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BEHAVIOUR OF TENSEGRITY CELLS ASSEMBLY DURING SINGLE CELL GROWTH

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ABSTRACT

The tensegrity structures form the cells of living organisms. We will deal with the cell monolayer. The particular cell is an elementary icosahedron. We are interested in the influence of the cell growth on the displacement and stress patterns in the cell matrix. The problem is geometrically nonlinear and visco-elastic.

Key Words: *Tensegrity, cell colonies, cell growth*

1. INTRODUCTION

The tensegrity structures form the cells of living organisms [1]. The role of cytoskeleton (CSK) is continuously discovered. The cytoskeleton can be modelled as a tensegrity structure consisting of equivalent microtubules acting as struts and actins which are the tendons [2]. We are observing the behaviour of the assembly of the cells in the situation of growth of a single cell. The cell growths because of the extension of the struts. We observe three exemplary modes of the extension, namely, all struts are growing, two parallel struts are growing and one strut is growing. We are not answering the question why the struts are starting to grow. The signal can come from an external agent [3].

2. MATHEMATICAL FORMULATION

We adopt the incremental formulation in the Updated Lagrangian frame. The equation of equilibrium reads

$$\left(\int_{\Omega'} \mathbf{B}_{L}'^{T}\,{}_{t}^{\prime}\bar{\boldsymbol{\sigma}}\mathbf{B}_{L}'\,d\Omega'\right)\Delta\mathbf{q}+\int_{\Omega'}\mathbf{B}_{L}^{T}\Delta\mathbf{S}\,d\Omega'=\int_{\Omega'}\mathbf{N}^{T}\Delta\mathbf{f}\,d\Omega'+\int_{\partial\Omega_{\sigma}'}\mathbf{N}^{T}\Delta\mathbf{t}\,d\left(\partial\Omega_{\sigma}'\right) \quad (1)$$

where $\mathbf{B}_{L}^{T}$ and $\mathbf{B}_{L}^{T}$ are the nonlinear and linear operators, $\mathbf{N}$ is the shape functions matrix, $\Delta\mathbf{S}$ is the stress increment, $\bar{\boldsymbol{\tau}}$ is the Cauchy stress matrix, $\Delta\mathbf{q}$ is the displacement increment, $\Delta\mathbf{f}$ and $\Delta\mathbf{t}$ are the body forces and the boundary tractions increments. The integration is done over the domain Ω and its boundary Ω_{σ}.

The constitutive model is visco-elastic such as the stress increment depends on total stress S, the shear modulus (G), the bulk modulus (K) and the strain increment ΔE as follows

$$\Delta \mathbf{S} = \mathbf{D}^{const}\left(\mathbf{S}, G, K\right)\Delta\mathbf{E} \qquad (2)$$

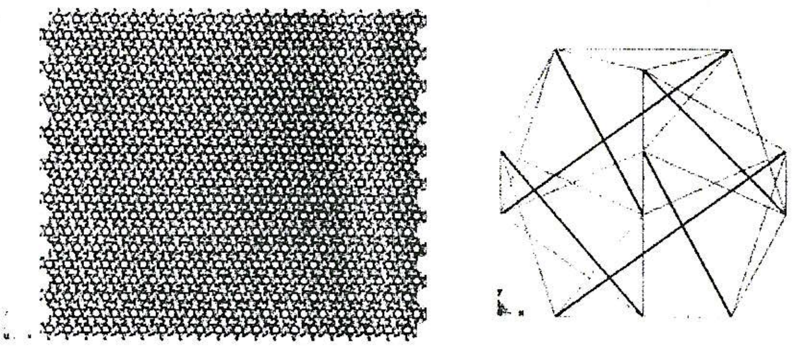
with the relaxation function

$$G(t) = G_o + \sum_{i=1}^{n} G_i \exp\left(\frac{-t}{\lambda_i}\right) \qquad (3)$$

where t is the time and λ_i are the relaxation times of the particular parallel dampers. These above describe the generalized Maxwell model.

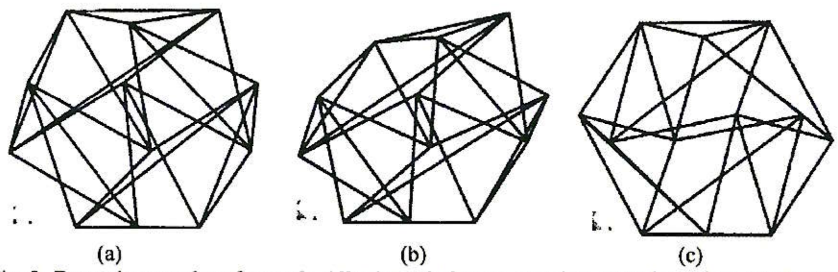
3. NUMERICAL EXAMPLE

The icosahedral tensegrity structure is presented in Fig. 1. The cytoskeleton can slide on the surface. The right side of cellural matrix is fixed. We adopt the following data, namely, height of the cell 19 μm, cross-sectional areas of the tendons (filaments) $10nm^2$, cross-sectional areas of the struts (microtubules) 190 nm^2, Young's modulus of the tendons 2.6GPa and the struts 1.2GPa, initial prestressing forces 20 nN, maximum loading 0.1N, relaxation time 1.0 sec, G_i/G_o ratio 0.91. The honeycomb pattern of the cells is shown in Fig 1a and the elementary icosahedral tensegrity structure which models the individual cells is shown in Fig 1b. The 6 pairs of the parallel struts are marked in blue color and the remaining bars are the tendons.



(a) (b)

Fig. 1. Honeycomb pattern of the cells (a) Elementary cell (b)



(a) (b) (c)

Fig. 2. Exemplary modes of growth. All microtubules are growing, growth mode A (a) Two parallel microtubules, growth mode B, (b) One microtubule, growth mode C (c).

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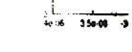

Fig. 4. Load fa
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The three considered elementary modes of growth are shown in Fig. 2. It is interesting to note that the growth of two parallel struts ovals the cell which is the growth mode B. The same happens to mode C, however it is less intence. The modes presented above correspond with Fig. 3 where the growing cell is placed close to the middle of the cellular matrix.

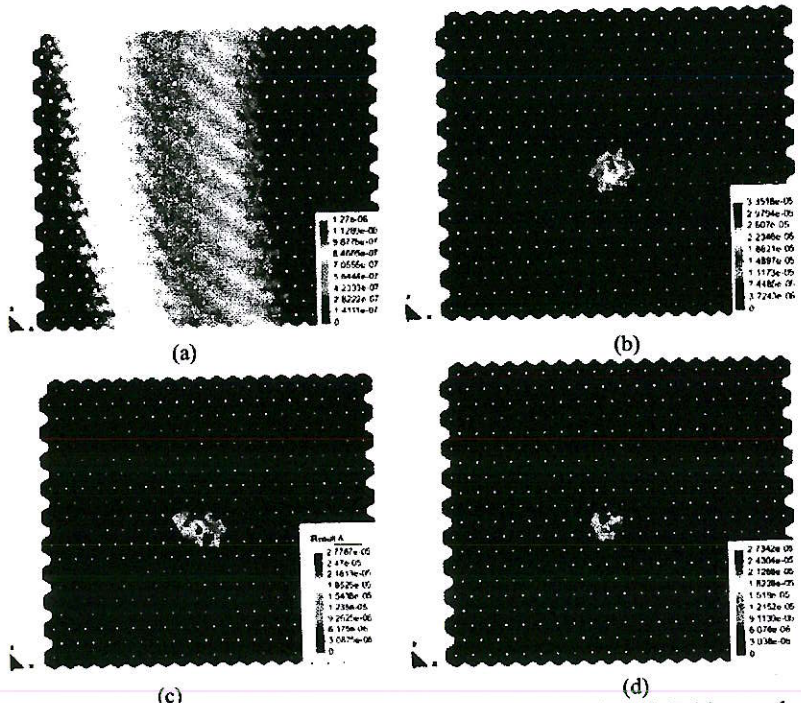

Fig 3. Displacement patterns. Tension (a) growth mode A (b), growth mode B (c), growth mode C (d).

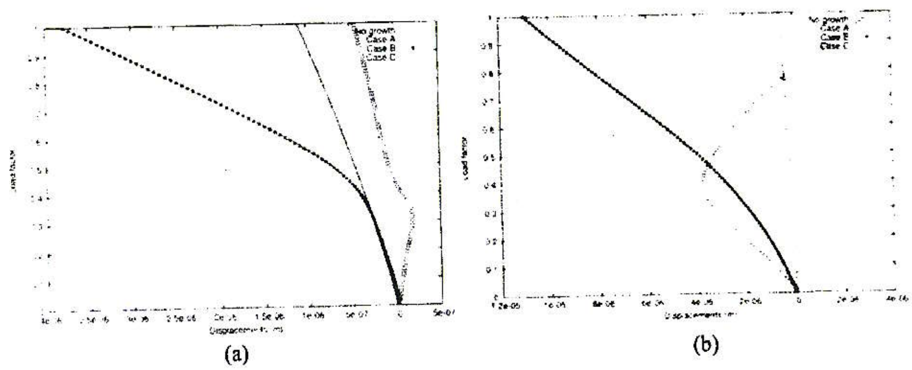

Fig. 4. Load factor versus horizontal displacement, point on the loaded edge (a) and the point on the surface close to the growing cell (b).

The cellular matrix is loaded with horizontal load and the single cells which are growing are introduced into. We may notice that the symmetry of the displacement pattern of the homogeneous cellular structure (Fig 1a) is slightly skewed. This is because of unsymmetry of the particular cells what makes the equivalent material anisotropic. We may notice that the two close to symmetry modes B and C gives relatively symmetric displacement patterns. The difference is mostly in the intensity of the perturbation of the displacement fields, Figs 1b and 1c. The most irregular pattern is introduced by mode B.
We observe the two characteristic horizontal displacements, Fig. 4. The points are chosen in the middle of the horizontally loaded edge and on the surface close to the growing cell. We may note that the introducing of the growing cell causes irregularities in the path of equilibrium. The less irregular path is in the case when the growing cell does not exist and the most irregular is in the case A.

4. CONCLUSIONS

Analysing all the above we may note that the behaviour of the living tissue is significantly different from the performance of the dead one. The tissue where the divisions do not occur behaves in regular and predictable way. The behaviour of the living tissue where the growing cells appearing is not as regular. The growing cells stand for appearing and disappearing of the inclusions and the dislocations in the material.

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