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Report on the doctoral dissertation
Dynamics of Systems of Charged Particles Sedimenting in a Viscous Fluid
by M.Sc. Christopher I. Trombley.

DOCUMENTS RECEIVED FROM THE INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH OF THE POLISH ACADEMY OF SCIENCES:

1. Letter on the resolution of the Scientific Council of the Institute of Fundamental Technological Research of the Polish Academy of Sciences of July 5, 2021 on a doctoral review.
2. Doctoral dissertation.
3. PhD student's CV.
4. Opinion of the dissertation supervisor, Prof. dr hab. Maria Ekiel-Jeżewska, about the doctoral student and his doctoral dissertation.

EVALUATION OF THE DOCTORAL DISSERTATION.

The direct motivation of the presented dissertation is to investigate the influence of electrostatic interaction on the dynamics of a pair of particles falling by gravity in a viscous fluid. The author shows that such an interaction has nontrivial stabilizing effects. Consequently, a further goal is to study the conditions for existence of different types of stable configurations of a given pair of particles with a large pool of attraction, in the context of the broader motivations of many important potential applications in the world of microparticles.

The main results of the dissertation have their source in the cooperation of the doctoral student with Prof. dr hab. Maria Ekiel - Jeżewska from IPPT PAN. The dissertation consists of three papers [1, 2, 3], quoted below:

[1] Trombley, C.I. and Ekiel-Jeżewska, M.L., 2019. Basic Concepts of Stokes Flows. In Flowing Matter (pp. 35-50). Editors: Toschi, F. and Sega, M., Springer.

[2] Trombley, C.I. and Ekiel-Jeżewska, M.L., 2018. Stable Configurations of Charged Sedimenting Particles. Physical Review Letters, 121(25), p.254502.

[2a] Stable Configurations of Charged Sedimenting Particles. Supplemented Material.

[3] Trombley, C.I. and Ekiel-Jeżewska, M.L., 2021. Relative trajectories of two charged sedimenting particles in a Stokes flow. Journal of Physics Communications. Accepted Manuscript online 27 May 2021.

The first four chapters of the dissertation play the role of a general presentation of the dissertation, the motivations for undertaking research, the aims of the dissertation, the methods used and the results obtained.

The paper [1] presents the theoretical foundation of the entire dissertation. Two nonlinear ordinary differential equations of the first order, coupled with each other, with variables expressing the mutual position of the molecules and the angle formed by a straight line connecting their centers with the direction perpendicular to the gravity field, were derived. After presenting the equations in dimensionless variables, the properties of the system turn out to be dependent on the three factors β, γ and δ. They express in turn: the ratio of the characteristic electrostatic force to the characteristic force of gravity, the ratio of the rays of the particles and the ratio of relative densities.

The analysis of the problem comes down to examining the stability of the stationary points of the system. This is achieved by the method of linearization of the system and consideration of many possible scenarios depending on the size of the parameters.

The paper [2] is devoted to the examination of the situation when the particles are one above the other, and therefore the angle is fixed, while the paper [3] considers a more general situation that also takes into account various mutual positions of particles in relation to the direction perpendicular to gravity.

Both situations are studied very meticulously and exhaustively, and the results are very interesting and surprising if we consider their counterparts (or lack thereof) for uncharged spherical particles of different densities and radii falling by gravity in the Stokes flow.

It is showed, inter alia, that

- The electrostatic interaction qualitatively changes the dynamics of a pair of particles falling by gravity in a viscous fluid.

- Two charged particles falling in a fluid described by the Stokes approximation can create asymptotically stable configurations with respect to perturbations (which is surprising, in view of Earnshaw's theorem about the absence of such configurations for charged particles in a vacuum).

- A pair of charged particles can form a stable configuration with a vertical or inclined line connecting the centers of the particles, but cannot perform periodic movements (the last two stable configurations do not exist for uncharged particles).

- For fixed parameter values, it is possible to have more than one stable stationary configuration of the relative positions of the particles, and the straight lines connecting the centers of the particles may position themselves both along the direction of gravity and sloped with respect to it.

–Stable pairs can only form charged particles that have different densities and radii, while their charge can be arbitrarily small.

In a stable stationary configuration, the larger particle is always above the smaller particle, and in addition, both particles must be denser than the fluid or both less dense than the fluid.

- If the particles are denser than the fluid, then in a stable stationary configuration

the smaller particle must be denser than the larger particle.

- If the particles are less dense than the fluid, then in a stable stationary configuration the smaller particle must be less dense than the larger particle.

- When, in a stable steady state, the straight line joining the centers of the particles is inclined with respect to the direction of gravity, the Stokes velocities are such that the insulated larger particle would descend faster than the smaller one.

- There are stable, stationary relative positions for arbitrarily large distances between particles.

- For long distances, the upper particle must have a slightly higher Stokes speed than the lower particle, and the characteristic electrostatic force must be large compared to the characteristic force of gravity.

- If the Stokes velocities of the two particles are very slightly different from each other, and their densities are almost equal, the stable stationary configurations are achieved for low values of the characteristic force ratio described above.

- The pool of attraction of the stationary states of relative positions is not limited from above, the radius of the horizontal cross section may be large compared to the sum of the particle radii.

There are also precise limits on the values of the system parameters - the ratio of the relative particle density, the ratio of rays and the ratio of the characteristic electrostatic force to the characteristic force of gravity (β, γ, and δ) - conditioning the possibility of the charged particles forming a stable pair.

Moreover, a phase diagram in the space of the ratio of rays and the Stokes velocity ratio was determined, giving areas for which stable stationary states can exist as well as areas where there are stable configurations only because of a certain subclass of disturbances.

Additionally, it has been shown what values of the distance between particles in a stable steady state can be expected for a given Stokes velocity and radii ratios.

SUMMARY.

The logical structure of the dissertation is coherent and clear, and the whole is presented in an elegant way. The subsequent sections contain the necessary remarks and explanations, placing the considerations in a broader context of the field of research under consideration. It proves the high mathematical culture of the doctoral student and his ability to present complex issues.

The dissertation is at a high professional level, competently uses auxiliary theorems and refers to relevant publications within the research field under consideration. The doctoral student showed a very good knowledge of the subject in a broader scope than just the dissertation itself, justifying in detail the considerations and their limitations.

The dissertation as a whole is written very carefully. It contains helpful drawings and graphs illustrating mutual relations between crucial parameters.

The results of the doctoral student are confirmed by three publications, including one as a chapter of a book (“ Flowing Matter ”, Springer 2019), and two in major journals of international scope (Physical Review Letters, Journal of Physics Communications).

An important problem in mechanics, the high professional level and the results of the

dissertation, as well as the clarity of the presentation place the reviewed work in a number of doctoral theses, which are not only an exercise but also a major scientific contribution to mechanics.

The attached CV of the doctoral student testifies to his large and varied (teaching, conferences, grants, publications) activity in the scientific community.

CONCLUSION

The presented doctoral dissertation of Mr. Christopher I. Trombley is an original solution to a scientific problem in the discipline of Mechanical Engineering. The doctoral student showed considerable freedom in using the methods of mathematics and mathematical physics to solve the problem under consideration. The dissertation has a logical and lucid structure, it is written in clear language.

In my opinion, the presented dissertation meets all the requirements for a doctoral dissertation (in accordance with the requirements contained in Article 13 (1) of the Act of scientific title, in connection with Art. 179 of the Act of July 3, 2018 Introducing the Act - Law on Higher Education and Science Journal Of Laws of 2018, item 1669). Thus, I am applying for the admission of Mr. Christopher I. Trombley to the next stages of posting for a doctoral degree. At the same time, I am in favour of awarding Mr. Trombley with the doctoral dissertation award, the justification for which is given in the above review, and which is further supported by the importance of the issues considered and the results obtained in the light of potential applications in biology, medicine and industry, a comprehensive description of the problems undertaken and an excellent presentation of the results obtained.

G. Łukaszewicz

Grzegorz Łukaszewicz Warszawa, September 28, 2021 r.