Y.K.Cheung, J.H.W.Lee & A.Y.T.Leung (eds.) 90 5410 029 X
Computational mechanics – *Proceedings of the Asian Pacific conference on computational mechanics, Hong Kong, 11 – 13 December 1991*
1991, 25 cm, c.1700 pp., 2 vols., Hfl.275 / $150.00 / £85
Computational mechanics is a relatively new discipline which has emerged from the parallel developments of the numerical approximation methods and the high speed modem computers. These developments have revolutionarized the manner in which engineers and scientists approach their problems. Many challenging problems which have never been solved can today be dealt with quantitatively by computational mechanics. The proceedings contains a collection of keynote and invited papers, together with contributions selected for presentation at the conference, covering a wide range of problem areas in solid and fluid mechanics. The papers summarize recent research developments in numerical techniques including semi-analytical, finite element and boundary element methods. They also offer a rich variety of examples of the application of these powerful methods to practical problems: structural analysis, nonlinear dynamics, soil-structure-fluid interaction, geotechnical engineering, crack and fracture, composite materials, mechanical and aeronautical engineering, geohysical, environmental and industrial fluid flows. Editors: Univ. of Hong Kong.

FROM THE SAME PUBLISHER:

Yu.K.Zaretskiy 90 6191 174 5
Viscoplasticity of soils and analyses of structures
GEOTECHNIKA 4 – Selected translations of Russian geotechnical literature
1992, 25 cm, 348 pp.,Hfl.165 / $90.00 / £52
A translation of *Vyazkoplastichnost' gruntov i raschoty sooruzheniy*, Moscow, 1988. A new complex approach to assessment of the stress-strain condition of foundations and earth structures which permits limit-state computations. Soil plasticity and dynamic consolidation are discussed in terms of rheological processes due to static and dynamic loading. Results of experiments on soil strength and deformability under static and dynamic loading are discussed. Algorithms are provided for computations of elastoviscoplastic soil deformations, and results of numerical stress-strain experiments employing those algorithms are analysed for foundations and footings of structures, dykes, dams, and other structures. Topics: Cohesionless soil; Clayey soils; Subsiding soils; Thawing soils; Basic equations & algorithms of engineering design; Prediction of subsidence & stress strain condition of natural foundations of industrial & civil engineering structures; Design of drilled-cast piles; Stress-strain design for earth dam.

Arogyaswamy, R.N.P. 90 5410 208 X
Geotechnical application in civil engineering
1992, 25 cm, 313 pp., Hfl.125 / $70.00 / £39
This book emphasis the need for close and effective cooperation of all relevant geotechnical disciplines. These include engineering geology, engineering geophysics, soil and rock mechanics and civil and mining engineering. Although each of these is a recognised disciplinein its own right, each one depends to a certain extent on the other for effective application to many engineering undertakings. The book discusses competently natural processes, such as weathering, sedimentation, seismotectonics, river and coastal mechanics and the hazards resulting from these. Methods of field and laboratory study of soils and clay minerals, field practices, and the role of geologists in engineering are also examined. The book will serve as a handbook to the site geologist as well as to the site engineer. (No rights India)

Sørum, Geraldine (ed.) 90 5410 025 7
Field measurements in geomechanics – *Proceedings of the 3rd international symposium, Oslo, 9 – 11 September 1991*
1991, 25 cm, 965 pp., 2 vols., Hfl.295 / $160.00 / £90
The 90 papers cover the 3 main themes of the symposium: Instruments and measurement techniques; Data acquisition, processing and interpretation; Use of field measurements as an aid in problem solving. The general high quality of the papers ensures that the reader is quickly engaged by the topics discussed. The number and variety of the papers indicate a high activity around the world within many areas of instrumentation and geomechanics and the papers presented should be an encouragement and challenge to those working within the field. For those not familiar with the advantages of instrumentation the publication will be an inspiration to find out more.

Rakowski, Z. (ed.) 90 5410 039 7
Geomechanics 91 – *Proceedings of an international conference, Hradec/Ostrava, Czechoslovakia, 24 – 26 September 1991*
1992, 25 cm, c.500 pp., Hfl.135 / $75.00 / £43
The proceedings cover two main groups of papers: Stability problems with special attention to numerical methods and the latest achievements in water jet technology with special attention to rock cutting. Theoretical approaches and practical experience are presented parallelly. Significant contributions were done by famous scientists from around the world. This book will serve very good for those interested in non linear geomechanical problems and rapidly expanding water jet technology. 52 papers.

A.S.Balasubramaniam et al. (eds.) 90 5410 019 2
Developments in geotechnical aspects of embankments, excavations and buried structures – *Proceedings of the symposium held in 1988 and 1990 at Bangkok on underground excavations in soils and rocks, including earth pressure theories, buried structures and tunnels, and developments in laboratory and field tests in geotechnical engineering practice, respectively*
1991, 25 cm, 608 pp., Hfl.195 / $99.00 / £62
This volume contains a total of 47 papers contributed by experts on recent developments in geotechnical aspects of embankments, excavations and buried structures. For ease of reference, this volume has been divided into five sections, namely: Laboratory and field tests; Embankments, excavations, slopes and earth retaining structures; Tunnels and buried structures; Soil and rock improvementys; In situ investigation and selected topics. The majority of papers in the first section deal with 1 g and centrifugal model tests, and they refer to a wide range of geotechnical problems. Papers in the second section refer to laboratory model tests and full-scale tests. A wide section of topics on buried pipelines and tunnels are covered in section three. All papers in section four relate mainly to soil improvements.

Balasubramaniam, A.S. et al. (eds.) 90 6191 116 8
Geotechnical aspects of restoration works on infrastructures and monuments – *Proceedings of a symposium, Bangkok, December 1988*
1990, 25 cm, 384 pp., Hfl.125 / $65.00 / £40.50
The 32 papers are classified under 5 sections. The first section on ancient structures & historical monuments contains 8 valuable contributions related to the ancient tomb damage in the valley of the Kings, the Leaning Tower of Pisa, the Venice Lagoon, the ancient city of Kausambi in India, the Mohenjodaro in Pakistan, the Buddhist temples in Thailand, and several other historical monuments in Italy, Egypt, and South Africa. Other sections: Recent structures - Buildings and water tower; Risk assessment & safety of dams; Rehabilitation of dams; Materials & miscellaneous topics.

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Incremental finite element formulation for nonlinear structural design sensitivity analysis

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ABSTRACT: The purpose of the paper is to provide a finite element formulation for path-dependent structural design sensitivity problems of nonlinear systems. The discussion is restricted to multi-degrees of freedom systems with fixed overall configuration. For consistency with the incremental description of equilibrium, the Taylor expansion about the current values of nodal displacements is made for the response functional and state functions of the systems. Both the direct differentiation and adjoint variable methods, as well as a mixed differentiation-adjoint technique are employed to evaluate 1st- and 2nd-order sensitivity increments during the load step. The total sensitivity is obtained by accumulating the incremental sensitivities.

Effectiveness and computational aspects of the procedures are discussed and compared. All the numerical algorithms are shown to be readily implemented in existing finite element codes.

A number of test examples illustrates the paper.

1 INTRODUCTION

Structural design sensitivity (SDS) concerns the relationship between structural response functional design variables. As an extension of the general sensitivity theory, see Frank [1], the development of SDS was initiated by Zienkiewicz and Campbell [2]. Further, the adjoint and direct methods were generalized by Haug, Arora, Choi and Komkov [3]–[5], Ryu et al [6], Yang and Botkin [7] and Raju et al [8]. Dems and Mroz [9,10], Haftka and Mroz [11,12] and Haug [3] presented the variational approaches to 1st- and 2nd-order nonlinear SDS. Using the fuzzy set theory Kleiber [13] estimated nonlinear SDS to imperfections in terms of natural language expressions. Discussions on the finite element (FE) analysis of the deterministic and stochastic SDS were offered by Hien and Kleiber [14]–[16]. Some computer codes have been supported with nonlinear SDS options, see Arora, Cardoso and Haririan [17]–[19] on the basis of incremental formulations. These procedures, however, is not fully incremental and thus not general enough to deal with any nonlinear behaviour. A general incremental formulation for path-dependent problems was proposed by Cardoso, Tsay and Arora [20]–[22] in an analytical way. Such a description in the context of the FE modelling seems to be non-existing in the literature. The purpose of the text is to provide a fully incremental formulation for nonlinear SDS problems.

2 PROBLEM STATEMENT

Consider response of the nonlinear system of N degrees of freedom defined by the functional

$$^{(t+\Delta t)}\mathcal{G} = {}^{(t+\Delta t)}G[\,^{(t+\Delta t)}\mathbf{q}(\mathbf{h}), \mathbf{h}] \tag{1}$$

where $^{(t+\Delta t)}G$ is a given function of its arguments at time $t+\Delta t$, $\mathbf{h} = \{h^e\}$, $e = 1, 2, \ldots, E$, is the vector of design variables, $^{(t+\Delta t)}\mathbf{q}(\mathbf{h}) = \{\,^{(t+\Delta t)}q_\alpha(h^e)\}$, $\alpha = 1, 2, \ldots, N$, is the vector of nodal displacement-type parameters. The displacements satisfy the FE equilibrium equation described in the incremental form as

$$K_{\alpha\beta}[\,^{(t+\Delta t)}\mathbf{q}(\mathbf{h}), \mathbf{h}]\,\Delta q_\beta(\mathbf{h}) = \Delta Q_\alpha(\mathbf{h}) \tag{2}$$

where $K_{\alpha\beta}$ is the system stiffness matrix, and

$$\Delta q_\alpha = {}^{(t+\Delta t)}q_\alpha - {}^{(t)}q_\alpha \tag{3}$$

is the vector of the displacement increments due to the load increments ΔQ_α from t to $t+\Delta t$. Assume that $K_{\alpha\beta}$ and ΔQ_α are S times continuously differentiable with respect to h^e. According to the implicit function theorem, [23,24], ${}^{(t+\Delta t)}q_\alpha$ is then also S continuously differentiable. Further, assume that ${}^{(t+\Delta t)}\mathcal{G}$ has also S continuous variations with respect to h^e and $S+1$ continuous variations with respect to ${}^{(t+\Delta t)}q_\alpha$. The computation up to S variations of ${}^{(t+\Delta t)}\mathcal{G}$ with respect to h^e can then be performed. We shall consider the cases when S is equal to 1 and 2, i.e., evaluate the 1st- and 2nd-order SDS functions, denoted by ${}^{(t+\Delta t)}\mathcal{G}^{\cdot e}$ and ${}^{(t+\Delta t)}\mathcal{G}^{\cdot ef}$, $e,f = 1,\ldots,E$.

Let us express ${}^{(t+\Delta t)}\mathcal{G}$, ${}^{(t+\Delta t)}\mathcal{G}^{\cdot e}$ and ${}^{(t+\Delta t)}\mathcal{G}^{\cdot ef}$ respectively as

$$ {}^{(t+\Delta t)}\mathcal{G} = {}^{(t)}\mathcal{G} + \Delta\mathcal{G} \qquad {}^{(t+\Delta t)}\mathcal{G}^{\cdot e} = {}^{(t)}\mathcal{G}^{\cdot e} + \Delta\mathcal{G}^{\cdot e} \qquad {}^{(t+\Delta t)}\mathcal{G}^{\cdot ef} = {}^{(t)}\mathcal{G}^{\cdot ef} + \Delta\mathcal{G}^{\cdot ef} \tag{4}$$

where $\Delta\mathcal{G}^{\cdot e}$ and $\Delta\mathcal{G}^{\cdot ef}$ denote the so-called 1st- and 2nd-order sensitivity increments. Assume that ${}^{(t)}\mathcal{G}^{\cdot e}$ and ${}^{(t)}\mathcal{G}^{\cdot ef}$ are known, the objective of the incremental formulation for nonlinear SDS analysis is to evaluate $\Delta\mathcal{G}^{\cdot e}$ and $\Delta\mathcal{G}^{\cdot ef}$. (From now on, let us use for simplicity the shortened notation with no the left upper index (t) to indicate all the quantities evaluated at t).

3 FIRST-ORDER SENSITIVITY INCREMENTS

By using the Leibnitz's rule differentiation of Eq. (1) leads to

$$ {}^{(t+\Delta t)}\mathcal{G}^{\cdot e} = {}^{(t+\Delta t)}G^{\cdot e} + {}^{(t+\Delta t)}G_{.\alpha}\,{}^{(t+\Delta t)}q_\alpha^{\cdot e} \tag{5}$$

where $(.)^{\cdot e}$ and $(.)_{.\alpha}$ denote the first partial derivatives with respect to h^e and q_α. The 1st-order Taylor expansion is made for ${}^{(t+\Delta t)}G$ about q_α (evaluated at t) gives

$$ {}^{(t+\Delta t)}G = G + G_{.\alpha}\Delta q_\alpha \tag{6}$$

Substituting the expanded equations for ${}^{(t+\Delta t)}G$ and its derivatives with respect to h^e and q_α into Eq. (5), and noting Eqs. (3),(4)$_2$ and $\partial\,{}^{(t+\Delta t)}G/\partial\Delta q_\alpha = G_{.\alpha}$ the e-th 1st-order sensitivity increment reads

$$\Delta\mathcal{G}^{\cdot e} = \left(G^{\cdot e}_{.\alpha} + G_{.\alpha\beta}q^{\cdot e}_\beta\right)\Delta q_\alpha + \left(G_{.\alpha} + G_{.\alpha\beta}\Delta q_\beta\right)\Delta q^{\cdot e}_\alpha \tag{7}$$

In this equation the only $\Delta q^{\cdot e}_\alpha$ remains to be calculated. This can be done alternatively by direct differentiation method (DDM) or by the adjoint variable method (AVM).

By the use of DDM we take differentiation of Eq. (2) with respect to h^e to get

$$K_{\alpha\beta}\Delta q^{\cdot e}_\beta = \Delta Q^{\cdot e}_\alpha - (K^{\cdot e}_{\alpha\beta} + K_{\alpha\beta.\gamma}q^{\cdot e}_\gamma)\Delta q_\beta \qquad e = 1,2,\ldots,E \tag{8}$$

Having solved Eq. (8) for $\Delta q^{\cdot e}_\alpha$ the expression for $\Delta\mathcal{G}^{\cdot e}$ can be obtained explicitly. It is seen from Eqs. (2),(8) that the number of equations to be solved is equal to $E+1$.

The main idea behind AVM is to define an adjoint vector $\lambda = \{\lambda_\alpha\}$, $\alpha = 1,2,\ldots,N$

$$K_{\alpha\beta}\lambda_\beta = G_{.\alpha} + G_{.\alpha\beta}\Delta q_\beta \tag{9}$$

so that

$$\Delta\mathcal{G}^{\cdot e} = \Delta Q^{\cdot e}_\alpha\lambda_\alpha + \left[G^{\cdot e}_{.\alpha} + G_{.\alpha\beta}q^{\cdot e}_\beta - (K^{\cdot e}_{\beta\alpha} + K_{\beta\alpha.\gamma}q^{\cdot e}_\gamma)\lambda_\beta\right]\Delta q_\alpha \tag{10}$$

Noting that the right-hand side of Eq. (9) is the 1st-order expansion about q_α of ${}^{(t+\Delta t)}G_{.\alpha}$ the incremental formulation gives the adjoint equation of identical form to that from the other formulations.

4 SECOND-ORDER SENSITIVITY INCREMENTS

From Eq. (5) the second derivatives of ${}^{(t+\Delta t)}\mathcal{G}$ with respect to h^e can be expressed as

$$\begin{aligned} {}^{(t+\Delta t)}\mathcal{G}^{\cdot ef} &= \left({}^{(t+\Delta t)}G^{\cdot e} + {}^{(t+\Delta t)}G_{.\alpha}\,{}^{(t+\Delta t)}q^{\cdot e}_\alpha\right)^{\cdot f} \\ &= {}^{(t+\Delta t)}G^{\cdot ef} + {}^{(t+\Delta t)}G^{\cdot e}_{.\alpha}\,{}^{(t+\Delta t)}q^{\cdot f}_\alpha + {}^{(t+\Delta t)}G^{\cdot f}_{.\alpha}\,{}^{(t+\Delta t)}q^{\cdot e}_\alpha \\ &\quad + {}^{(t+\Delta t)}G_{.\alpha\beta}\,{}^{(t+\Delta t)}q^{\cdot f}_\beta\,{}^{(t+\Delta t)}q^{\cdot e}_\alpha + {}^{(t+\Delta t)}G_{.\alpha}\,{}^{(t+\Delta t)}q^{\cdot ef}_\alpha \end{aligned} \tag{11}$$

Employing in Eq. (11) the expansion (6) for ${}^{(t+\Delta t)}G$ and for its first and second derivatives with respect to h^e and q_α, and noting Eqs. (3),(4)$_3$ we arrive at

$$\begin{aligned} \Delta\mathcal{G}^{\cdot ef} &= \left(G^{\cdot ef}_{.\alpha} + G^{\cdot e}_{.\beta\alpha}q^{\cdot f}_\beta + G^{\cdot f}_{.\beta\alpha}q^{\cdot e}_\beta + G_{.\beta\gamma\alpha}q^{\cdot e}_\beta q^{\cdot f}_\gamma + G_{.\beta\alpha}q^{\cdot ef}_\beta\right)\Delta q_\alpha \\ &+ \left[G^{\cdot e}_{.\alpha} + G_{.\beta\alpha}q^{\cdot e}_\beta + \left(G_{.\alpha\beta} + G_{.\gamma\alpha\beta}q^{\cdot e}_\gamma\right)\Delta q_\beta\right]\Delta q^{\cdot f}_\alpha \\ &+ \left[G^{\cdot f}_{.\alpha} + G_{.\alpha\beta}q^{\cdot f}_\beta + \left(G_{.\alpha\beta} + G_{.\alpha\gamma\beta}q^{\cdot f}_\gamma\right)\Delta q_\beta\right]\Delta q^{\cdot e}_\alpha \\ &+ \left(G_{.\alpha\beta} + G_{.\alpha\beta\gamma}\Delta q_\gamma\right)\Delta q^{\cdot f}_\beta\Delta q^{\cdot e}_\alpha + \left(G_{.\alpha} + G_{.\alpha\beta}\Delta q_\beta\right)\Delta q^{\cdot ef}_\alpha \end{aligned} \tag{12}$$

In Eq. (12) the only $\Delta q^{\cdot e}_\alpha$ and $\Delta q^{\cdot ef}_\alpha$ remain to be evaluated. This can be done by DDM or AVM, or by a mixed differentiation-adjoint scheme.

4.1 *Direct Differentiation Technique*

By differentiating Eq. (8) with respect to h^e to obtain

$$\begin{aligned} K_{\alpha\beta}\Delta q^{\cdot ef}_\beta &= \Delta Q^{\cdot ef}_\alpha - \left[K^{\cdot ef}_{\alpha\beta} + K^{\cdot e}_{\alpha\beta.\gamma}q^{\cdot f}_\gamma + \left(K^{\cdot f}_{\alpha\beta.\gamma} + K_{\alpha\beta.\gamma\zeta}q^{\cdot f}_\zeta\right)q^{\cdot e}_\beta + K_{\alpha\beta.\gamma}q^{\cdot ef}_\gamma\right]\Delta q_\beta \\ &+ \left(K^{\cdot e}_{\alpha\beta} + K_{\alpha\beta.\gamma}q^{\cdot e}_\gamma\right)\Delta q^{\cdot f}_\beta - \left(K^{\cdot f}_{\alpha\beta} + K_{\alpha\beta.\gamma}q^{\cdot f}_\gamma\right)\Delta q^{\cdot e}_\beta \qquad e,f = 1,2,\ldots,E \end{aligned} \tag{13}$$

Eqs. (2),(8),(13) and Eq. (12) provide the complete solution. Since Eq. (13) is symmetric with respect to e and f this problem requires $E(E+3)/2+1$ equations to be solved.

4.2 *Adjoint Variable Technique*

Differentiating Eq. (10) with respect to h^e yields

$$\begin{aligned} \Delta\mathcal{G}^{\cdot ef} &= \Delta Q^{\cdot ef}_\alpha\lambda_\alpha + \Big\{G^{\cdot ef}_{.\alpha} + G^{\cdot e}_{.\alpha\beta}q^{\cdot f}_\beta + \left(G^{\cdot f}_{.\alpha\beta} + G_{.\alpha\beta\gamma}q^{\cdot f}_\gamma\right)q^{\cdot e}_\beta + G_{.\alpha\beta}q^{\cdot ef}_\beta \\ &- \left[K^{\cdot ef}_{\beta\alpha} + K^{\cdot e}_{\beta\alpha.\gamma}q^{\cdot f}_\gamma + \left(K^{\cdot f}_{\beta\alpha.\gamma} + K_{\beta\alpha.\gamma\zeta}q^{\cdot f}_\zeta\right)q^{\cdot e}_\gamma + K_{\beta\alpha.\gamma}q^{\cdot ef}_\gamma\right]\lambda_\beta\Big\}\Delta q_\alpha \\ &+ \left[G^{\cdot e}_{.\alpha} + G_{.\alpha\beta}q^{\cdot e}_\beta - \left(K^{\cdot e}_{\beta\alpha} + K_{\beta\alpha.\gamma}q^{\cdot e}_\gamma\right)\lambda_\beta\right]\Delta q^{\cdot f}_\alpha + \left[\Delta Q^{\cdot e}_\alpha - \left(K^{\cdot e}_{\alpha\beta} + K_{\alpha\beta.\gamma}q^{\cdot e}_\gamma\right)\Delta q_\beta\right]\lambda^{\cdot f}_\alpha \end{aligned} \tag{14}$$

where $\Delta q^{\cdot f}_\alpha$ and $\lambda^{\cdot f}_\alpha$ remain to be computed. To do this, let us first take differentiation of Eq. (9) with respect to h^e, and then define two sets of E adjoint variable vectors $\vartheta^e = \{\vartheta^e_\alpha\}$ and $\xi^e = \{\xi^e_\alpha\}$, $e = 1,2,\ldots,N$, as the solution of the equations

$$\begin{aligned} K_{\alpha\beta}\vartheta^e_\beta &= \Delta Q^{\cdot e}_\alpha - \left(K^{\cdot e}_{\alpha\beta} + K_{\alpha\beta.\gamma}q^{\cdot e}_\gamma\right)\Delta q_\beta \qquad e = 1,2,\ldots,E \\ K_{\alpha\beta}\xi^e_\beta &= G^{\cdot e}_{.\alpha} + G_{.\alpha\beta}q^{\cdot e}_\beta - \left(K^{\cdot e}_{\alpha\beta} + K_{\alpha\beta.\gamma}q^{\cdot e}_\gamma\right)\lambda_\alpha + G_{.\alpha\beta}\vartheta^e_\alpha \end{aligned} \tag{15}$$

Having solved $2(E+1)$ equations. (2),(9),(15) for λ_α, ϑ^e_α and ξ^e_α we arrive at

$$\begin{aligned} \Delta\mathcal{G}^{\cdot ef} &= \Delta Q^{\cdot ef}_\alpha\lambda_\alpha + \Big\{G^{\cdot ef}_{.\alpha} + G^{\cdot e}_{.\alpha\beta}q^{\cdot f}_\beta + \left(G^{\cdot f}_{.\alpha\beta} + G_{.\alpha\beta\gamma}q^{\cdot f}_\gamma\right)q^{\cdot e}_\beta + G_{.\alpha\beta}q^{\cdot ef}_\beta \\ &- \left[K^{\cdot ef}_{\beta\alpha} + K^{\cdot e}_{\beta\alpha.\gamma}q^{\cdot f}_\gamma + \left(K^{\cdot f}_{\beta\alpha.\gamma} + K_{\beta\alpha.\gamma\zeta}q^{\cdot f}_\zeta\right)q^{\cdot e}_\gamma + K_{\beta\alpha.\gamma}q^{\cdot ef}_\gamma\right]\lambda_\beta\Big\}\Delta q_\alpha \\ &+ \left[G^{\cdot f}_{.\alpha} + G_{.\alpha\beta}q^{\cdot f}_\beta + \left(G^{\cdot f}_{.\alpha\beta} + G_{.\alpha\beta\gamma}q^{\cdot f}_\gamma\right)\Delta q_\beta - \left(K^{\cdot f}_{\alpha\beta} + K_{\alpha\beta.\gamma}q^{\cdot f}_\gamma\right)\lambda_\beta\right]\vartheta^e_\alpha \\ &+ \left[\Delta Q^{\cdot e}_\alpha - \left(K^{\cdot e}_{\alpha\beta} + K_{\alpha\beta.\gamma}q^{\cdot e}_\gamma\right)\Delta q_\beta\right]\xi^f_\alpha \end{aligned} \tag{16}$$

4.3 *Mixed Differentiation-Adjoint Technique*

Observing that the last term on the right-hand side of Eq. (12) is the product of the first adjoint load (Eq. (9)) multiplied by the 2nd-order sensitivity of the displacement increments (Eq. (13)) we arrive at

$$\begin{aligned} \Delta \mathcal{G}^{\cdot ef} = & \left(G^{\cdot ef}_{\cdot\alpha} + G^{\cdot e}_{\cdot\beta\alpha} q^{\cdot f}_{\beta} + G^{\cdot f}_{\cdot\beta\alpha} q^{\cdot e}_{\beta} + G_{\cdot\beta\gamma\alpha} q^{\cdot e}_{\beta} q^{\cdot f}_{\gamma} + G_{\cdot\beta\alpha} q^{\cdot ef}_{\beta} \right) \Delta q_{\alpha} \\ + & \left[G^{\cdot e}_{\cdot\alpha} + G_{\cdot\beta\alpha} q^{\cdot e}_{\beta} + \left(G_{\cdot\alpha\beta} + G_{\cdot\gamma\alpha\beta} q^{\cdot e}_{\gamma} \right) \Delta q_{\beta} \right] \Delta q^{\cdot f}_{\alpha} \\ + & \left[G^{\cdot f}_{\cdot\alpha} + G_{\cdot\alpha\beta} q^{\cdot f}_{\beta} + \left(G_{\cdot\alpha\beta} + G_{\cdot\alpha\gamma\beta} q^{\cdot f}_{\gamma\cdot} \right) \Delta q_{\beta} \right] \Delta q^{\cdot e}_{\alpha} \\ + & \left(G_{\cdot\alpha\beta} + G_{\cdot\alpha\beta\gamma} \Delta q_{\gamma} \right) \Delta q^{\cdot f}_{\beta} \Delta q^{\cdot e}_{\alpha} \\ + & \left\{ \Delta Q^{\cdot ef}_{\alpha} - \left[K^{\cdot ef}_{\alpha\beta} + K^{\cdot e}_{\alpha\beta\cdot\gamma} q^{\cdot f}_{\gamma} + \left(K^{\cdot f}_{\alpha\beta\cdot\gamma} + K_{\alpha\beta\cdot\gamma\zeta} q^{\cdot f}_{\zeta} \right) q^{\cdot e}_{\gamma} + K_{\alpha\beta\cdot\gamma} q^{\cdot ef}_{\gamma} \right] \Delta q_{\beta} \right. \\ + & \left. \left(K^{\cdot e}_{\alpha\beta} + K_{\alpha\beta\cdot\gamma} q^{\cdot e}_{\gamma} \right) \Delta q^{\cdot f}_{\beta} - \left(K^{\cdot f}_{\alpha\beta} + K_{\alpha\beta\cdot\gamma} q^{\cdot f}_{\gamma} \right) \Delta q^{\cdot e}_{\beta} \right\} \lambda_{\alpha} \end{aligned} \qquad (17)$$

Eqs. (2),(8),(9) and Eq. (17) give the complete solution. By using the mixed differentiation-adjoint method the total number of solutions reduces to $E+2$.

5 NUMERICAL RESULTS

To illustrate the above formulation we shall consider below the 1st-order sensitivity response of some geometrically nonlinear systems. The response functional is assumed as the displacement limits at various degrees of freedom, $\mathcal{G} = \mid q_\alpha \mid / q^{(A)}_\alpha - 1 \leq 0$, where $q^{(A)}_\alpha$ is an admissible value; and the member cross-sectional areas are defined as design variables.

Let us first consider two-bar truss given in Fig. 1, where $h = 34.202$, $l = 93.969$; Young modulus

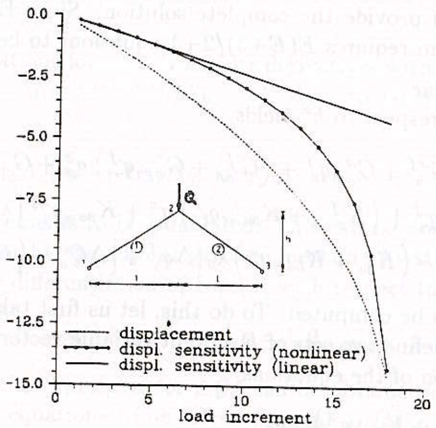

Fig. 1

$E = 50$, cross-sectional area $A = 1.0$, $Q = 0.9$. The problem was solved adopting 18 load increments. The functional is defined at node 2 with $q^{(A)}_z = 1$. The z-displacement at node 2 and the sensitivity to the variation of each cross-sectional area are shown in Fig. 1, when compared against that of the linear system. The results may be easily verified in an analytical way.

Example 2 deals with the 25-member truss. Its geometry is given in Fig. 2. The following data are adopted: $E = 6.9 \times 10^8$, $A = 3.225 \times 10^{-4}$, $Q = \{Q_x, Q_y, Q_z\} = \{-19.2, -192.0, -24000.0\}$. The functional is defined at node 3 with $q^{(A)}_z = 1.0$. The problem is considered in two cases: (i) structure with ideal geometry and (ii) structure is slightly twisted by moving node 3 by 0.1

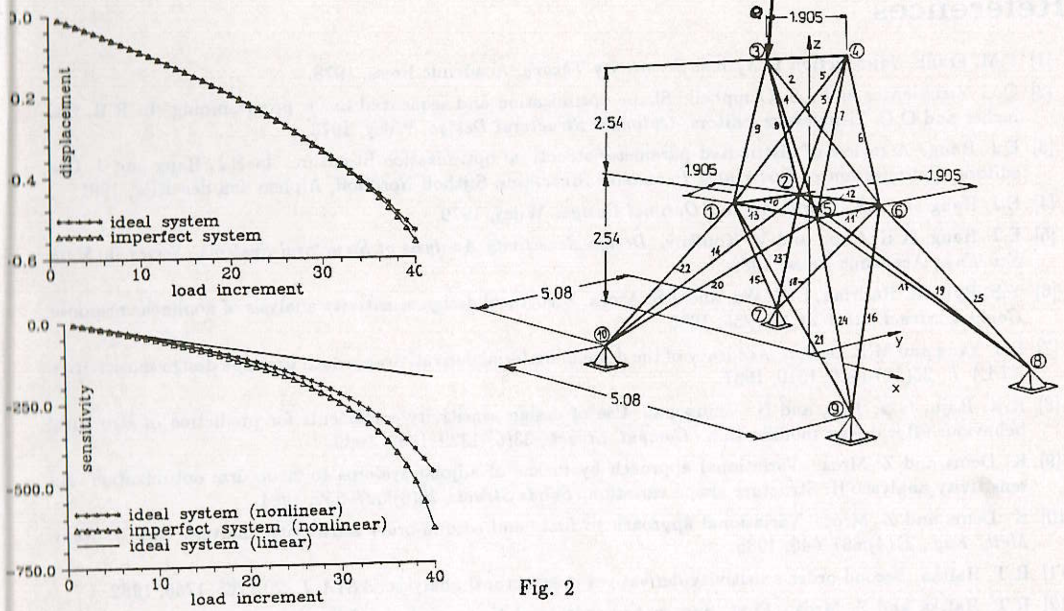

Fig. 2

and node 4 by −0.1 along x-direction. The computed result obtained for 40 load increments is shown in Fig. 2. There is kept attention to response of the z-displacement at node 3 for ideal and imperfect systems and to the rate of the sensitivity to the change of cross sectional area of the cross-bar 4.

6 CONCLUDING REMARKS

The fully incremental formulation for 1st- and 2nd-order nonlinear SDS problems seems to have advantages over other approaches which require finding of internal nodal forces. It may be directly developed for non-shape nonlinear SDS with no additional difficulties. The procedures developed can readily be adapted to existing FE-based codes and would offer a cost-effective alternative in nonlinear SDS analysis.

For the case of the 1st-order SDS effectiveness of DDM and AVM depends on the total number of design variables, of active constraints and of load conditions in the problem on hand. In the 2nd-order SDS analysis AVM is generally more efficient than DDM; the mixed differentiation-adjoint technique far exceeds the both AVM and DDM but requires some sophistication in computer implementation resolution.

In contrast to the primary system the 1st- and 2nd-order equations of DDM and the adjoint equations in AVM are always linear. This enables one to obtain the solution of these equations simply by a typical forward-backward substitution procedure.

In dependence of the FE model employed the derivatives with respect to design variables can be implemented in explicit way by direct differentiation, or in implicit way by a finite difference technique or by the least square fit method. The derivatives with respect to displacements would be preferably evaluated by an implicit technique since the stiffness matrix is generally an implicit function of displacements. It should be pointed out that on contrary to the stiffness matrix almost all the entries derivative matrices are zero; in fact, their computation as well as calculations of sensitivity coefficients can be carried out at the element level.

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