eta = \eta_0 \quad \text{for } t = 0, \quad \text{in } \Sigma^0$$

$$\frac{\partial \eta}{\partial t} = \frac{\partial \eta_0}{\partial t} \quad \text{for } t = 0, \quad \text{in } \Sigma^0$$

$$\sigma(\eta).n_0 = \hat{\phi} \quad \text{on } \Gamma^0_{\omega}$$

$$\sigma(\eta).n_0 = 0 \quad \text{on } \Gamma^0_{\Sigma,ext}$$

$$\left(\sigma(\eta).n_0\right).\tau_{out} = 0 \quad \text{on } \Gamma^0_{\Sigma,out}$$

$$\eta = 0 \quad \text{on } \Gamma^0_{\Sigma,out}$$

$$\eta = 0 \quad \text{on } \Gamma^0_{\Sigma,in}$$

\+
compatibility conditions

$$\frac{\partial \eta_0}{\partial t} = u_0 \quad \text{on } \Gamma^0_{\omega}$$

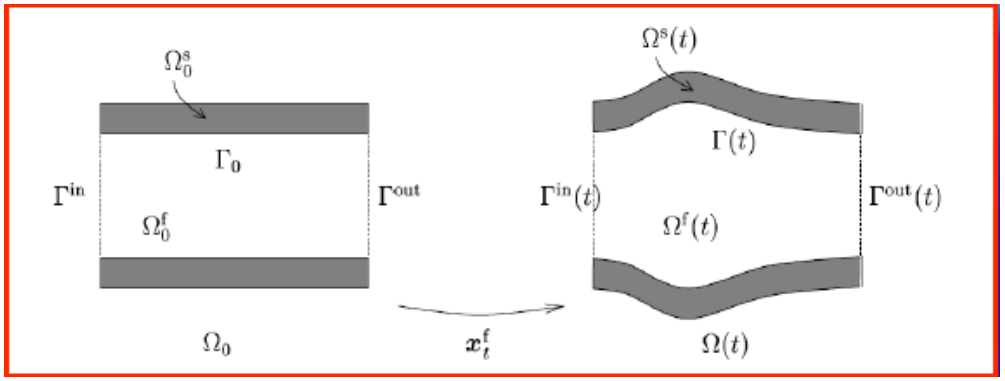
&
interface conditions

(fixed structure)

Mechanical Interaction - FSI

ALE Formulation

The vessel wall should be in Lagrangian coordinates and the fluid in Eulerian coordinates



$\Omega^0 \longrightarrow$ Reference configuration for the fluid domain

The motion of the fluid domain is described by the **ALE map** defined by

$$A^t : \Omega^0 \to \Omega^t$$

$$A^t(\hat{x}) = x(t, \hat{x}), \quad \hat{x} \in \Omega^0$$

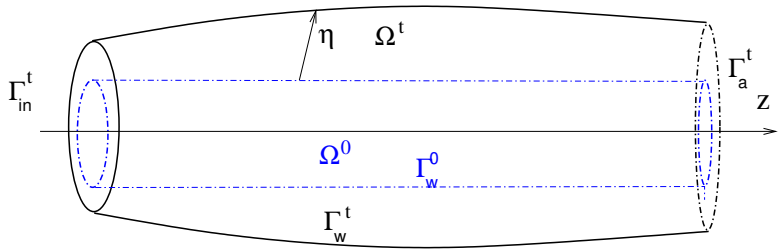
and the computational domain is recovered because $\Omega^t = A^t(\Omega^0)$

The velocity of the fluid domain is defined by

$$w(t,x) = \frac{\partial A^t}{\partial t}$$

Mechanical Interaction - FSI

The FSI model – the fluid equations in the ALE frame



Remark: The ALE map is arbitrary. It is possible to define it using an harmonic extension of the boundary domain, by solving

$$\begin{cases} \Delta A^t = 0, \quad em \quad \Omega^t \\ A^t = \eta \quad em \quad \Gamma_w^t \\ A^t . n = 0, \quad \dfrac{\delta A^t}{\delta n} . \tau = 0 \quad em \quad \Gamma_{in}^t \cup \Gamma_{out}^t \end{cases}$$

Boundaries are fixed in the longitudinal direction, but they freely move in the radial (and tangential) direction

$$\begin{cases} \rho \left. \dfrac{\partial u}{\partial t} \right|_{\hat{x}} + \rho(u - w.\nabla)u + \nabla p - 2div(\mu(\dot{\gamma})D(u)) = 0 \quad em\ \Omega^t \\ div\ u = 0 \quad em\ \Omega^t \\ \text{interface cond.} \quad em \quad \Gamma_w^t \qquad \forall t \in (0,T], \\ \text{with initial condition} \quad u = u_0 \end{cases}$$

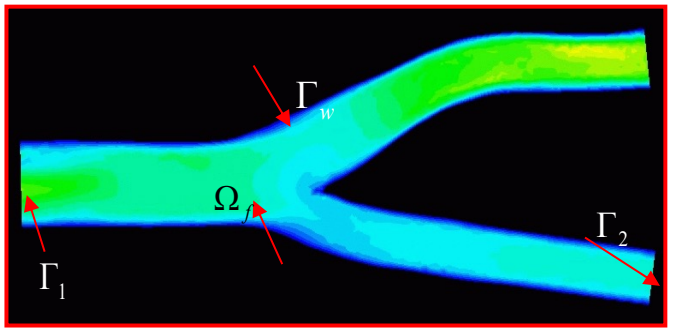
Blood Flow Simulations - FSI

Blood flow: Generalized Newtonian flow (ALE frame)

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t}+(\boldsymbol{u}-\boldsymbol{w})\cdot\nabla\boldsymbol{u}\right)+\nabla p-\nabla\cdot\boldsymbol{\tau}(\boldsymbol{u})=0 \quad \text{in } \Omega_f$$

$$\nabla\cdot\boldsymbol{u}=0 \quad \text{in } \Omega_f$$

$$\boldsymbol{\tau}(\boldsymbol{u})=2\mu(\dot{\gamma})\boldsymbol{D}(\boldsymbol{u}) \quad \text{in } \Omega_f$$



Deformation of the vessel wall

$$\rho_w\frac{\partial^2\eta}{\partial t^2}-\nabla_0.\sigma(\eta)=0 \quad \text{in } \Sigma^0$$

Interface conditions

$$\boldsymbol{\sigma}(\eta)\cdot\boldsymbol{n}=-p\boldsymbol{n}+\boldsymbol{\tau}(\boldsymbol{u})\cdot\boldsymbol{n} \quad \text{at } \Gamma_w$$

$$\boldsymbol{u}=\frac{\partial\eta}{\partial t} \quad \text{at } \Gamma_w$$

u **= blood velocity**
w = domain velocity
p **= pressure**
ρ_f **= density**
μ **= viscosity**
η **= wall displacement**

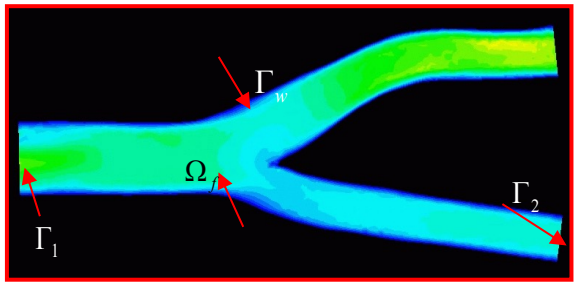
+ initial and boundary conditions at Γ_i (i=0,1,2)

Blood Flow Simulations - FSI

Interface conditions

$$u = \frac{\partial \eta}{\partial t}, \quad \forall t \in I, \quad \text{at } \Gamma_\omega^t$$

$$\sigma(\eta)\cdot n = -pn + \tau(u)\cdot n, \quad \forall t \in I, \quad \text{at } \Gamma_\omega^t$$



using the Piola transform

$$-(\det \nabla_0 \eta)\tau(u,p)(\nabla_0^{-T}\eta)\cdot n_0 = \sigma(\eta)\cdot n_0, \quad \forall t \in I, \quad \text{on } \Gamma_\omega^t$$

Boundary conditions in Γ_i

$$u = h \quad on\ \Gamma_1$$

$$-\tau^{wr}(u,p).n = pn + \frac{\rho_f}{2}|u|^2 n + 2\mu(\gamma)D(u).n$$

$$= \left(\bar{p}_{1D} + \frac{\rho_f}{2}|\bar{u}_{1D}|^2\right)n \quad on\ \Gamma_2$$

$$Q_{3D} = \int_{\Gamma_2} u.n\, d\gamma = Q_{1D}$$

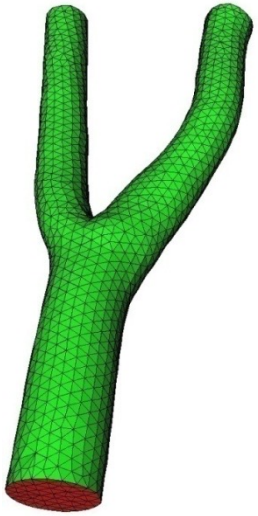
Blood Flow Simulations - FSI

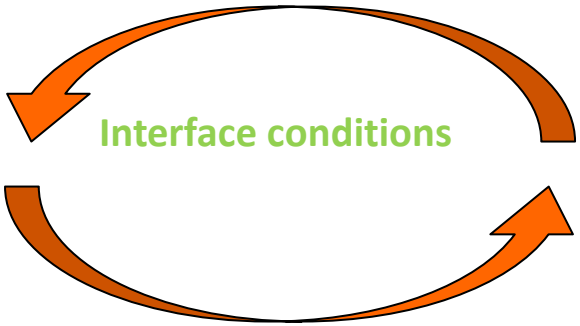
Blood:

Newtonian or non-Newtonian fluid

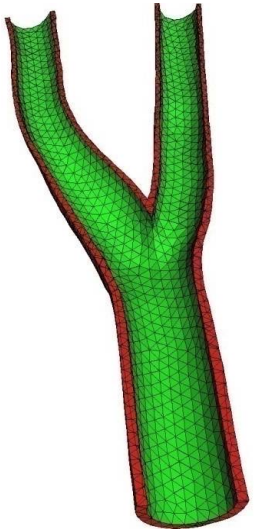
Deformation of the Vessel Wall:

3D (nonlinear) elasticity or 2D shell type models





implicit coupling
(iterative procedure)



Open problems:

Well posedness of the FSI problem

Contributions given by e.g. : D.Coutand, S. Shkoller, Y.Maday, C.Grandmont, B.Desjardins, M.Esteban, G.P. Galdi, H.Beirão da Veiga, S. Canic, among others

Devise efficient numerical algorithms

Contributions given by e.g. : P. le Tallec, F.Nobile, M.A.Fernandéz, M.Moubachir, J-F.Gerbeau, S.Deparis, W.A.Wall, among others

Mechanical Interaction - FSI

Regularity Assumptions:

$\Omega^t \subset \mathbf{R}^3$ is an open connex domain

$\partial\Omega^t = \Gamma^t_\omega \cup \Gamma^t_{in} \cup \Gamma^t_{out}$ is locally Lipschitz $\partial\Omega^t \in C^{1,1}$

$\Omega^t \Longrightarrow$ satisfies the cone property (to apply the Korn inequality)

$\frac{\partial \eta}{\partial t} \in H^{1/2}(\Gamma^t_\omega) \Longrightarrow u(t) \in H^1(\Omega^t), \forall t$

Mechanical Interaction - FSI

An Energy Estimate for the Coupled Problem

[A. Moura, A. S, , J. Janela, 2009 – generalization of L. Formaggia, A. Moura, F. Nobile, 2007]

$$\mathrm{E}(t) = \frac{\rho}{2}\left\|u\right\|^2_{L^2(\Omega^t)} + \frac{\rho_w}{2}\left\|\frac{\partial \eta}{\partial t}\right\|^2_{L^2(\Sigma^0)} + \mu(\dot{\gamma})\left\|E(\eta)\right\|^2_{L^2(\Sigma^0)} + \frac{\lambda}{2}\left\|trE(\eta)\right\|^2_{L^2(\Sigma^0)}$$

Theorem: The coupled FSI problem, with homogeneous Dirichlet BC at the boundary $u = 0 \text{ at } \Gamma^t_{in} \text{ and } \Gamma^t_{out}$ satisfies the following energy inequality

$$\frac{d}{dt}(\mathrm{E}(t)) + 2\mu_\infty \left\|D(u)\right\|^2_{L^2(\Omega^t)} \le 0$$

and, therefore, the energy decay

$$\mathrm{E}(t) + 2\mu_\infty \int_0^t \left\|D(u)\right\|^2_{L^2(\Omega^t)}\, dt \le \mathrm{E}(0)$$

where E(0) is a constant that only depends on the initial data $u_0, \eta_0, \dot{\eta}_0$

REMARK: $\int_{\Gamma^t_{in}} |u|^2\, u.n > 0, \int_{\Gamma^t_{out}} |u|^2\, u.n > 0$ for homogeneous Neumann BC

Blood Flow Simulations - FSI

FSI Algorithm: (adapted from Fernandéz & Moubachir, 2005)

ALE formulation to account for the evolution of the computational domain

Efficient solvers for each fluid and structure subproblems to ensure accurate and fast convergence of the FSI nonlinear coupled system

Fluid equations: **Discretization in time:** **implicit Euler scheme**

Discretization in space: **Stabilized P1 buble / P1 FE**

Structure equations: **Discretization in time:** **mid-point Newmark method**

Discretization in space: **P1 FE**

Coupling strategy: fully implicit coupling based on a Newton algorithm with the exact computation of the Jacobian

Blood Flow Simulations - FSI

Implementation issues

- Solve the whole problem simultaneously ***(monolithic approach):***
  - **Pros:** no stability issue !
  - **Cons:** huge system, develop a new solver

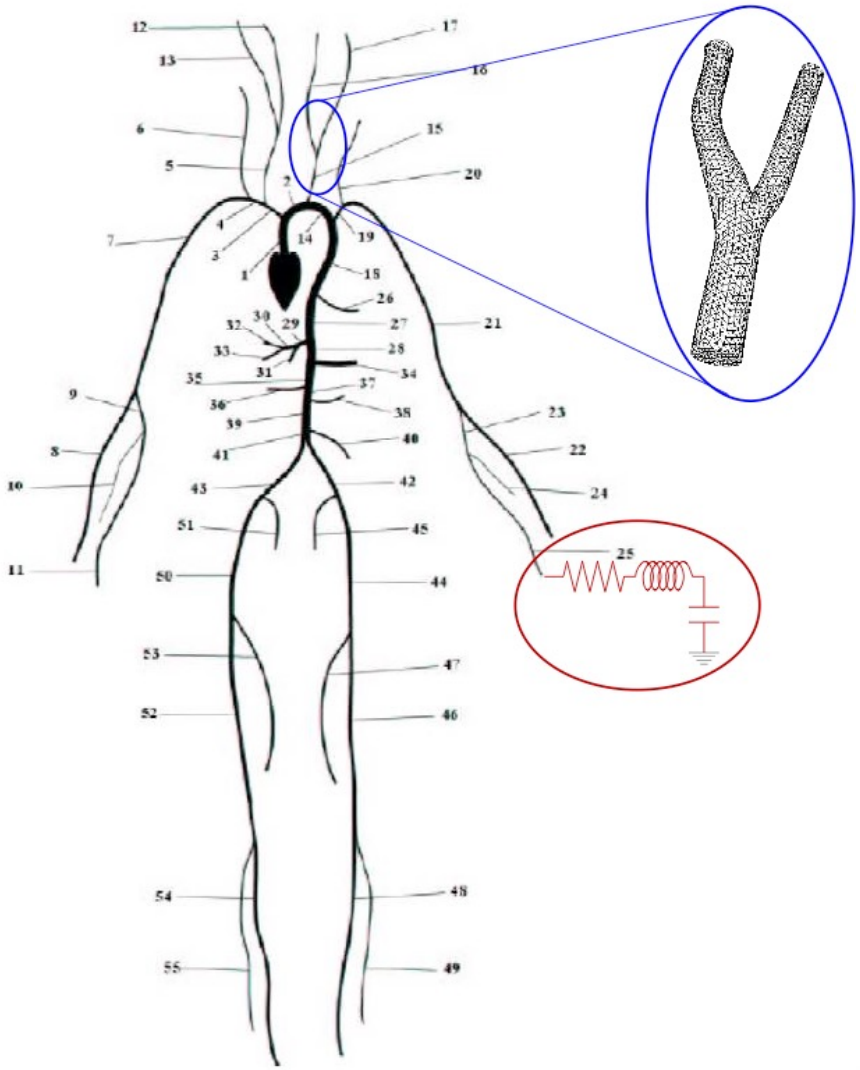
- Use independent solvers for fluid and structure (**partitioned approach):**
  - **Pros:** re-usability of state of the art algorithms, easy to change solvers
  - **Cons:** possible troubles with the coupling algorithms

- **Important remark:** in the ***partitioned approach***, we have the choice

Strong coupling: sub-iterations per time step (no spurious energy)
➡ The results are the same as for the **monolithic approach !**

- **Weak coupling:** 1 or 2 iterations per time step (possible spurious energy)
➡ Possible source of instabilities (due to the added-mass effect)

Geometrical Multiscale



- Global features have influence on the local fluid dynamics

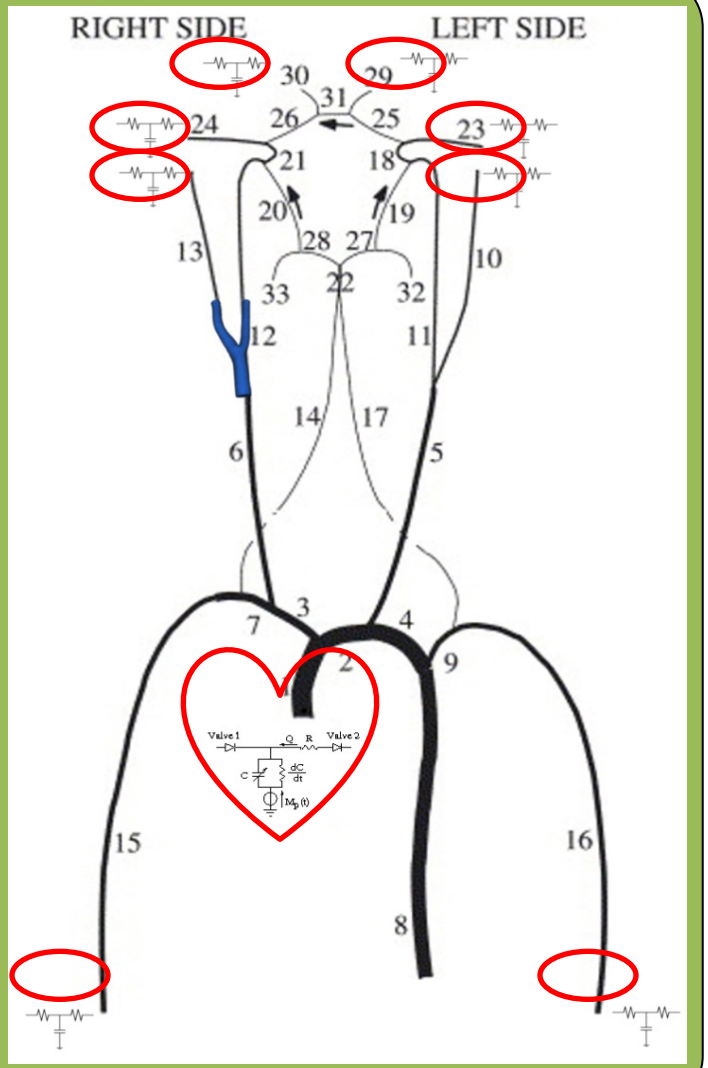
- Local changes in geometry or material properties (e.g. due to surgery, aging, stenosis, …) may induce pressure waves reflections
  ➔ **global effects**

Modeling strategy

- use the expensive 3D model only in the region of interest

- couple with network models that include peripheral impedances to account for global effects

Geometrical Multiscale



Allows to take into account the global circulation in localized simulations and set proper boundary conditions

3D

- Very detailed simulations
- Very complex
- Computationally very costly

1D

- Evolution of mean pressure and flux in arteries
- System of hyperbolic equations
- Low computational cost

0D

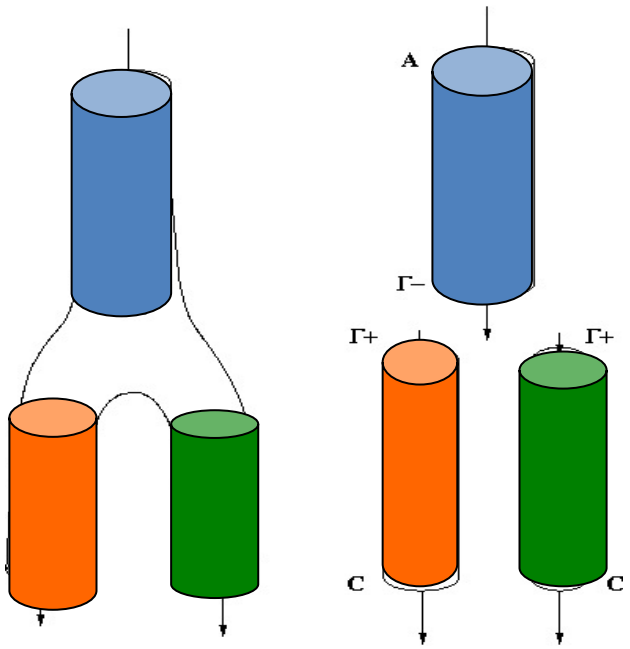
- Evolution in time of mean pressure and flux in wide compartments
- System of ODEs
- Very low computational cost

Geometrical Multiscale

1D Model

- Describes the **wave propagation** nature of blood flow
- Allows for the simulation of complex arterial networks!

Domain decomposition



$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial z}=0$$

$$\frac{\partial Q}{\partial t}+\alpha\frac{\partial}{\partial z}\left(\frac{Q^2}{A}\right)+\frac{A}{\rho}\frac{\partial P}{\partial z}+K\frac{Q}{A}=0$$

$$P-P_0=\Psi(A)$$

$K=8\pi\mu$ → friction parameter

$\propto$ → Coriolis coefficient

(initial terms are negligible and elastic stresses in the radial direction are dominant)

Area →
$$A(z,t)=\int_{\Omega\cap\Sigma(z)} d\gamma$$

Flux →
$$Q(z,t)=\int_{\Omega\cap\Sigma(z)} u_z(x,t)\,d\gamma$$

Mean Pressure →
$$P(z,t)=\frac{1}{|\Sigma(z)|}\int_{\Omega\cap\Sigma(z)} p(x,t)\,d\gamma$$

The 3D (FSI) - 1D Coupling

At the coupling interface we impose the **continuity** of the:

➔ **Flux:**

$$Q_{3D} = \int_{\Gamma_{3D}} u.n\, d\gamma = Q_{1D}$$

➔ **Normal stress:**

$$-\tau^{tot}.n = pn + \frac{\rho}{2}|u|^2 n - 2\mu(\dot{\gamma})D(u).n = \left(\bar{p}_{1D} + \frac{\rho}{2}|\bar{u}_{1D}|^2\right)n$$

Homogeneous Neumann conditions on the structure at the interface gives a stable coupling

Absorbing Boundary Conditions

The 3D - 1D Coupling:

[+] Allows to integrate 3D (FSI) models into lower order (1D) models that can represent large parts of the vascular system

[+] Acts as physiological boundary condition, partially filtrating spurious pressure wave reflections

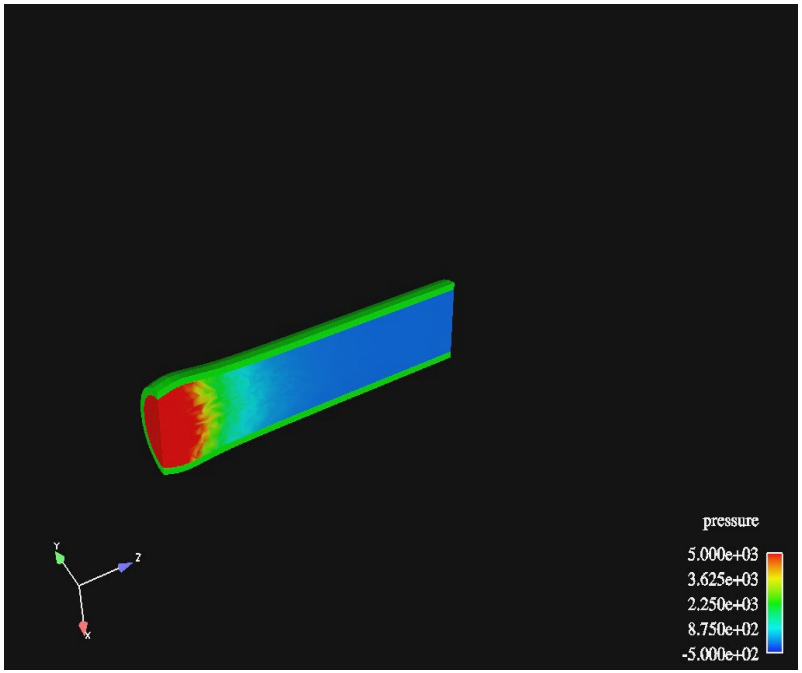
[-] If the 1D hyperbolic problem is solved explicitly, a CFL condition imposes a time step much smaller than the one required by the 3D FSI algorithm

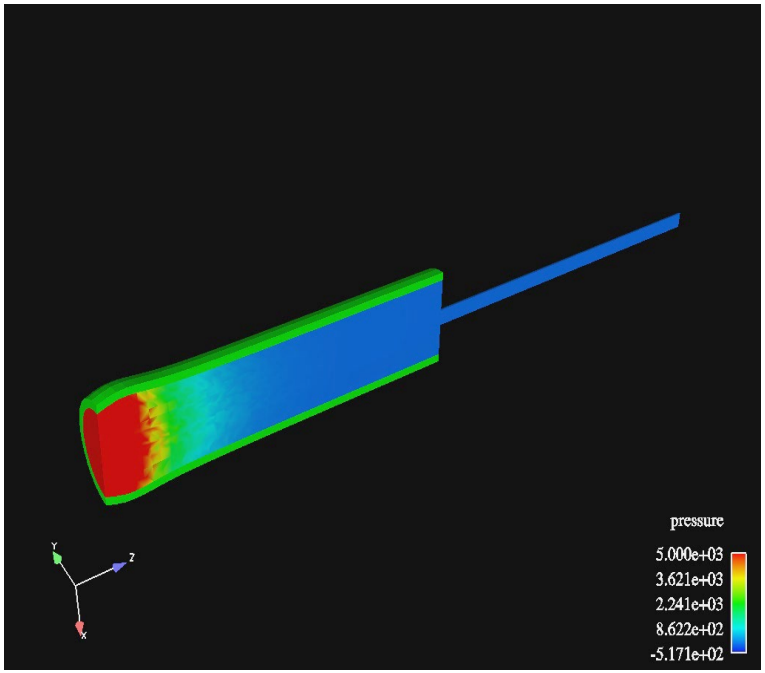
[-] May be impossible or nontrivial to implement in many widely used commercial CFD codes

IDEA: To impose a condition on the characteristic variable $W_2(Q,\bar{p}) = 0$ directly on the 3D FSI model [Janela, Moura, Sequeira, 2010]

Geometrical Multiscale

3D-1D for a cilindrical artery: pressure pulse





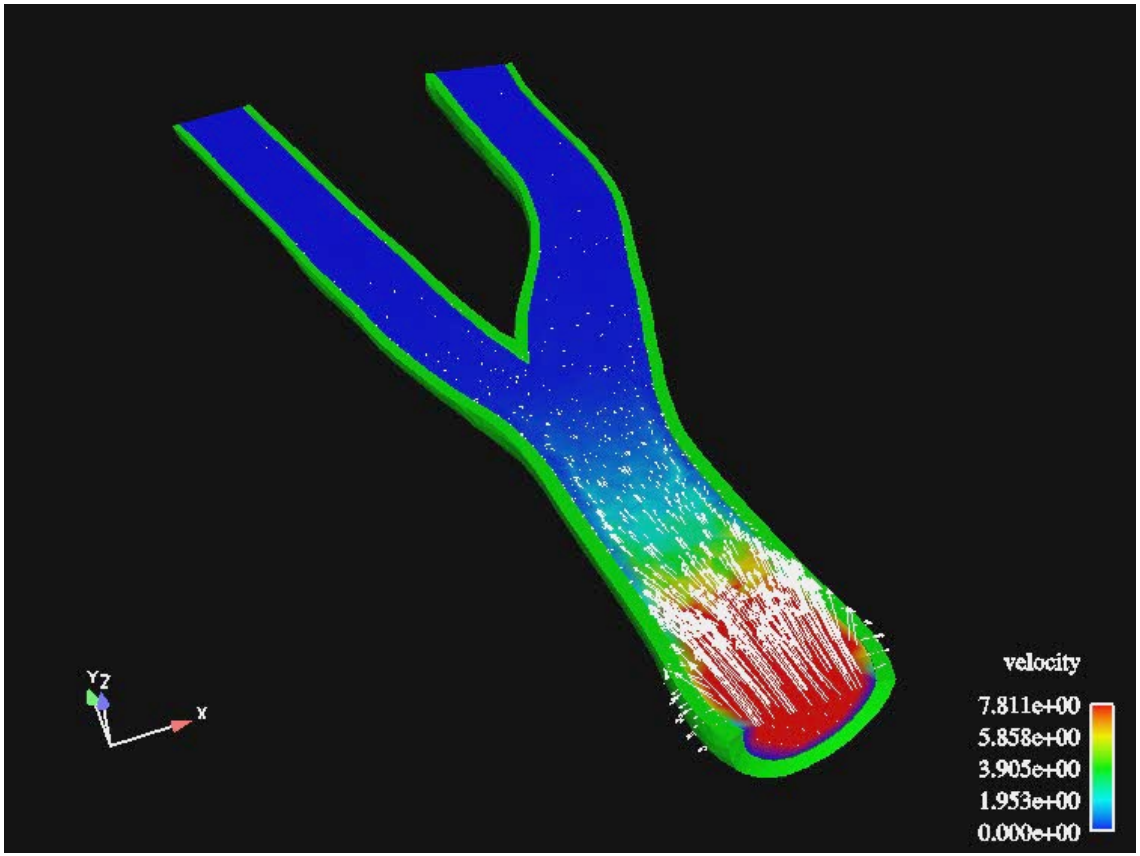
3D model (spurious reflections)

3D-1D coupled model

(A. Moura)

Geometrical Multiscale

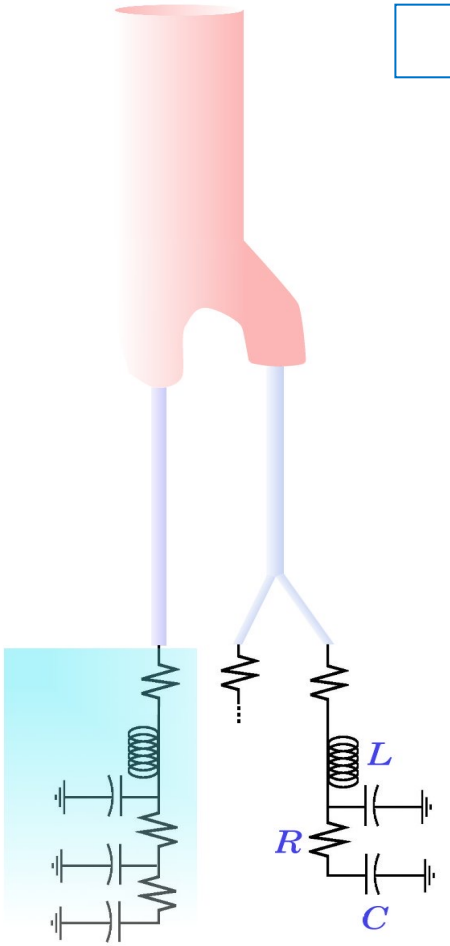
3D-1D for the carotid bifurcation: velocity field



(A. Moura)

Geometrical Multiscale

0D Model



0D Lumped parameters (system of linear ODE's)

$$C\frac{dP_i}{dt} = -(Q_{i+1} - Q_i),$$

$$L\frac{dQ_i}{dt} = -(P_i - P_{i-1}) - RQ_i$$

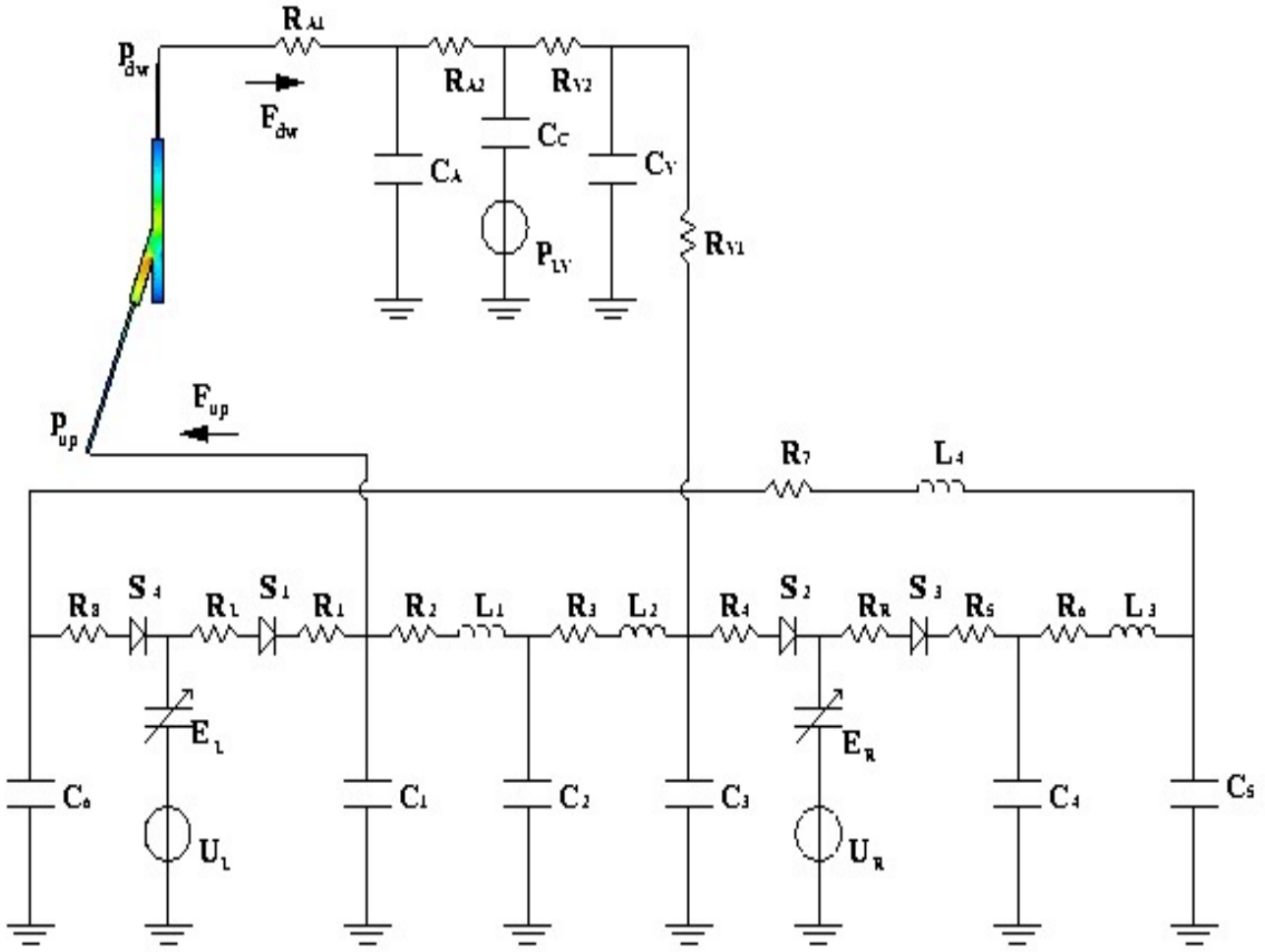
The analogy

Fluid dynamics	Electrical circuits
Pressure	Voltage
Flow rate	Current
Blood viscosity	Resistance R
Blood inertia	Inductance L
Wall compliance	Capacitance C

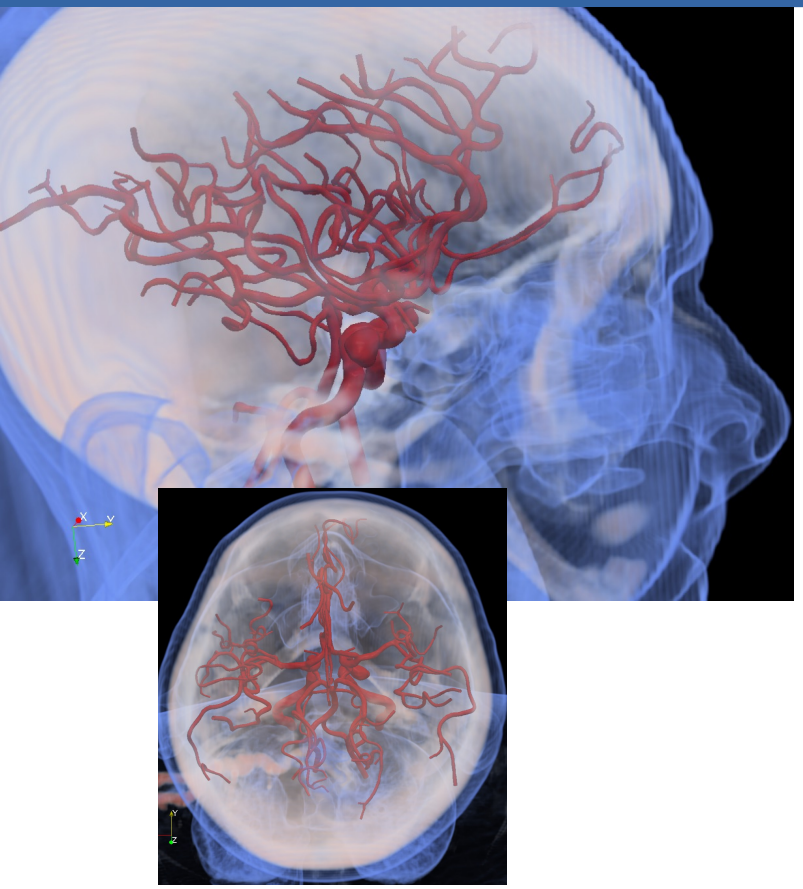
- RLC circuits model "large" arteries
- RC circuits account for capillary bed
- Can describe compartments (such as peripheral circulation)

Geometrical Multiscale

A full geometric multiscale model: 0D-1D-2D (or 3D) coupling

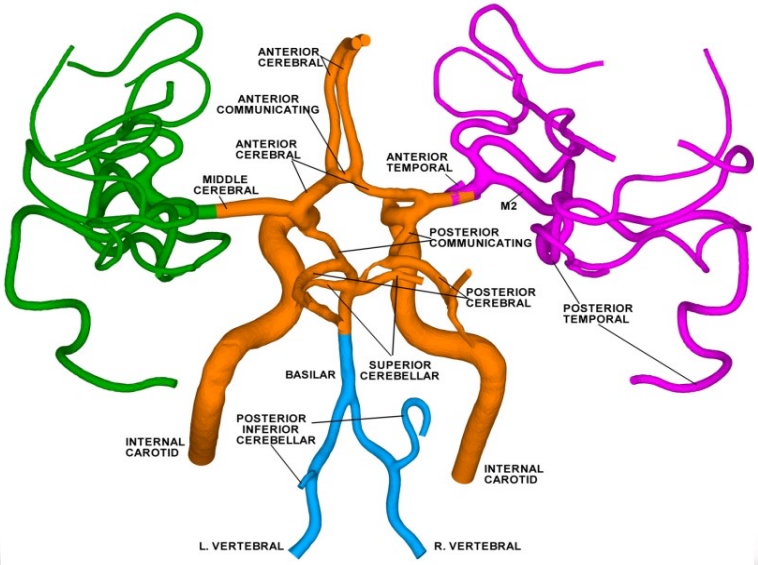


Clinical Study



Simulation-Based Medicine

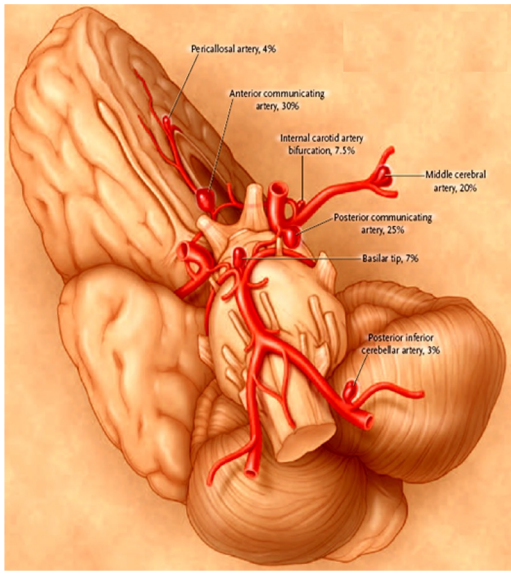
Computational Hemodynamics of Cerebral Aneurysms



Cerebral Aneurysms

Can CFD help in prognosis and therapy planning?

- Cerebral aneurysms are arterial dilations with a non uniform distribution: they are typically found at specific points of the arterial system, namely in the apex of **bifurcations** and at the outer bands of **curved segments** in and near the Circle of Willis
- In case of rupture they are the most common cause of **hemorrhagic strokes**
- There are typically no symptoms until rupture
- The mechanisms behind the development, growth and rupture of intracranial aneurysms are still not well understood
- A better understanding of these processes can lead to better patient evaluation and treatment



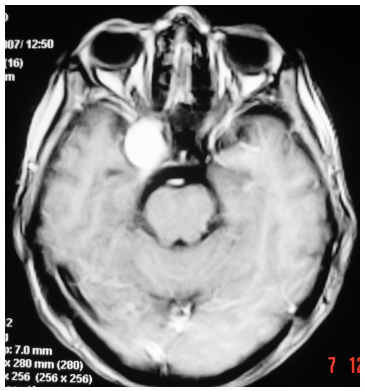
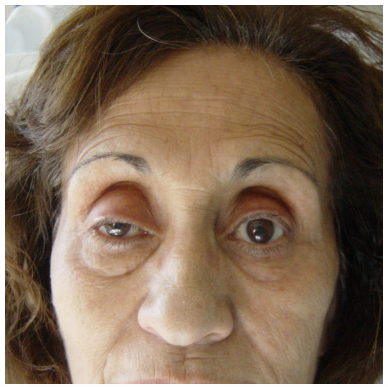
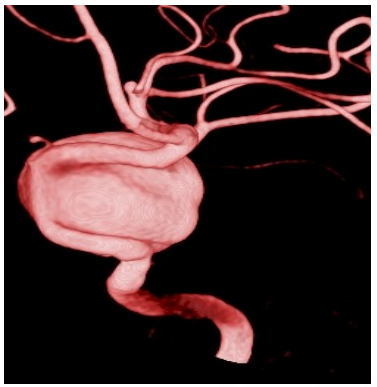
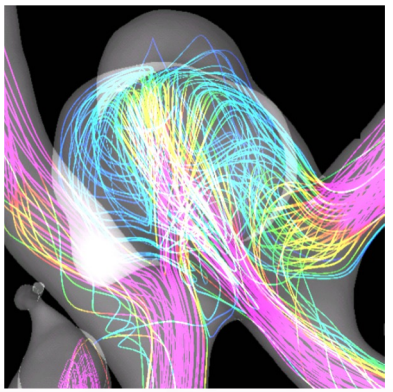
Correlation with Hemodynamics

- Two factors associated with increased risk for development of cerebral aneurysms **alter the geometry of the vessels** (and hence the flow): **Asymmetry** of the Circle of Willis and **Cerebral atherosclerosis.**

- **Hypertension alters load** on vessel and is associated with both increased development and rupture.

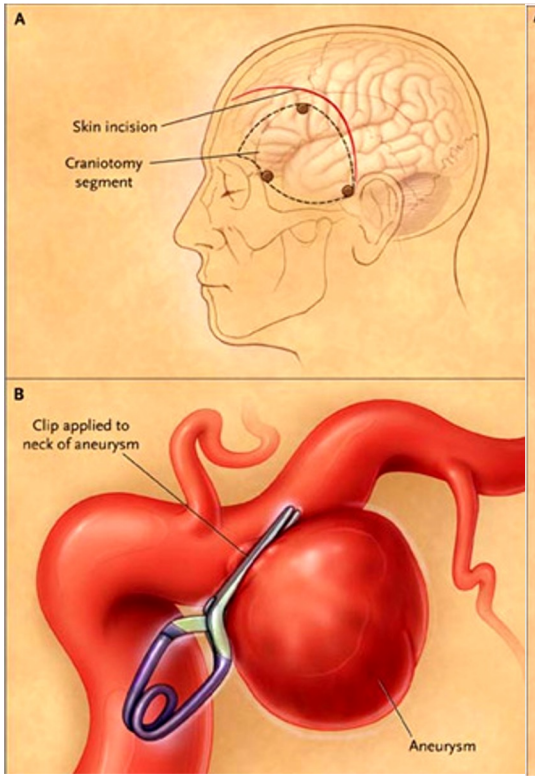
Main Goal

An extensive analysis of personalized clinical data and computer simulations **(CFD)** to study the possible relations between morphology, hemodynamics and the risk for development and rupture of cerebral aneurysms, helping to improve its evaluation & treatment.

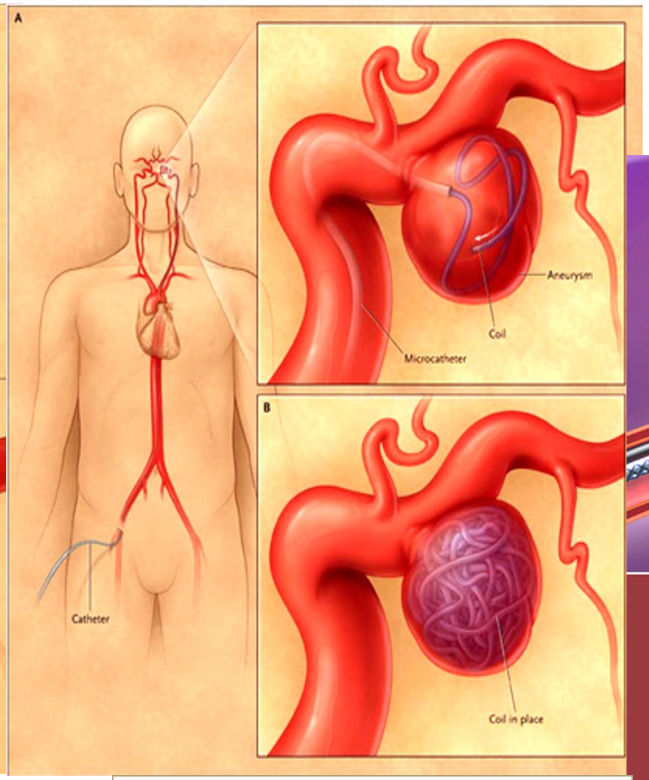





Cerebral Aneurysms - Treatment

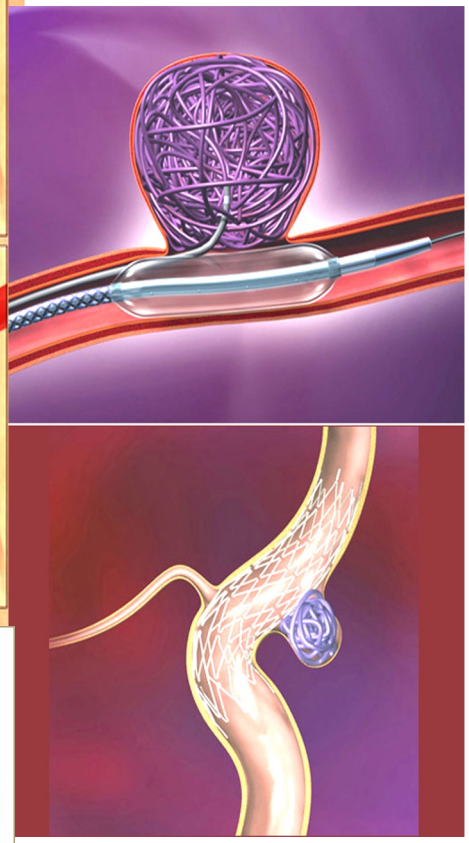
Clipping



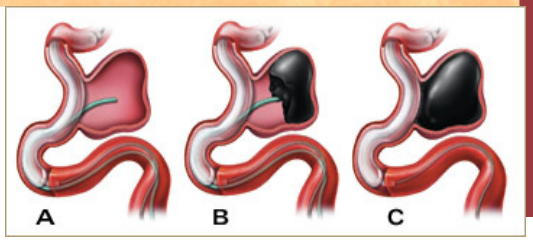
Coiling



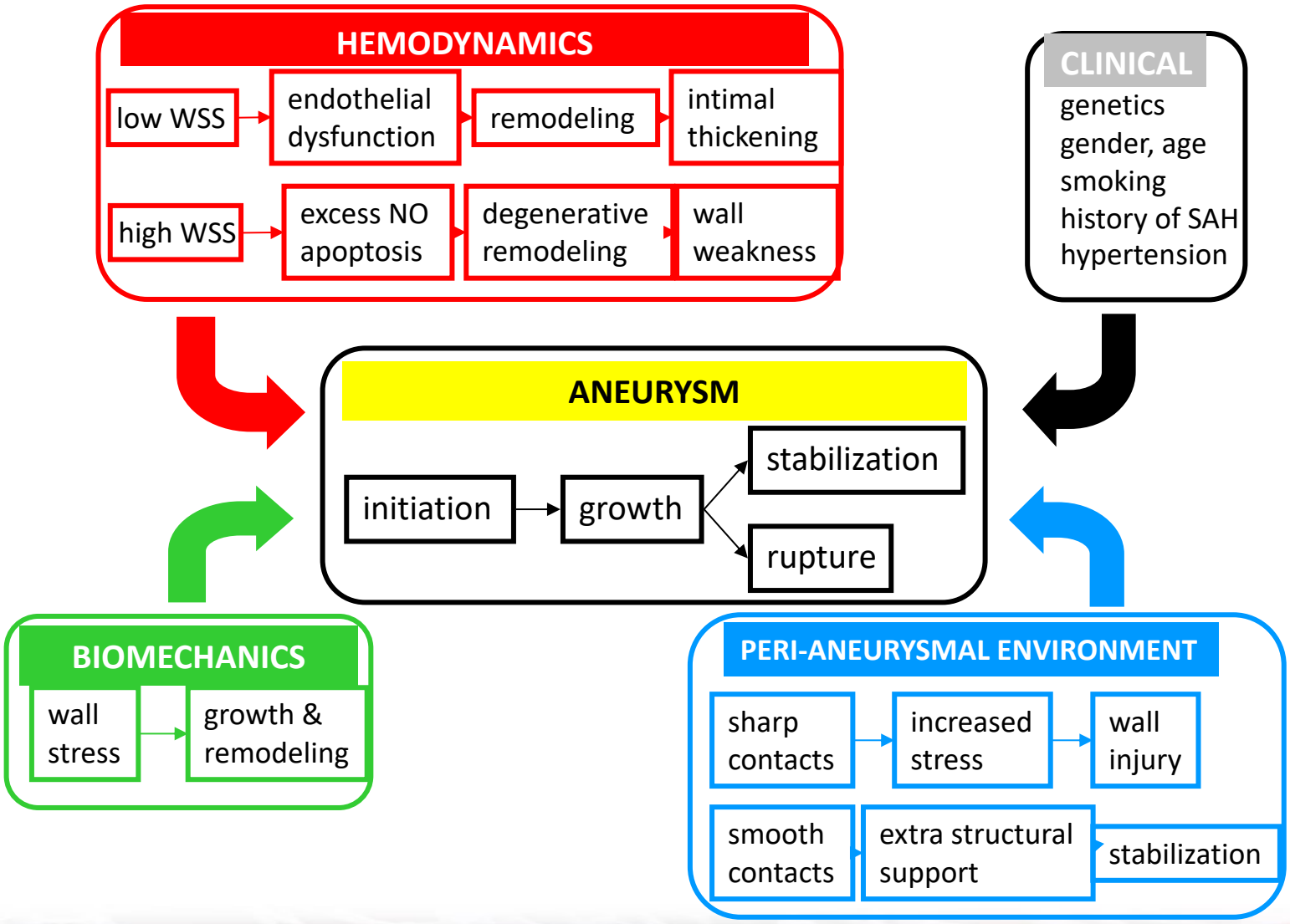
Balloon and stent assisted coiling



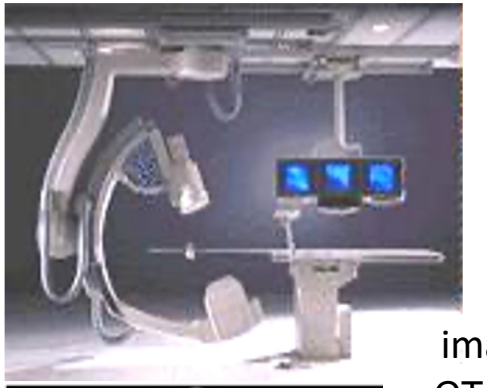
Balloon assisted Onyx embolization



Cerebral Aneurysms – Multi-Factorial Problem



A Case Study: From Medical Imaging to CFD



hemodynamics modeling

$$\nabla \cdot \mathbf{u} = 0$$
$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \nabla\cdot\tau$$

image acquisition
CTA - Circle of Willis

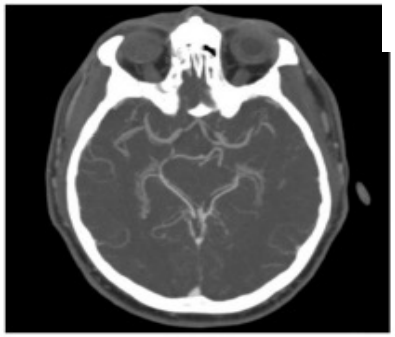
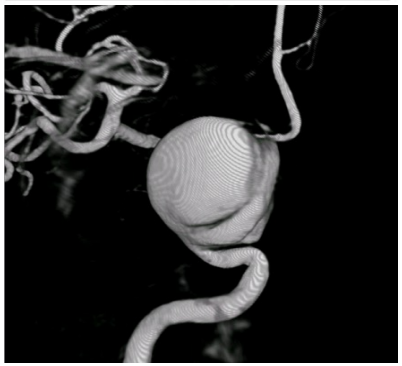
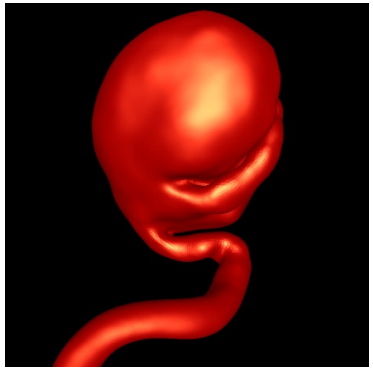


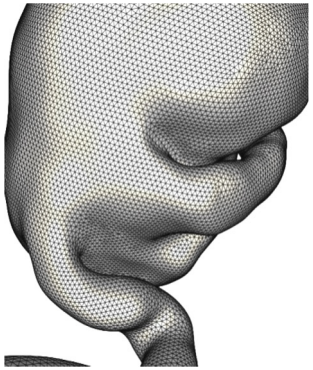
image processing
&
geometry modeling

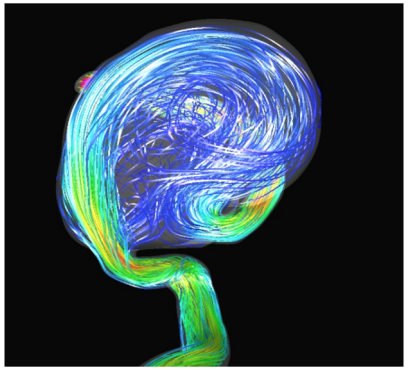
mesh generation

flow solution
&
visualization



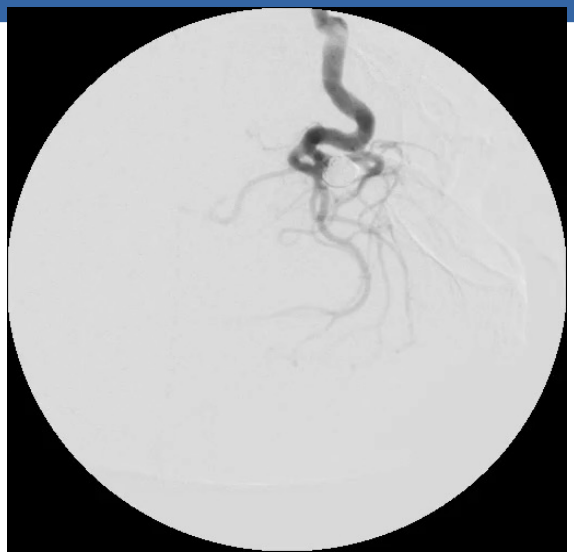




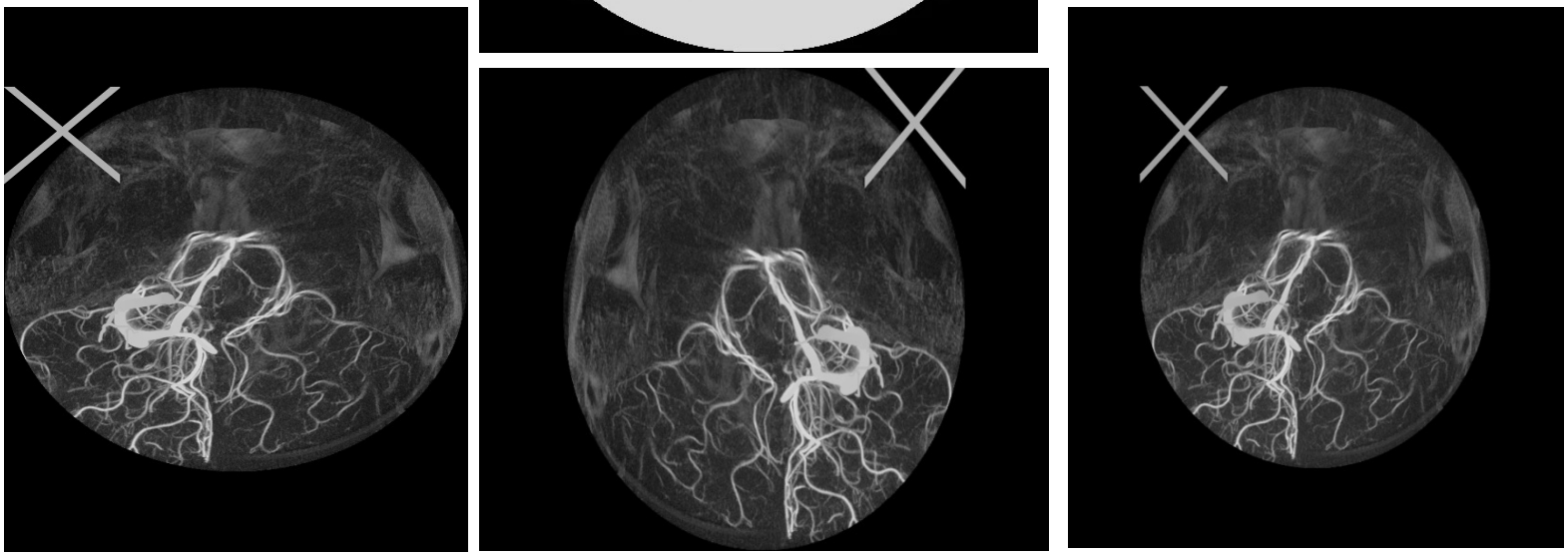


A Case Study: From Medical Imaging to CFD

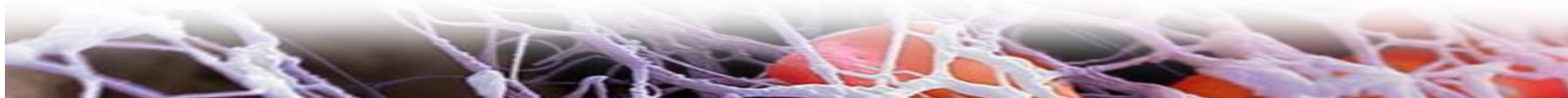
Rotational CTA



- voxel resolution of 0.8 mm size
- 512^3 grid

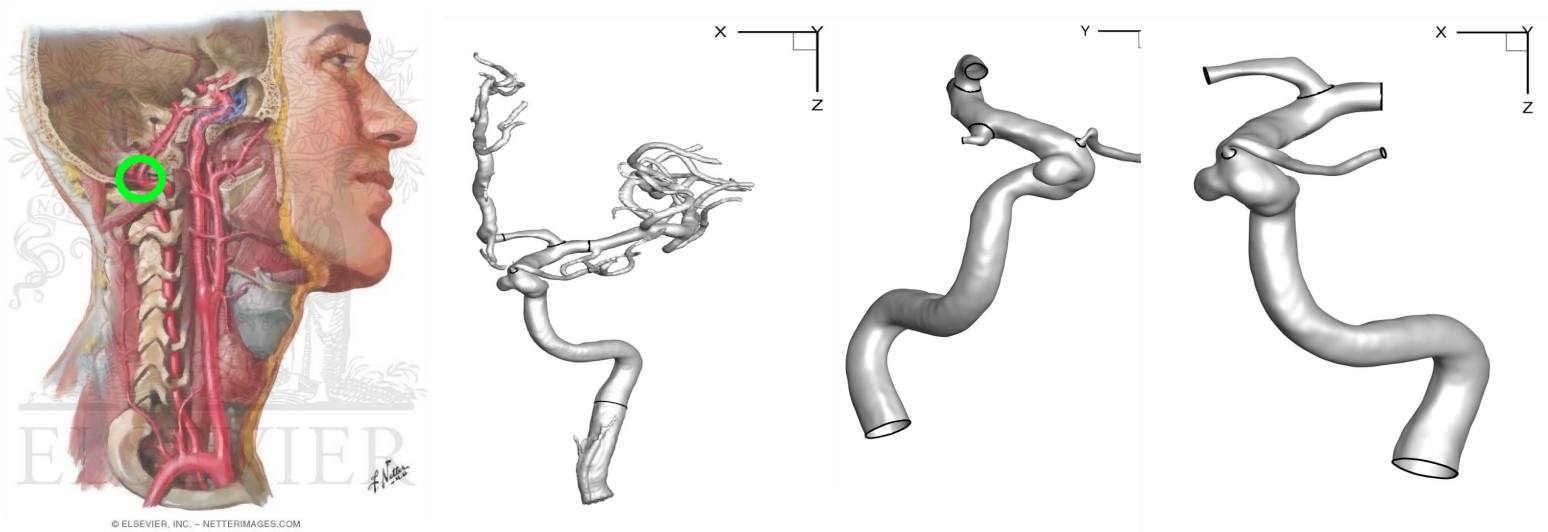


Maximum intensity projections



Medical Imaging and Virtual Model Reconstruction

Extracted domain for numerical simulations

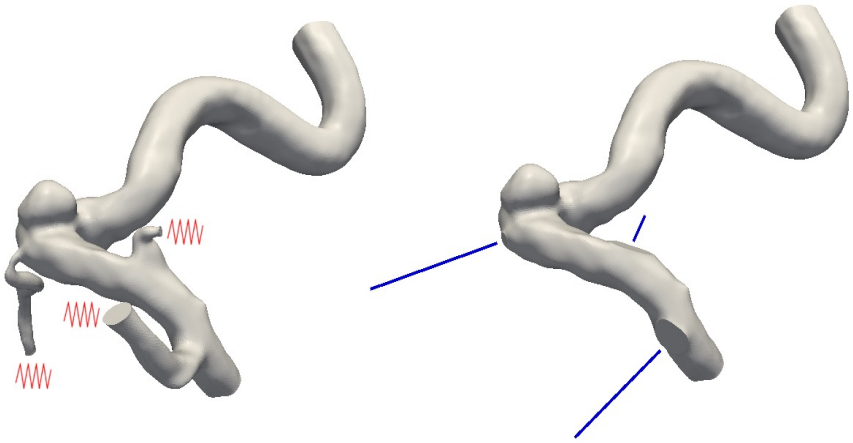


Vasculature in the neck (left www.netterimages.com) Cerebral arterial system showing a saccular aneurysm located on the outer bend

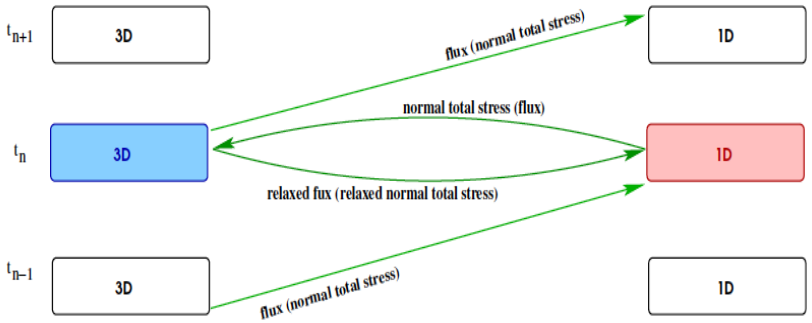
Model reconstruction:

- constant treshold segmentation
- marching tetrahedra algorithm for 3D surface extraction
- surface smoothing (200 iterations of the bi-Laplacian)

Outflow Boundary Conditions



Solving 3D-1D – Implicit coupling



Schematic of the coupling with the 0D model (left) and the 1D model (centre)

Scheme of the explicit coupling between the 3D and 1D models (right)

Four different outflow conditions analyzed for the side branches

- No slip: u=0 (neglect the side branch)
- Traction free
-Coupling with a 1D model equivalent to the 3D side branch
-Coupling with a 0D resistance model based on the 1D model

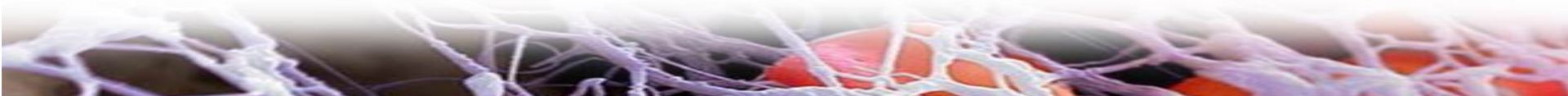
Software for Segmentation and Meshing

Automatic and manual segmentations - **ITKSnap**
http://www.itksnap.org/pmwiki/pmwiki.php?n=Main.HomePage

Surface smoothing suitable for simulations – **Meshlab**
http://www.meshlab.net/

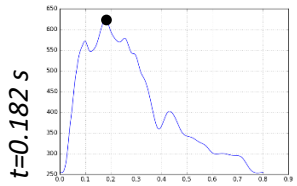
Creation of extensions – **MeshMixer** (geometry manipulation)
http://www.meshmixer.com/

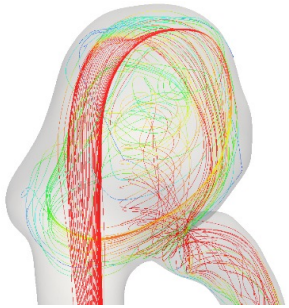
Meshing – **Gmsh** (3D FE mesh generator)
http://gmsh.info/

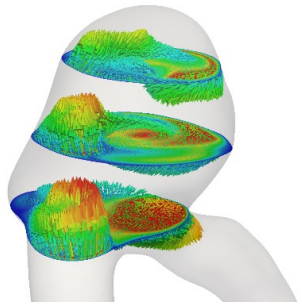


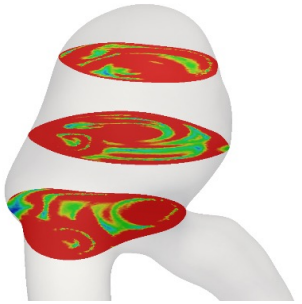
Hemodynamics Parameters - Velocity

Pic of systole

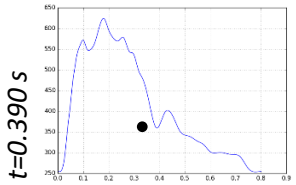


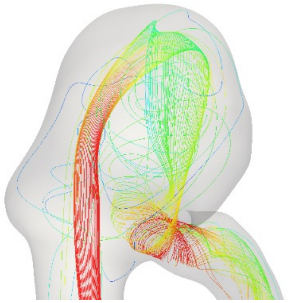


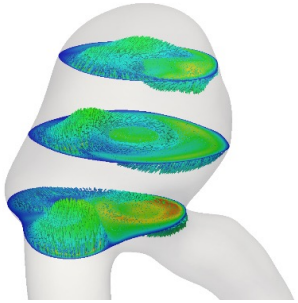


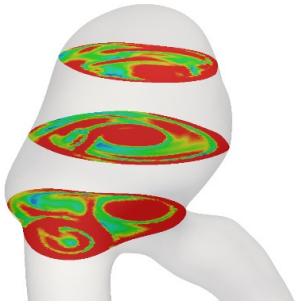


Mid of diastole

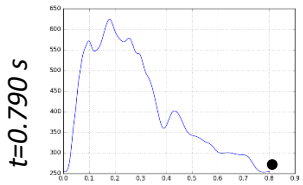


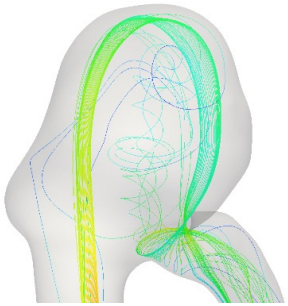


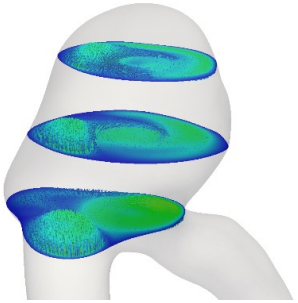


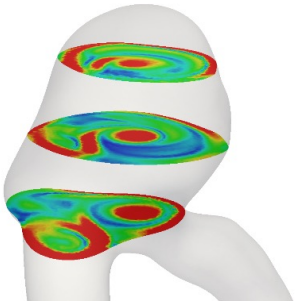


End of diastole



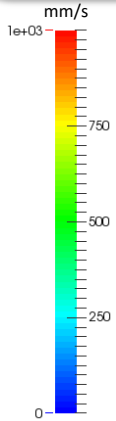


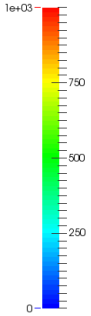




Blood: incompressible Newtonian fluid

Density: ρ=1.056x10^{-3} g/mm^3
Kinematic viscosity: ν =3.267 mm^2 /s
Time step: 1x10^{-3}
3 cardiac cycles were computed using 800 time-steps per cycle

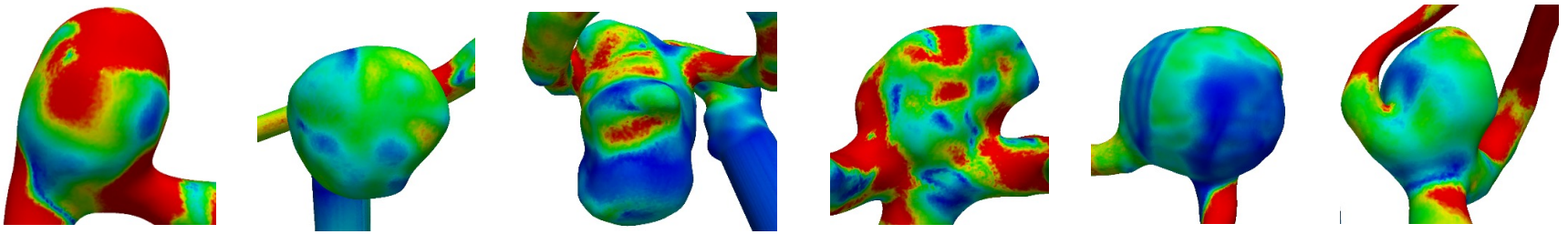




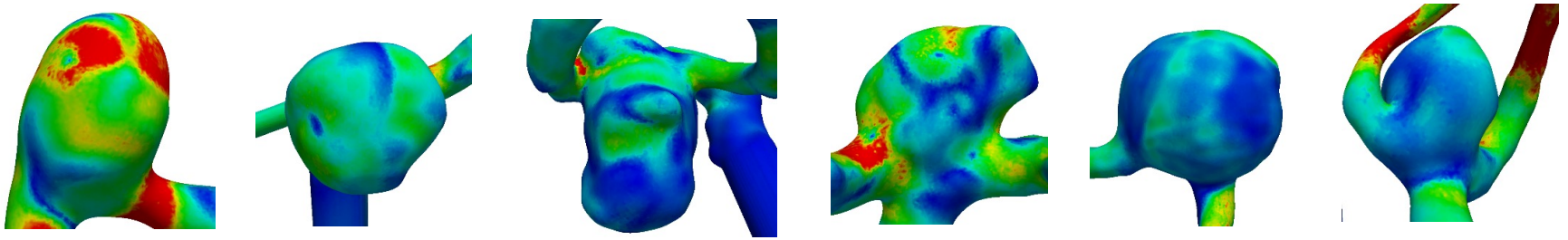
Offline post-processing with **ParaView**
https://www.paraview.org/

Hemodynamics Parameters – WSS

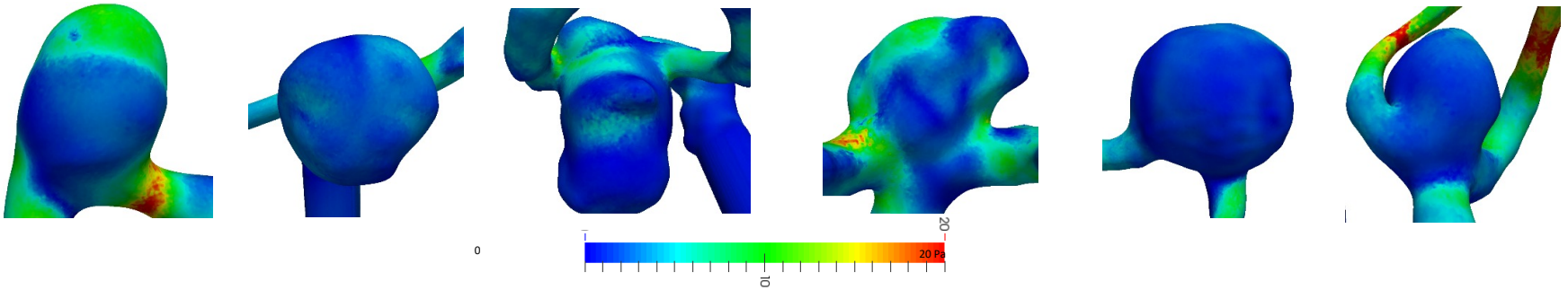
Pic of systole



Mid of diastole



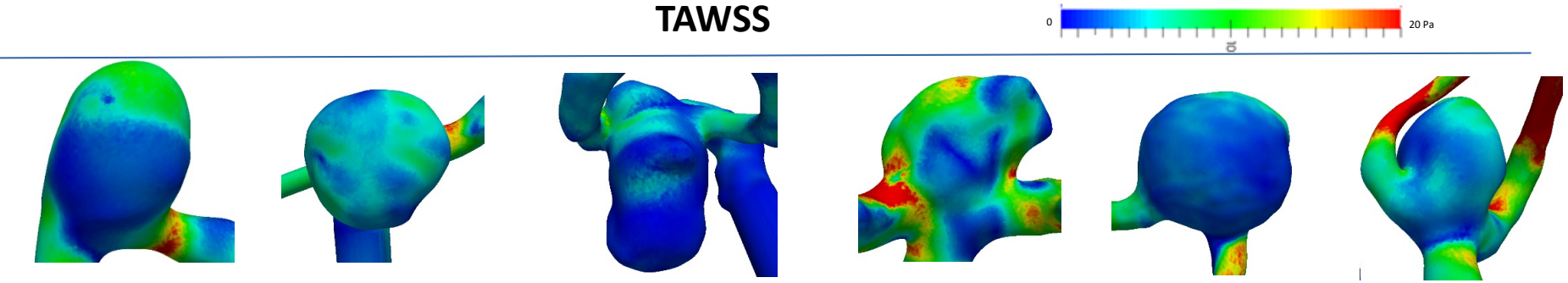
Mid of diastole



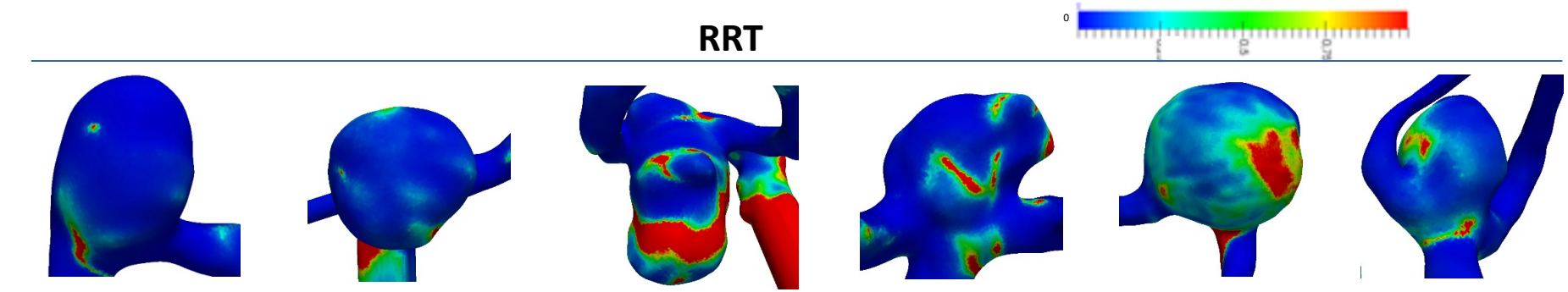
WSS $\tau_W = t - (t.n)\,n$

Other Hemodynamics Indicators

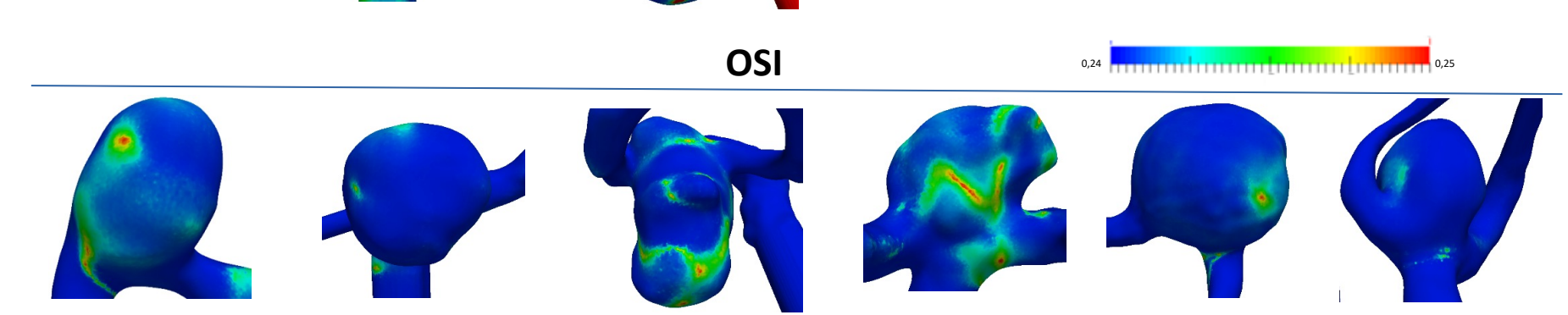
TAWSS



RRT



OSI



$$\textbf{TAWSS} = \frac{1}{T}\left|\int_0^T WSS\, dt\right|$$

$$RRT = \frac{1}{TAWSS}$$

$$\textbf{OSI} = 1/2 - \left(1 - \frac{\left|\int_0^T WSS\, dt\right|}{\int_0^T |WSS|\, dt}\right)$$

center for computational and stochastic mathematics

TÉCNICO LISBOA

ENUMATH 2023

European Conference on Advanced Mathematics and Numerical Applications

meets Lisbon

https://enumath2023.com/

ENUMATH 2021 has been canceled due to the COVID-19 Pandemic and postponed **to September 4-8, 2023**. It will take place in Lisbon, at the IST. (**April 30 – deadline for submission of abstract proposals for Contributed Talks** and **Poster Presentations)**

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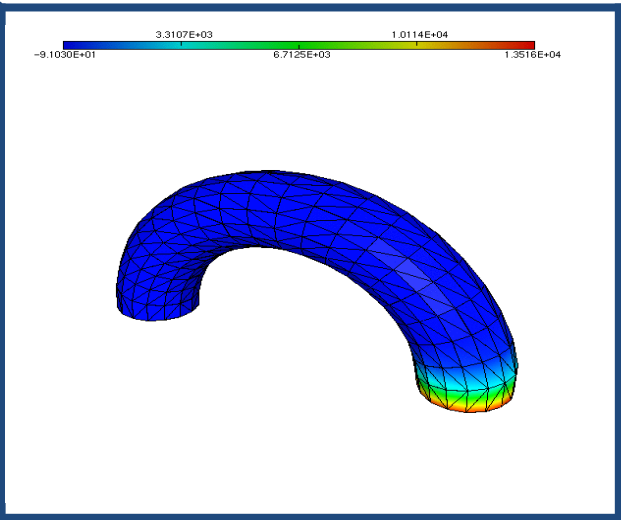
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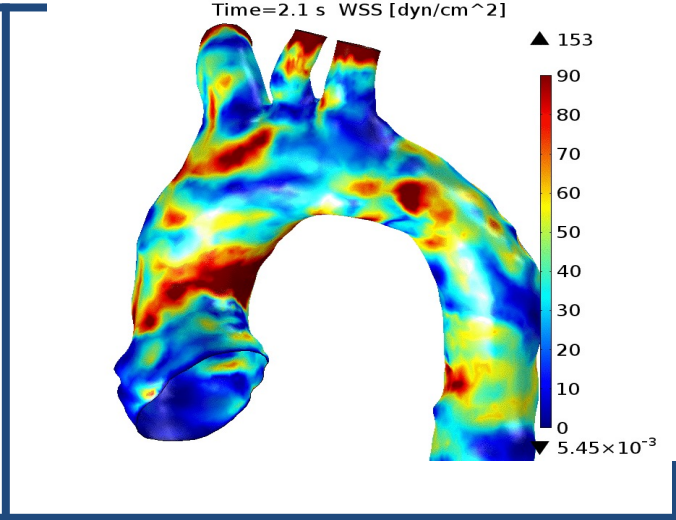
-9.1030E+01
6.7125E+03
1.3516E+04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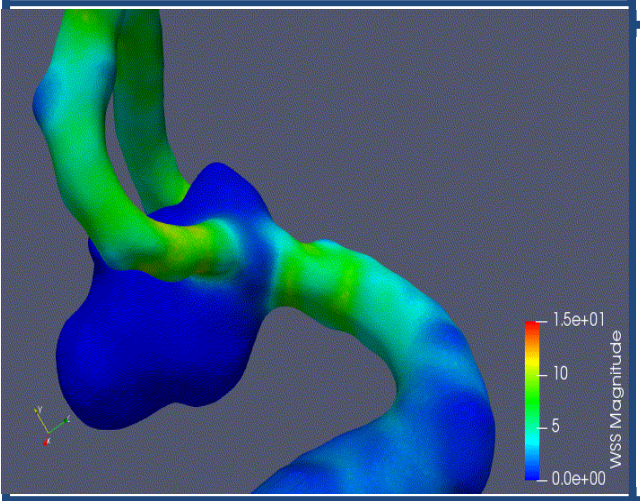
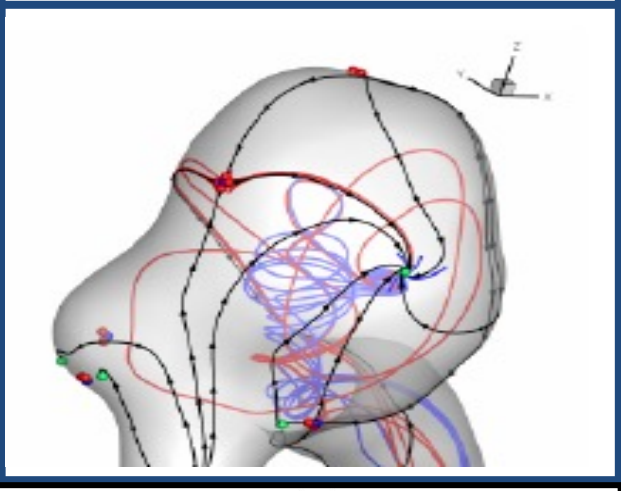
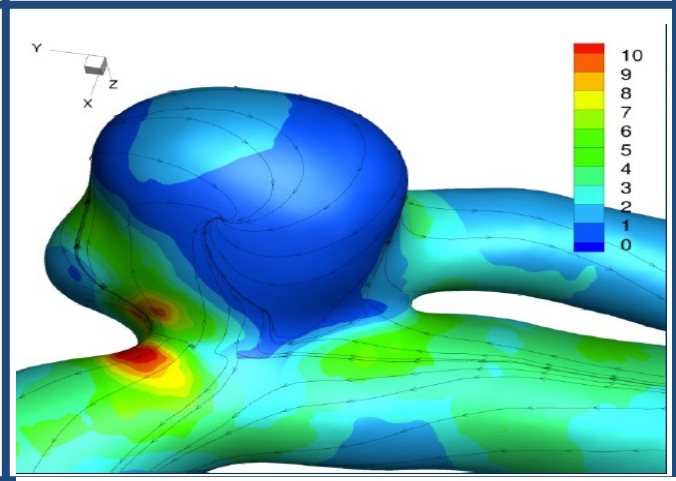
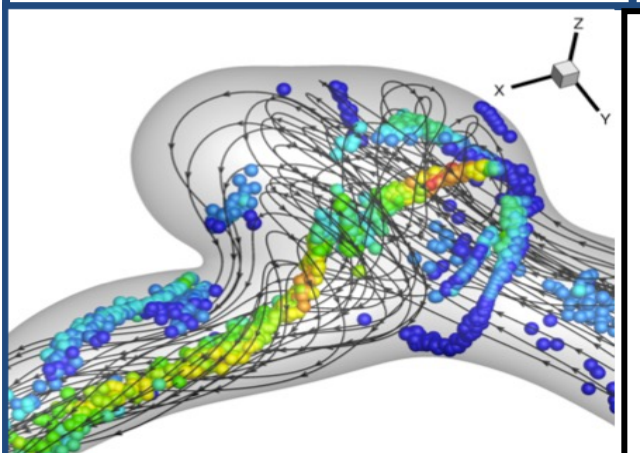
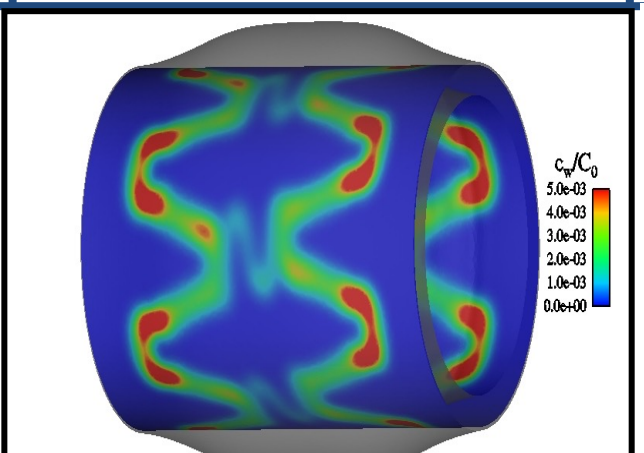
153
90
80
70
60
50
40
30
20
10
0
5.45×10-3


1.5e+01
10
5
0.0e+00
WSS Magnitude




10
9
8
7
6
5
4
3
2
1
0




c_w/C_0
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4.0e-03
3.0e-03
2.0e-03
1.0e-03
0.0e+00