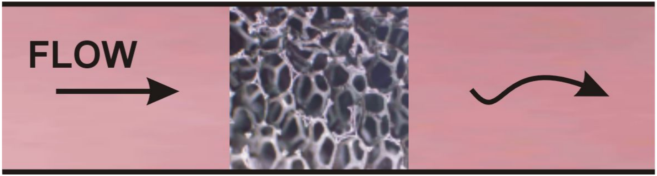
Direct numerical simulations of modulated turbulence

Arkadiusz K. Kuczaj



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Outline

1. Modulated turbulence
   - General description - turbulence problem
   - Application context - flow in complex geometries
   - Modeling idea - forcing
2. Turbulence simulations
   - Numerical method
   - Computational effort
3. Broadband forced turbulence
   - Energy dynamics
   - Mixing quantification
4. Conclusions

Outline

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   - General description - turbulence problem
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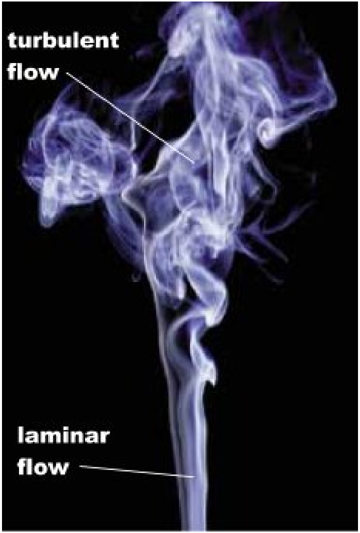
What is turbulence?



Properties of turbulence

- chaotic and random state of a fluid
- three dimensional and rotational
- space- and time-dependent
- deterministic
- sensitive to initial conditions
- wide range of nonlocally interacting degrees of freedom

Why turbulence is important?

Studies of turbulence

- physics - to understand
  - dispersion of pollution
  - ocean circulation
  - atmosphere dynamics (weather)
- engineering - to control/use
  - combustion, mixing
  - multiphase flows
  - catalyst processes
  - complex fluids: jets, sprays, bubbles/particles interactions

Why turbulence is so difficult?

Mathematical description:
Newton's law ($F = ma$) written for a viscous fluid leads to...

...the Navier-Stokes equations

- nonintegrable
  $\hookrightarrow$ uniqueness of solution
- nonlocal
  $\hookrightarrow$ sensitivity to small changes
- nonlinear
  $\hookrightarrow$ enormous amount of interacting scales

Navier-Stokes equations

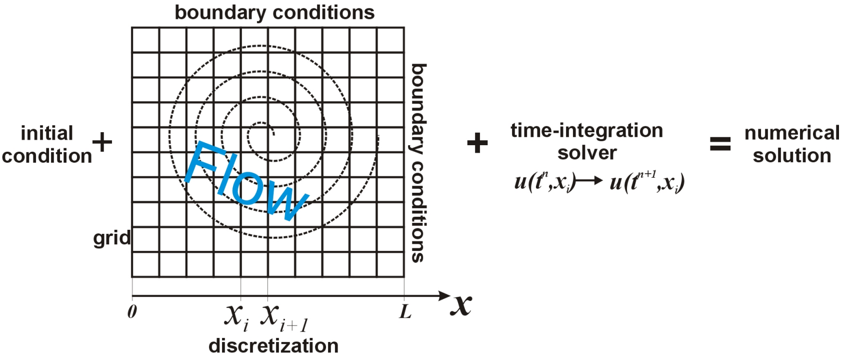
Incompressible and nondimensional form:

$$\frac{\partial \mathbf{u}(\mathbf{x},t)}{\partial t} + \underbrace{(\mathbf{u}\cdot\nabla)\mathbf{u}}_{\text{nonlinear convection}} - \underbrace{\frac{1}{\text{Re}}\nabla^2\mathbf{u}}_{\text{dissipation}} + \nabla p = 0$$

- Velocity $\mathbf{u}(\mathbf{x},t)$ and pressure $p(\mathbf{x},t)$
- Reynolds number: $\text{Re} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{UL}{\nu}$

Flow around a car:
$L = 1\ [m]$; $U = 10\ [\frac{m}{s}]$; $\nu = 10^{-5}\ [\frac{m^2}{s}] \rightarrow \text{Re} = 10^6$

How to solve these equations?



Direct Numerical Simulations

- numerically exact solution of NS equations
- need to capture all scales by resolution

Reynolds number sets the smallest scales of turbulent motion

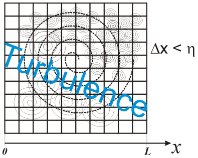
Computational challenge

- L - characteristic length
- η - size of the smallest scales (Kolmogorov scale)
- from dimensional analysis: $\frac{L}{\eta} \sim \mathrm{Re}^{3/4}$
- discretization accounts the smallest scales: $N > \frac{L}{\eta}$
- 3D problem: $N^3 > \mathrm{Re}^{9/4}$ computational points

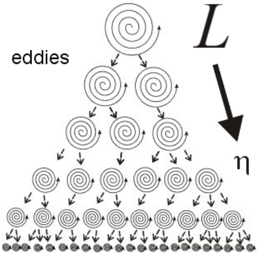
Flow around a car:
$\mathrm{Re} = 10^6 \to N > 32.000$; $N^3 > 32 \cdot 10^{12}$ points

$\hookrightarrow$ computationally not feasible

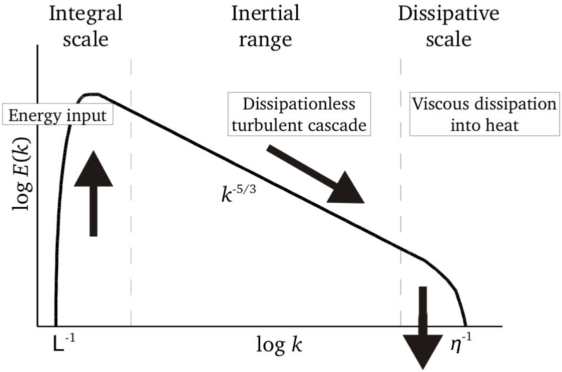
$\hookrightarrow$ need for a theory or modeling!



Kolmogorov K41 description - universal cascade of eddies



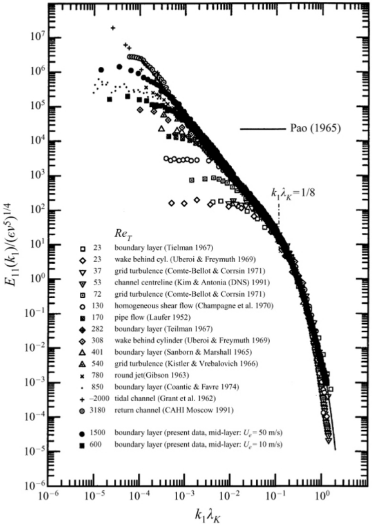
Richardson (1920)
eddies break up



Kolmogorov (1941) - equilibrium state

Motivation

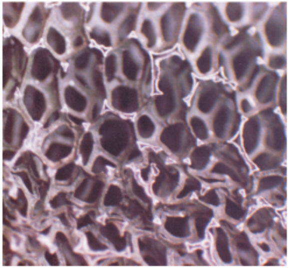
K41 theory serves well in many cases



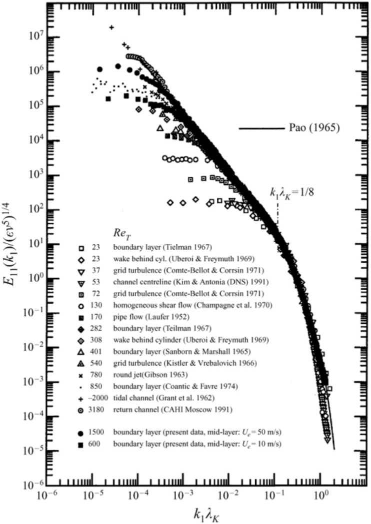
Motivation

K41 theory serves well in many cases

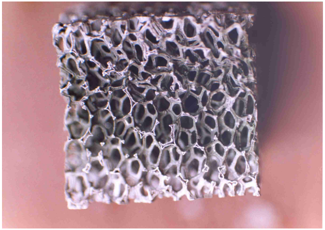
. . . but turbulent flows in complicated geometries



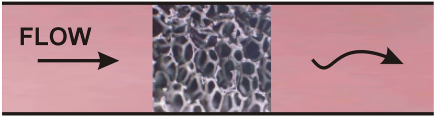
. . . do not follow K41 theory.



Flow through porous region



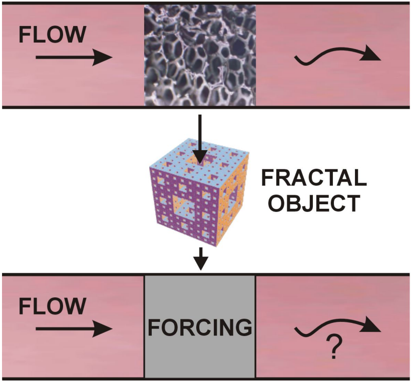
Porous object (metal foam)
↪ thermo-acoustic pump application



Modeling attempts

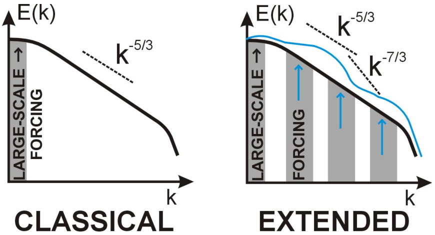
- Macroscopic approximations
  ↪ lack of incorporated scales
- Explicit boundary conditions
  ↪ computationally not feasible
- **Forcing**
  ↪ ?

Extended forcing strategy



Forcing as part of modeling

- Multi-scale application
- Energy spectrum modification
  ↪ controlled non-Kolmogorov turbulence



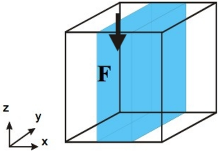
Spatially localized broadband forcing of turbulent flow

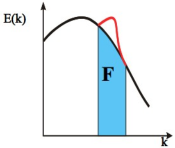
Forced Navier-Stokes equations

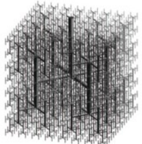
$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\mathrm{Re}}\nabla^2 \mathbf{u} + \nabla p = \mathbf{F}(\mathbf{x}, t)$$

$\mathbf{F}(\mathbf{x}, t)$ - force

- can be localized in physical space
- can explicitly agitate specified scales (fractal-like)
- can follow time-protocol (stirring, shaking)







Amalgamated Research Inc.

Fractal stirrer

"Fractal generated turbulence", B. Mazzi, J.C. Vassilicos, JFM, 2004

- drag force $\sim$ surface area
- forcing amplitude $\sim$ number of boxes of size k^{-1}
- fractal object described by the fractal dimension D_f

Forcing term in spectral space:

$$\mathbf{F}(\mathbf{k},t) = k^{D_f-2} f_\varepsilon \mathbf{e}(\mathbf{k},t)$$

$$f_\varepsilon = \frac{\varepsilon_W}{\sum_{\mathbf{k}\in\mathbb{K}} |\mathbf{u}(\mathbf{k},t)|\, k^{D_f-2}}$$

$$\mathbf{e} = \gamma \left(\frac{\mathbf{u}(\mathbf{k},t)}{|\mathbf{u}(\mathbf{k},t)|} + \imath \frac{\mathbf{k}\times\mathbf{u}(\mathbf{k},t)}{|\mathbf{k}|\,|\mathbf{u}(\mathbf{k},t)|} \right)$$

$\mathbf{e}$ - unit vector
ε_W - demanded energy input
γ - normalization parameter

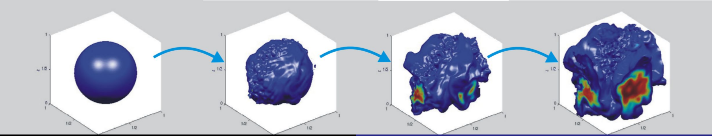
Influence of forced turbulence on transport properties

Passive scalar $T(\mathbf{x}, t)$

$$\frac{\partial T}{\partial t} + \underbrace{(\mathbf{u} \cdot \nabla) T}_{\text{convection}} - \underbrace{\frac{1}{\mathrm{Re}\,\mathrm{Sc}} \nabla^2 T}_{\text{diffusion}} = 0$$

Quantified turbulent dispersion

- Schmidt number Sc
- Developed level-set integration method:
  - surface-area at specified iso-levels
  - surface-wrinkling: small-scale characteristics



Outline

Numerical implementation

3D parallel Navier-Stokes solver

- Canonical problem
  - ↪ Incompressible Navier-Stokes equations
  - ↪ Periodic geometry with pseudo-spectral method
  - ↪ Compact storage 4-stage Runge-Kutta method
- Parallel processing for various CPU topologies
  - ↪ Message Passing Interface (MPI)
- Fast Fourier Transforms
  - ↪ 3D with FFTW/SCSL-SGI libraries
- Data storage and parallel I/O
  - ↪ Hierarchical Data Format (HDF5)

Computational effort

Discretization

- goal: to simulate flows at moderate Reynolds number
- $N = 512$ in each direction
- $N^3 > 10^8$ grid points
- 3 velocity components: 3.2 GB
- stationary statistics
  $\hookrightarrow$ long-time simulations

$\hookrightarrow$ parallel processing needed

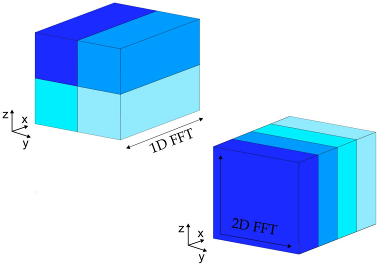
Memory requirements

N	Memory
32	0.8 MB
64	6 MB
128	50 MB
192	170 MB
256	0.4 GB
384	1.4 GB
512	**3.2 GB**
1024	26 GB
2048	206 GB
4096	1.6 TB

Computational speedup - how much we can gain with parallelization

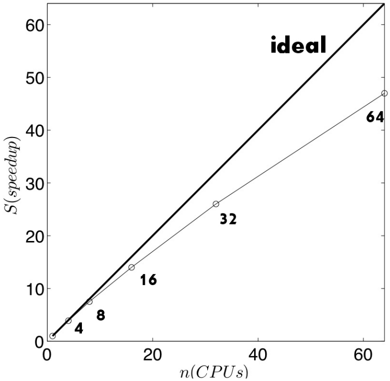
Amdahl's law

- Ideal speedup: $S(n) = n$
- p - parallelized code
- $S(n) = \frac{1}{\frac{p}{n}+(1-p)}$
- $p = 0.994$



Cost: communication between CPUs

Measured speedup at 4, 8, 16, 32 and 64 processors



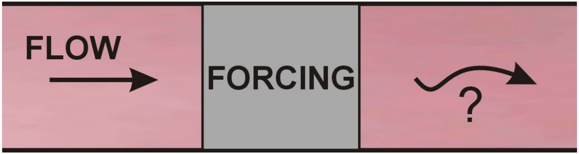
Simulations on SGI Origin and Altix supercomputers

SARA Supercomputing Center

- resolution: 128^3 up to 512^3
- simulations: 1 day up to a few weeks

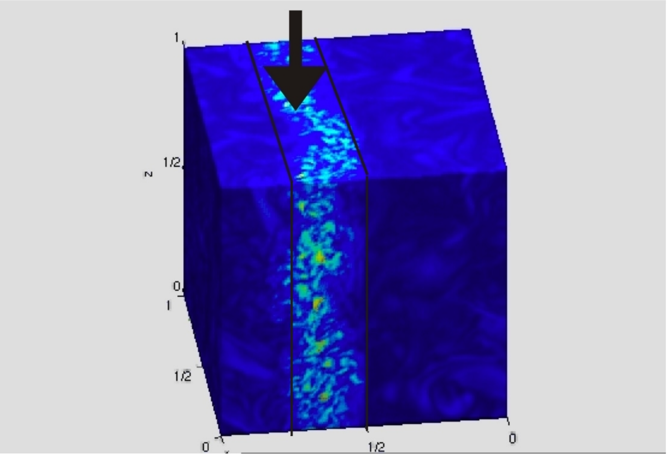
Outline

Key-research questions

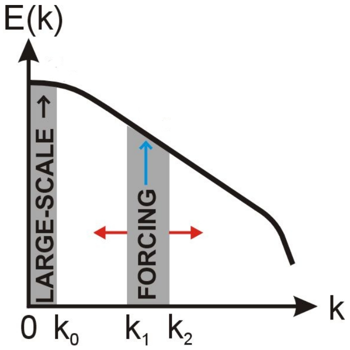


- How forcing influences turbulence (flow-structuring)?
- How forcing changes energy dynamics?
- How forcing modulates transport properties?
- Is there an efficient way to stir/force turbulence?

Spatially localized broadband forcing of tubulence



Numerical experiments

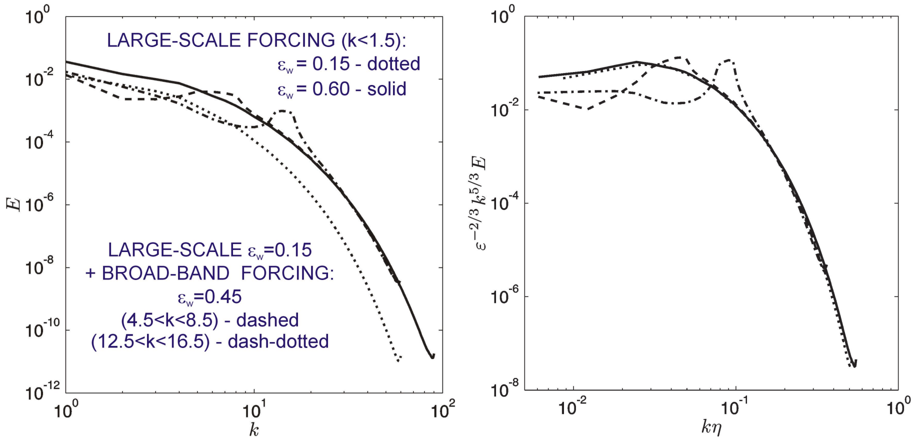


- How broadband forcing changes the energy dynamics?
  $\hookrightarrow$ varying the location (k_1, k_2) $\leftrightarrow$
  $\hookrightarrow$ varying the power ε_W $\uparrow$
- Consequences for mixing?
  $\hookrightarrow$ passive scalar simulations

Canonical problem ($R_\lambda \cong 50, 100$)

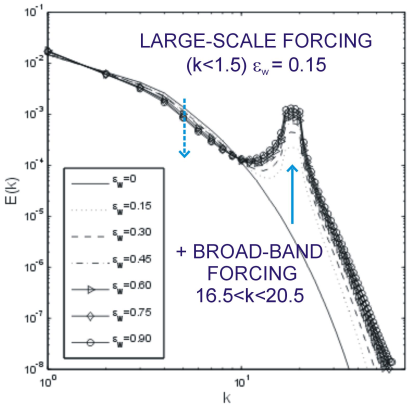
- Large-scale forcing $k_0 \leq 1$
  $\varepsilon_W = 0.15$ or $\varepsilon_W = 0.60$
- Broadband forcing in (k_1, k_2)
  supplementary $\varepsilon_W = 0.45$
  bands: (4,8) and (12,16)

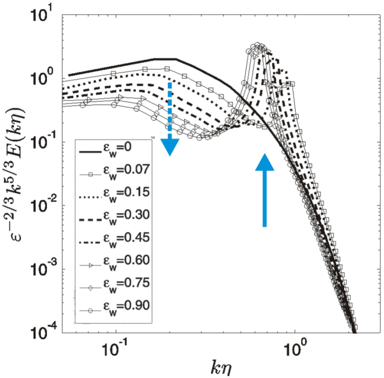
Energy spectra - varying the location ($k_1 < k < k_2$) ↔



Forcing modifies energy cascade → different scaling

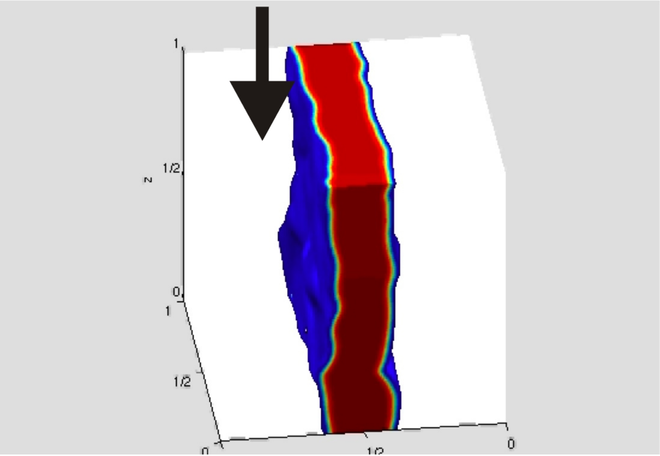
Energy spectra - varying the power $\varepsilon_w \uparrow$





Forcing removes energy from large scales → nonlocality

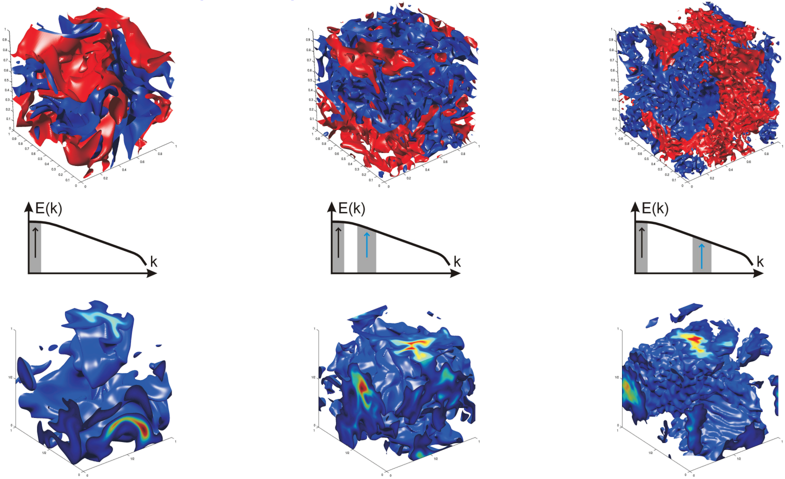
Dispersion of a tracer in forced turbulence



↪ forcing induces small-scale mixing

Influence of forcing on the flow and its transport properties

Velocity (top) and passive scalar (bottom) snapshots



Quantified turbulent dispersion

Mixing process in time

- instantaneous
- cumulative
- final total effect

Developed level-set integration method

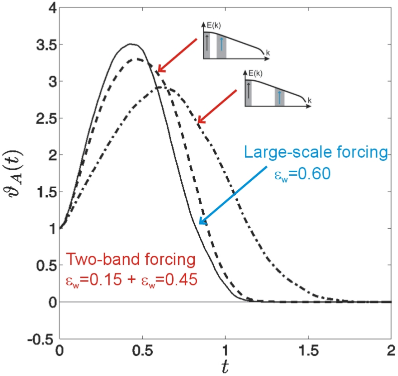
- surface area A at specified iso-levels
- surface wrinkling W – *small-scale characteristics*

Averaged growth parameters of:

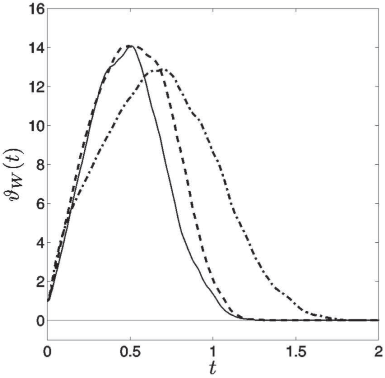
- surface area $\vartheta_A(t) = A(t)/A(0)$
- wrinkling $\vartheta_W(t) = W(t)/W(0)$
- accumulated area $\zeta_A(t)$ – *time-integrated*
- accumulated wrinkling $\zeta_W(t)$ – *time-integrated*

Surface area and wrinkling

Two-band forcing → different localization of the second band



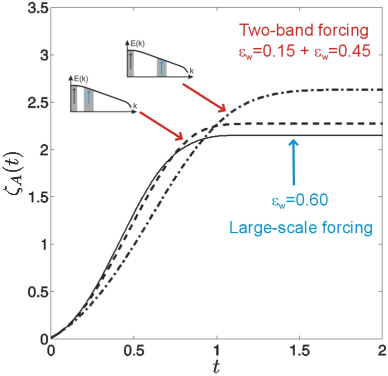
Surface area



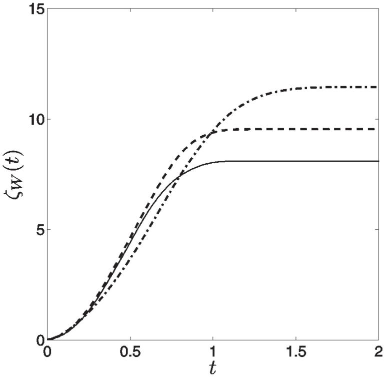
Wrinkling

Cumulative surface area and wrinkling

Time-integral over area and wrinkling as the total effect



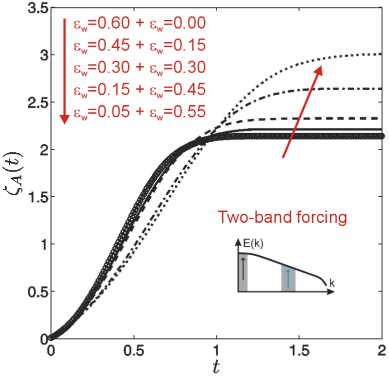
Surface area



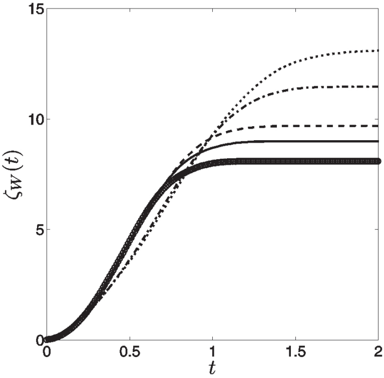
Wrinkling

Cumulative surface area and wrinkling

Different energy-input proportions between two forced bands



Surface area



Wrinkling

Outline

Summary

Conclusions

- Feasibility of forcing application as a modeling tool
- Modification of cascading process in turbulence
- Small-scale forcing → nonlocal large-scale effects
- Spatially localized broadband forcing → modeling method
- Quantified mixing → relevance for technological processes

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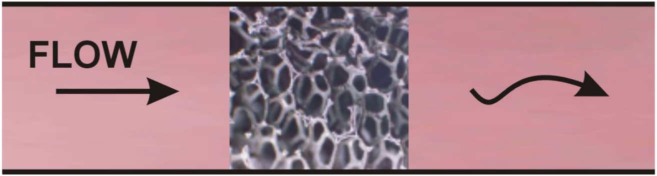

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