Slip flow structures in confined geometries

Steffen Jebauer
sjebauer@ippt.gov.pl

28.05.2008

Use *Navier Stokes* Flow Solver for Microdevices

- Simulate gas slip flows in complex (curved) geometries (e.g. MEMS)
- Use computationally efficient approach
- Study ways for mixing enhancement in micro-devices
- Study analogy between electro-osmotic flow and gas flow structures in micro-devices

Outline

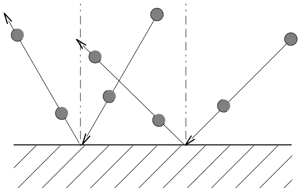
Wall Slip

- Collisions of gas molecules with solid walls
- Slip due to insufficient number of collisions
- Effect of accommodation –> accommodation coefficient σ

$$\sigma_V = \frac{\text{particles contributing streamwise momentum}}{\text{total number of impinging particles}}$$

$\sigma_V = 1$ - diffuse reflection of particles

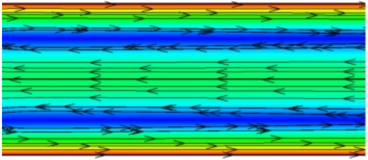
$\sigma_V < 1$ - diffuse/specular reflection



Thermal Creep Flow

- Also called thermal transpiration flow
- Momentum induced by temperature gradients, flow from cold to hot regions
- Can be used as a pumping mechanism
- Example case: Thermal creep flow between two heated tanks

left tank: 300 K



right tank: 400 K

Thermal Stress Flow

- Effect due to spatial differences in temperature gradients
- Present e.g. around curved boundaries, even if boundary temperatures are constant
- For walls with non-zero tangential temperature gradient induced flow is opposite to thermal transpiration flow direction
- Has been performed experimentally around curved objects

Knudsen Number

Knudsen number *Kn*

Classification of flows

$$Kn = \frac{\lambda}{D}$$

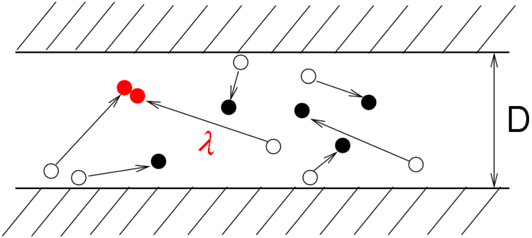
$D \ldots$ Length scale

$\lambda \ldots$ Mean free path:

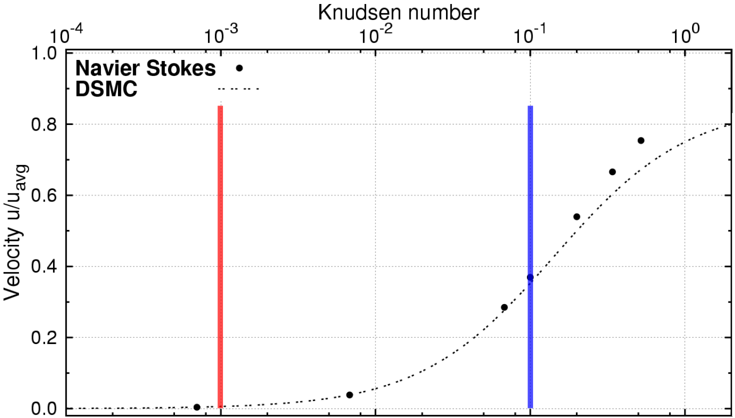
$$\lambda = \frac{k \cdot T}{\sqrt{2} \cdot \pi \cdot d^2 \cdot p}$$

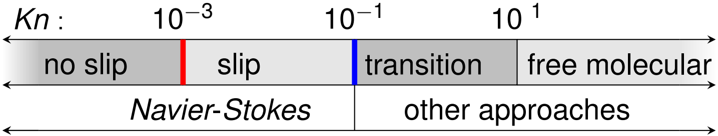
$k \ldots$ Boltzmann constant

$d \ldots$ Molecular diameter



Knudsen Number





Modelling

- Boundary conditions derived from kinetic theory of gases are applied for continuum description
- Using Navier Stokes equations with modified boundary conditions –> resulting in slip Navier Stokes equations
- Approach used with pressure-driven inflow or in periodic channels

Temperature Jump Expression

Equation

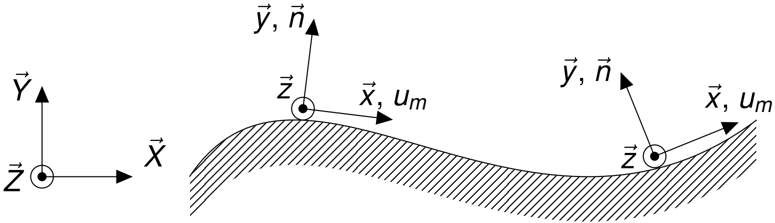
$$T_{fluid} - T_{wall} = \frac{2-\sigma_T}{\sigma_T} \frac{2\gamma}{\gamma+1} \frac{\lambda}{Pr} \frac{\partial T}{\partial y}$$

$$Pr = \frac{\mu \cdot c_p}{k_L}$$

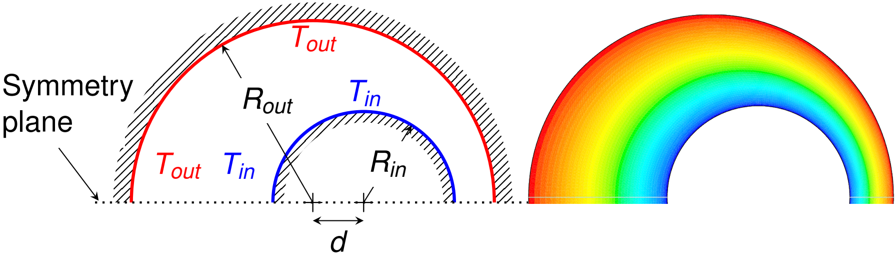
σ_T... Thermal accommodation coefficient
γ ... Specific heat ratio
c_p... Specific heat
k_L... Thermal conductivity

Modified slip velocity boundary condition

$$u_{fluid} - u_{wall} = \frac{2-\sigma_v}{\sigma_v}\lambda\left[\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x} + \frac{\mu}{\rho}\left(\frac{1}{\rho}\frac{\partial^2 \rho}{\partial x \partial y} - \frac{1}{T}\frac{\partial^2 T}{\partial x \partial y}\right)\right] + \frac{3}{4}\frac{\mu}{\rho T}\frac{\partial T}{\partial x}$$

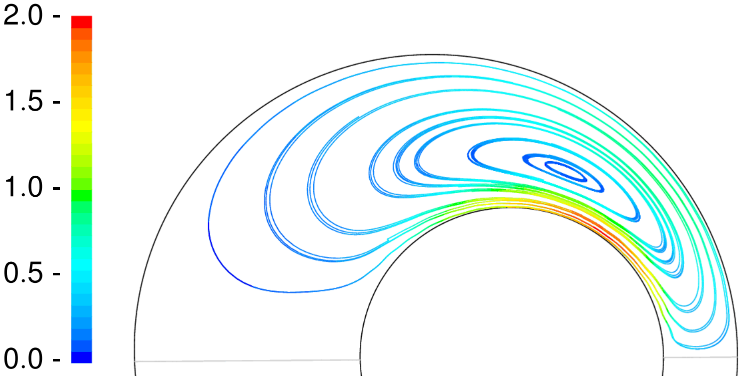


Thermal stress flow - Setup of Original case



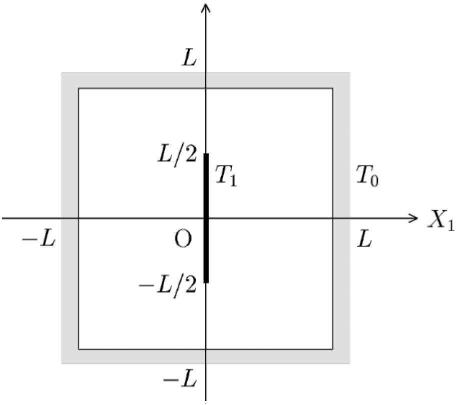
- Nominally two-dimensional setup
- Laminar case; ideal gas: air
- Temperature at walls: $T_{in} = 300\ K$, $T_{out} = 350\ K$
- $R_{in} = 2.5 \cdot 10^{-4} m$, $R_{out} = 5 \cdot 10^{-4} m$, $d = 1.3 \cdot 10^{-4} m$
- *Extended slip velocity condition* and *temperature jump* applied at walls

Thermal stress flow - Streamline plot



- Upper half of domain shown

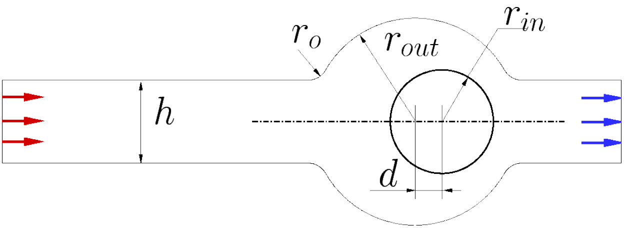
Thermal stress flow - Experiment



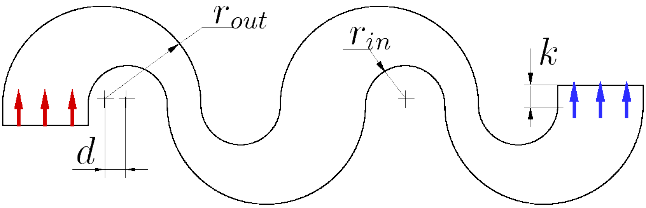
- Heated plate placed inside a heated box
- Thermal stress flow around the plate corners
- Sone, Yoshimoto, Phys Fluids, **9**, p. 3530, 1997

Thermal stress slip flow - Channel geometries I

- All setups resemble the basic setup geometry
- Setup combined with inlet/outlet section:

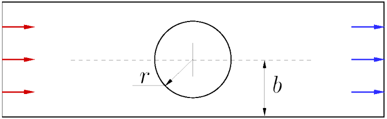


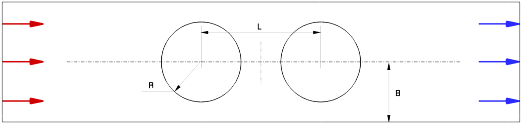
- Half-setup series combined to waveform channel:



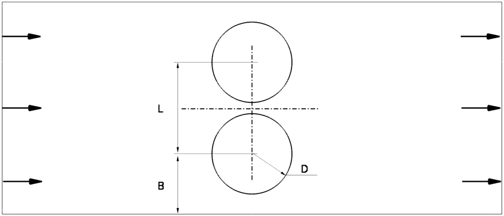
Table

Thermal stress slip flow - Channel geometries II





- One curved wall
- Two curved walls, horizontal

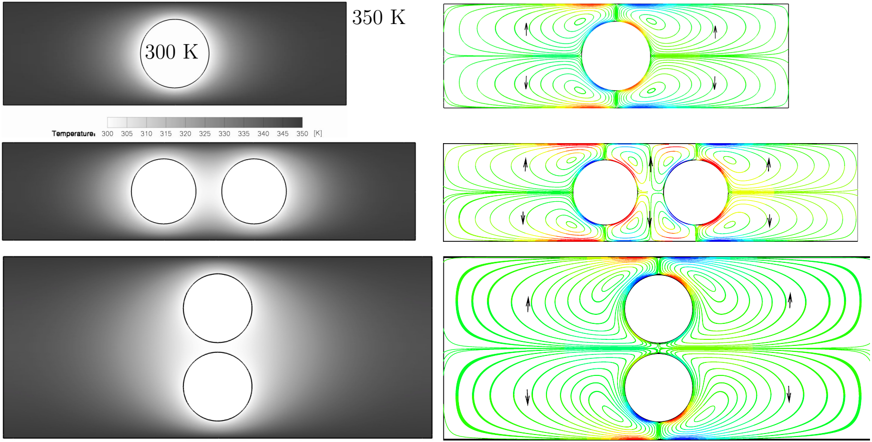


- Two curved walls, vertical

Thermal stress slip flow - Configuration

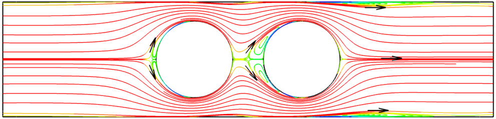
- Two-dimensional geometry
- Ideal gas: air
- Boundary conditions:
  - Wall temperature $T = 300$ K or 350 K, depending on setup
  - Inlet pressure $p = 10^{-7} \ldots 10^{-3}$ Pa or ...
  - Periodic boundary conditions, depending on setup
  - Temperature jump condition and modified slip velocity condition applied at all walls
- Reference pressure $p = 101325$ Pa
- *Kn* calculated using average mean free path and curved wall radius –> *Kn* defined by changes in curved wall radius (scaling)
- *Re* calculated using average inlet velocity and curved wall radius

Thermal stress flow - Temperature fields and streamline plots

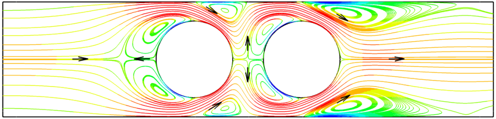


Thermal stress slip flow - Streamline plots

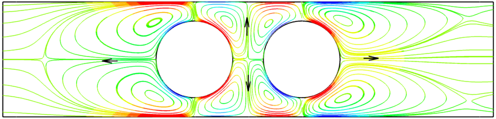
$\Delta p = 10^{-4}$ Pa



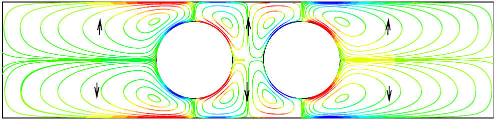
$\Delta p = 10^{-5}$ Pa



$\Delta p = 10^{-6}$ Pa

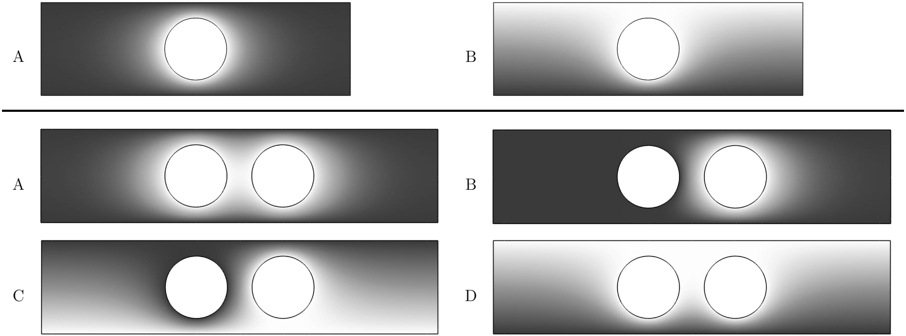


$\Delta p = 0$ Pa



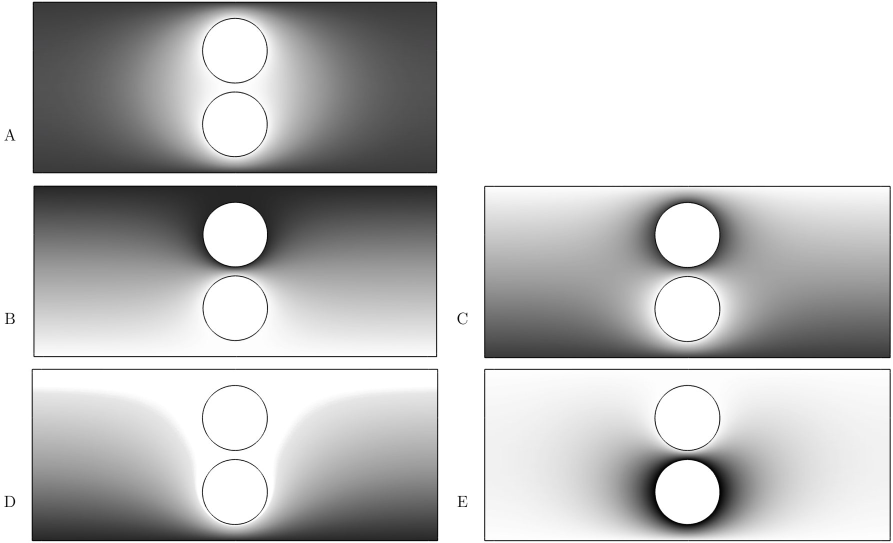
Thermal stress flow - Temperature field variations I

- Alter temperature distribution –> different flow patterns



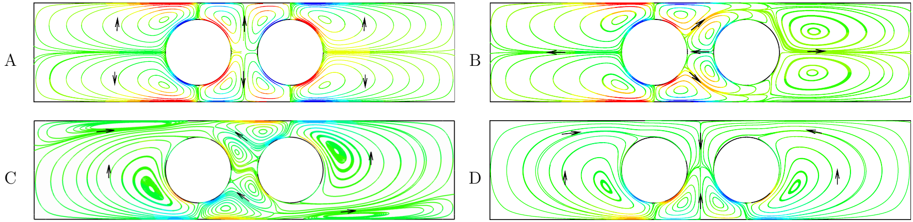
- Temperature distribution:
  Dark - 350 K
  Light - 300 K

Thermal stress flow - Temperature field variations II



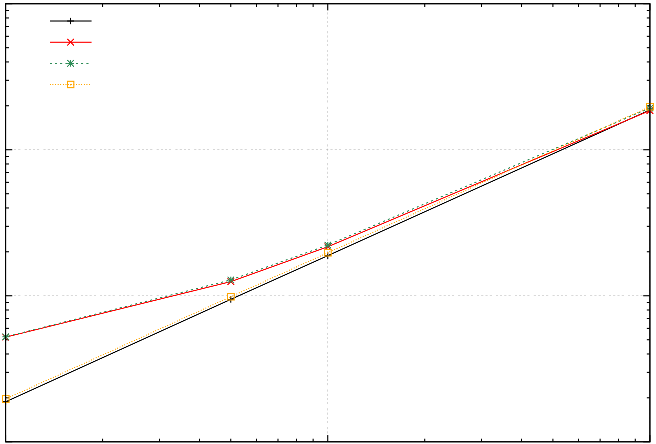
Mass flow

- Resulting streamline patterns for horizontally aligned curved walls



- Arrows indicate orientation of vortex rotation

Mass flow



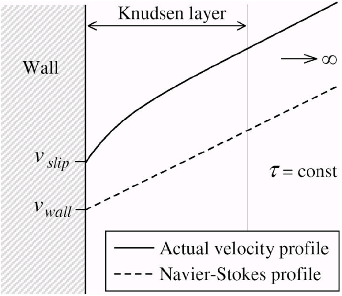
- Cases B and C have higher massflow for lower pressures
- As pressure increases massflows values become similar

Analogy Between Electro-Osmotic and Gas Flow structures

- Both phenomena are dependent on walls and wall-near layers
- Analogy in resulting flow structures is expected, due to temperature gradients <–> electric potential differences

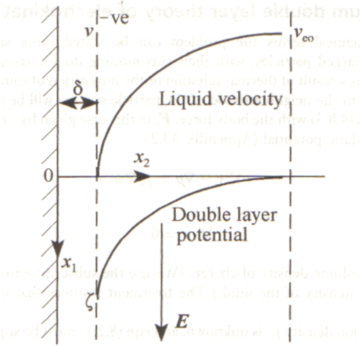
Knudsen Layer and Electrical Double Layer

- Knudsen layer in gas flow, thickness of order of mean free path λ



- Lockerby, D.A. et al., Phys. Fluids, 17, 100609, 2005

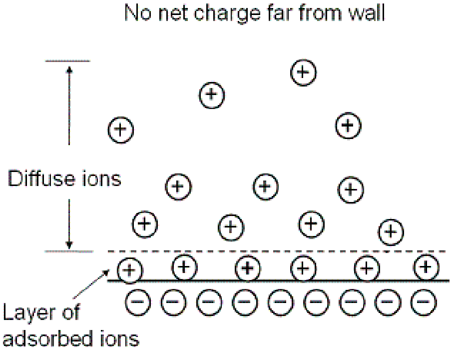
- Electrical double layer
- Surface charge acquired due to contact of solid surfaces with electrolyte solution



- Hunter, R. J., Foundations of Colloid Sciences, Oxford University Press

Electrical Double Layer

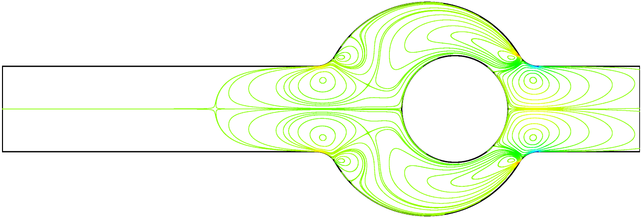
- Ions with counter-charge attach to the wall
- Mechanisms of generation of surface charge:
  - Surface dissociation, e.g. for glass walls
  - ion adsorption from solution
  - defects in the lattice



- http://microfluidics.stanford.edu/bioanal.htm

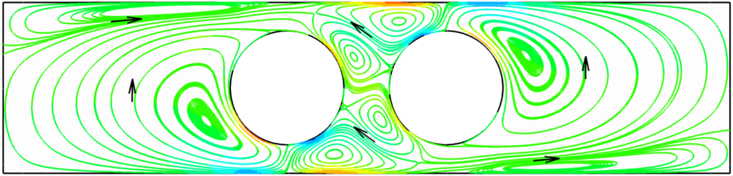
Test Cases I

- Mixing in electro-osmotic devices was studied experimentally
- Possible test cases for studies of analogy include
- Compare: Wang et al., Ind. Eng. Chem. Res. (2004), 43, 2902-2911

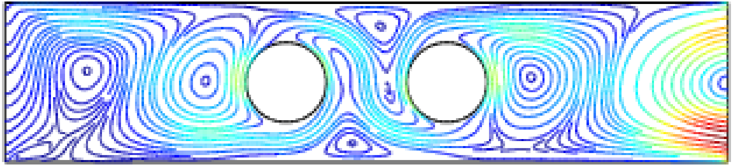


Test Cases II

- Comparative calculations –> streamline plots
- Gas slip flow



- Electroosmotic flow



Conclusions

- MEMS gas flow devices in the slip flow regime can be simulated with this implementation and the *Navier Stokes* equations
- Computationally efficient approach compared to other approaches
- Thermal stress flows on curved geometries can be modelled
- Temperature variations lead to different flow patterns that respond differently to pressure-driven inflow
- Vortex creation gives way to enhanced mixing (although turbulence is absent)
- Possible analogy between electroosmotic flow and gas slip flow give opportunity for more detailed studies of flow behaviour
- Perform experiments on electroosmosis in order to gain insight on gas slip flows or vice versa?