JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
44, 3, pp. 455-484, Warsaw 2006

MODELLING OF DAMAGE AND FRACTURE IN CERAMIC MATRIX COMPOSITES – AN OVERVIEW[1]

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This is a review paper on the existing approaches to modelling of discrete cracks (fracture) and diffuse microcracking (damage) in ceramic matrix composites under mechanical or thermal loading. The focus is on Ceramic Matrix Composites (CMC) with metal particle inclusions and on interpenetrating metal ceramic networks. The second phase in form of ceramic inclusions is not considered. The models of toughening mechanisms are discussed in considerable detail. Sections 2-5 deal with discrete cracks while Sections 6-9 with diffuse microcracking. The paper is concluded with identification of unresolved problems and topics for future research in the area of fracture and damage of CMC.

Key words: ceramic matrix composites, particles, interpenetrating networks, fracture, damage, toughening mechanisms, bridging, cracks, microcracks, cavitation, debonding

1. Preliminaries

The lack of ductility and the consequent low resistance to crack propagation of brittle ceramics have been an issue limiting their technological application for a long time. Much research effort has been devoted to enhance the fracture toughness of made materials. Apart from transformation toughening (e.g. ZrO_2), whisker, platelet or ceramic fibre reinforcement (e.g. SiC/Al_2O_3, or Al_2O_3/Al_2O_3) and microcrack shielding (e.g. Al_2O_3/ZrO_2), ductile particle

[1]Extended version of the state-of-the-art study performed within the EU project "Knowledge-based Multicomponent Materials for Durable and Safe Performance" (KMM-NoE, NMP3-CT-2004-502243)

toughening and toughening via metal infiltration into a ceramic matrix are the most common mechanisms bringing additional toughness to ceramics. Some other effects contributing to the enhanced toughness of Ceramic Matrix Composites (CMC) are crack trapping by the ductile phase and crack deflection. For example, by dispersing partly oxidized aluminium particles in a glass matrix, a sixtyfold increase in toughness was observed. By infiltrating Ni_3Al intermetallic phase into a porous preform of Al_2O_3, the fracture toughness of $9.2\,\mathrm{MPa}\sqrt{\mathrm{m}}$ was achieved, which is much enhanced when compared with monolithic aluminium oxide.

As for damage in CMC, the heterogeneity of such composites leads to diverse damage modes under thermomechanical loading. In contrast to monolithic ceramics, particulate CMC may reveal different types of microcracking such as matrix microcracking, interfacial debonding or intra-particle breakage (Krstic, 1983). While stability and growth of a well-developed macrocrack are treated by the fracture mechanics, the nucleation and evolution of diffuse microcracks and their effect on the material response is typically a subject of the damage mechanics. Somewhat arbitrarily, damage models may be arranged in two subgroups: continuum damage mechanics and micromechanical damage models.

The Continuum Damage Mechanics (CDM) models are phenomenological models that relate nonlinear macroscopic material behaviour to the process of internal microcracking. A majority of the CDM models start by introducing the damage variable of either scalar, vector, or tensor type (Lemaitre and Chaboche, 1985; Krajcinovic, 1996). Since damage is to be modeled as a process, the damage variable has to be endowed with an evolution law. It should be stressed that early CDM models were plagued by too many unidentifiable constants or even led to contradictory predictions. It was a direct consequence of ignoring the underlying micromechanisms of damage. The accomplishments and deficiencies of phenomenological CDM models are discussed at length in Krajcinovic (1996).

The outburst of micromechanical damage models in the past two decades (e.g. Krajcinovic, 1996; Mura, 1987; Nemat–Nasser and Hori, 1999; Kachanov, 1993; Aboudi, 1991; Basista, 2001) can, to some extent, be explained as a search for a remedy for the shortcomings of the CDM. The attribute "micromechanical" is commonly attached to the class of models and macroresponse of a material to its microstructure. As such, these models span two different scales, one of which is typically inhomogeneous (microscale) while the other (macroscale) is, for computational expedience, approximated by a homogeneous effective continuum. An unquestionable merit of micromechanical damage models resides in their ability to explain the physics of damage processes with minimum ambiguity and arbitrariness. Constants appearing in micromechanical models have clear physical interpretations. This is not to say, though, that their numerical values can always be measured using available experimental techniques.

2. Crack bridging models in particle reinforced CMC

In ductile particle reinforced CMC, there are several physical scenarios for crack extension: (1) the crack may avoid the particles and propagate in the matrix only, (2) stress concentration can induce plastic deformation of the particles as well as partial or complete debonding at the interfaces, (3) plastically deforming particles may form a bridging zone and the crack be advancing by failure of the stretched particles. Even a superficial perusal of the existing literature on CMC fracture leads to the conclusion that the crack bridging model is the most often used one when predicting the fracture toughness and strength of CMC with particulate second phase. This model was first introduced by Krstic (1983) assuming that once the crack has reached the crack-matrix interface it will be locally blunted and forced to circumvent the particle leaving the crack faces pinned together by ductile ligaments at some distance behind the advancing crack tip.

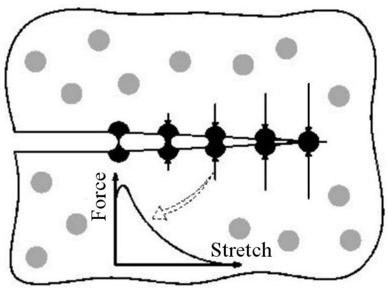

Fig. 1. Crack bridging by ductile particles

The physical mechanism of toughening is rather straightforward (Fig. 1). The ductile particles that span the advancing crack stretch as the crack opens until they fracture or decohere. The work of stretching contributes to the overall toughness of the composite. On the other hand, the bridging ligaments exert closing forces which reduce the stress intensity factor at the crack tip. The size of the bridged zone and the amount of toughening depend on the fracture strength of spanning particles and the plastic deformations they may undergo before they fail (Budiansky *et al.*, 1988).

For the toughening by crack bridging to be effective it is necessary that the following conditions be met:

- the elastic stiffness of the particle must be less than that of the matrix for the crack to be attracted by the particle; otherwise the crack will avoid the particle and grow only in the matrix
- there must be a sufficient number of particles for the toughening effect to occur
- there must be satisfactory bonding between the matrix and the particle to utilize the ductility of the particles in the toughening process.

As pointed out in Ashby *et al.* (1989), the force-displacement curve for a bonded (constrained) particle is quite different than that for an unconstrained material as measured in an ordinary tensile test. The degree of constraint is an important factor affecting the amount of energy absorbed in stretching, thus the fracture toughness. A series of experiments on glass reinforced with lead revealed typical failure modes for ductile inclusions in the brittle matrix as shown in Fig. 2.

In the crack bridging model the key point is to determine the bridging traction law $\sigma(u)$ relating the nominal stress supported across the bridging ligament and the ligament stretch u. Once the $\sigma(u)$ is determined, the toughness enhancement may be computed as

$$\Delta G_c = f \int\limits_0^{u^*} \sigma(u)\, du \tag{2.1}$$

where f is the area fraction of ductile particles intercepted by the crack, u^* is the crack opening at failure ($\sigma = 0$). The $\sigma(u)$ function was postulated in constant or linear form (Bannister *et al.*, 1992; Bao and Zok, 1993) or as a bi-linear function (Budiansky *et al.*, 1988). In more in-depth models it was estimated using the Bridgman plasticity solution for a cylindrically necked bar, the slip line solution, and the finite element solution (Sigl *et al.*, 1988). These solutions complement each other. The finite element calculations are restricted to early stages with small geometry changes of the particle. The Bridgman and the slip line solutions are pertinent to the advanced stages of the deformation process when a stretch zone in the particle is formed.

In Evans and McMeeking (1986), Mataga (1989) and Sigl *et al.* (1988), perfect bonding was assumed in analyses of plastic stretching of ductile particles

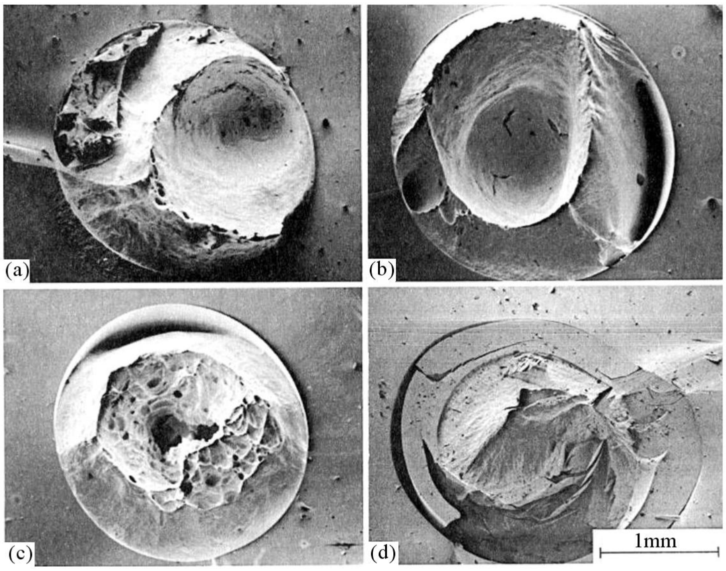

Fig. 2. Failure modes of lead inclusions in glass matrix: (a) single void, (b) decohesion and internal void, (c) decohesion and multiple voiding, (d) matrix cracking (reprinted from *Acta Metall.*, **37**, Ashby M.F. *et al.* (1989), with permission from Elsevier)

in a ceramic matrix. The perfect bonding imposes lateral constraints on the inclusions prohibiting the full advantage of the particle's ductility. In contrast, the interaction of partial debonding with plastic stretching leads to a larger crack opening in the bridging zone and is closer to the experimental findings than the perfect bonding predictions. By modifying the neck geometry used by Sigl *et al.* (1988), Bao and Hui (1990) proposed a coupled debonding-stretching model on the assumption of particle incompressibility.

To appreciate the effect of debonding on the fracture toughness enhancement, we shall outline the models promoted in Sigl *et al.* (1988) and Bao and Hui (1990). These two models are based on the Bridgman solution making the analyses tractable.

Consider the blunting geometry shown in Fig. 3. The objective, as set in Sigl *et al.* (1988) and Bao and Hui (1990) was to find the explicit expression $\sigma(u)$ for the integrand in Eq. (2.1). The Bridgman result for the mean axial stress in the necked region reads

$$\overline{\sigma}_z = \sigma_f\Big(1 + \frac{2R}{a}\Big)\ln\Big(1 + \frac{a}{2R}\Big) \tag{2.2}$$

where R is the radius of neck curvature, a the neck radius, σ_f the uniaxial yield stress. According to Sigl *et al.* (1988), the mean axial stress $\overline{\sigma}_z$ and the nominal stress σ in Eq. (2.1) are interrelated as follows

$$\sigma = \overline{\sigma}_z\Big(\frac{a}{a_0}\Big)^2 \tag{2.3}$$

with a_0 being the initial particle radius. The ductile particle is assumed to behave as a power law hardening material. Hence, the stress σ and the plastic tensile strain ε are related as

$$\frac{\sigma}{\sigma_0} = \Big(\frac{\varepsilon}{\varepsilon_0}\Big)^n \qquad \text{and additionally} \qquad \varepsilon = 2\ln\frac{a_0}{a} \tag{2.4}$$

where σ_0 and ε_0 are the initial yield stress and strain. If volume constancy of the ductile particle is assumed and the matrix is allowed to undergo only rigid displacement (no elastic deformations admitted) then it can be shown (Sigl *et al.*, 1988) that the volume removed from the voids (V_1) may be added to the neck (V_2), where obviously $V_2 = V_1$.

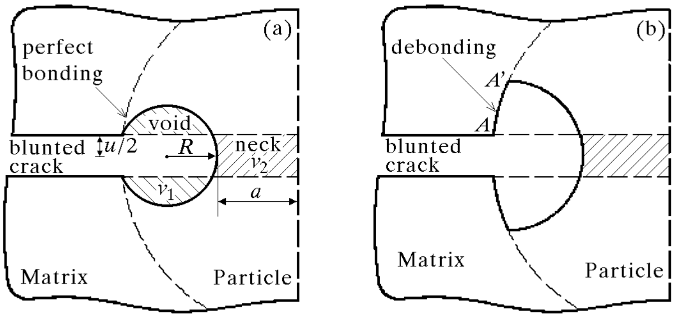

Fig. 3. Neck geometry in crack bridging: (a) perfect bonding, based on Sigl *et al.* (1988), (b) debonding with stretching, based on Bao and Hui (1990)

From the geometry of the angular void crack it follows that the crack opening and the void radius are interrelated as:
— for perfect bonding

$$\frac{u}{a_0} \approx 2\pi\Big(\frac{R}{a_0}\Big)^2 \tag{2.5}$$

— for partial debonding

$$\frac{u}{a_0} \approx \Big(\frac{R}{a_0}\Big)^2\Big(\pi - \frac{4R}{3a_0}\Big) \tag{2.6}$$

Combining Eqs (2.3), (2.4) and (2.5) or (2.6), it was possible to solve the system explicitly for $\sigma(u)$ and then integrate Eq. (2.1) to get the toughness enhancement ΔG_c. Details of the derivation are provided in the original papers by Sigl *et al.* (1988) and Bao and Hui (1990). As the comparison in Bao and Hui (1990) shows, the effect of debonding increases the fracture toughness by ca. 80%.

In addition to ΔG_c, other fracture parameters sought in Sigl *et al.* (1988), Bao and Zok (1993), Bannister *et al.* (1992), Ashby *et al.* (1989), Mataga (1988) included the length of the bridged zone L and the crack opening at failure u^*. In Bao and Zok (1993), a continuous model based on the micromechanics of plastically deformed particles and the small-scale bridging approximation were proposed to determine the length of the bridging zone. In Sigl *et al.* (1988), these parameters were verified experimentally for two different composites: Al_2O_3/Al and WC/Co. Only the bridged zone length was reproduced satisfactorily while the crack opening at failure and the toughness increase were appreciably less than their experimentally measured counterparts. The reasons for these discrepancies were attributed to inadequate modelling of the bridging traction law $\sigma(u)$.

The approximate analysis of debonding based on circular neck shapes and the Bridgman solution for rigid-plastic materials proposed in Bao and Ui (1990) was further advanced in Tvergaard (1992) by considering large elastic-plastic deformations and using a cohesive zone model to represent the debonding. A rather important result of the numerical analysis in Tvergaard (1992) was that the predicted values of ΔG_c were much higher that those obtained using the Bridgman rigid-plastic solution for the stress in the neck.

Apart from the models employing the plasticity-based bridging traction laws, a distribution of nonlinear springs was also proposed to mimic the bridging action (Budiansky *et al.*, 1988). However, as admitted by the authors themselves "*...the present theory seems to imply suspiciously high particle strengths. If such strengths are not confirmed, toughening mechanisms in additions to crack bridging may be operative in particulate-reinforced ceramics*" (Budiansky *et al.*, 1988).

In the papers reviewed so far, the modelling of particle bridging consisted of the following main steps: the system of discretely distributed particles was replaced by the action of smeared forces; the crack opening constraining forces were modelled via a specific bridging traction law (stress-crack opening displacement relationship) obtained for a single particle bridge; the resulting nonlinear integral equation was solved numerically. Additionally, it was assumed that the material outside the bridging zone be homogeneous and non-deforming or linear elastic with the properties obtainable from the rule of mixture (matrix with lower stiffness inclusions) or using the effective media techniques.

Rubinstein and Wang (1998) proposed a comprehensive two-dimensional model accounting for discrete particle distribution and particle-matrix interface properties. No approximation of the bridging traction law was introduced in their model. What is also important, the interface properties were directly incorporated into the model. The particles were assumed elastic-ideally plastic with their volumes remaining constant during plastic deformation. The plastic deformation was modelled using a plastic zone reminiscent of the Dugdale-Barenblatt model. The basic methodology applied in Rubinstein and Wang (1998) was that of the Muskhelishvili stress potentials – hence the confinement of the analysis to a 2D case. From the stress potential, the corresponding crack opening displacement was obtained for the bridging region. Imposing plastic incompressibility, the shapes of bridging cross sections of initially spherical particles were obtained. Additionally, a parabolic shape of the neck was assumed. A specific dimensionless parameter A was introduced to specify the curvature of the selected parabolic profile. The physical meaning of A is that it characterizes the strength of the inclusion-matrix interface; $A = 0$ corresponds to a weak, while $A = 50$ to a strong interface (Fig. 4).

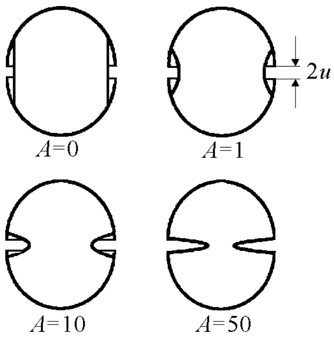

Fig. 4. Neck deformation profiles for different values of interface strength parameter A (reprinted from *J. Mech. Phys. Solids*, **46**, Rubinstein A.A. and Wang P. (1998) with permission from Elsevier)

Several interesting conclusions were drawn in Rubinstein and Wang (1998). For example, it turned out that the fracture toughness of CMC with a weak interface may be significantly higher that those with strong interfaces – this effect was earlier experimentally observed by other authors. The optimal property for the interface seems to be that of $A = 0$. Negative values of A correspond to such weak interfaces that the growing crack may bypass the reinforcing particles.

The interaction of the bridged main crack with microcracks was studied in Kotoul and Profant (2000) who used smeared Bridgman solution in the bridged zone. The SIFs and the COD were estimated for the bridged crack interacting with an arbitrarily located single microcrack. The obtained values of SIFs and COD were further used to determine the effective fracture toughness of the composite. When compared with the available experimental data, the obtained theoretical results differed only by 0.5%-3.5% depending on the matrix vs. particle stiffness.

3. Thermal crack problems in CMC

The CMC are fabricated at elevated temperatures up to 1300°C. When cooled down to the ambient temperature, residual thermal stresses may develop due to the mismatch in thermal expansion coefficients between the ceramic matrix and the metallic reinforcement. These residual stresses are capable of triggering crack propagation at the matrix-particle interface and are thus of vital importance in the material design and various applications employing CMC like engines and turbines. Metal particle reinforced CMC are promising candidates for future high temperature applications due to superior fracture toughness over monolithic ceramics and excellent high temperature strength properties when compared with metal alloys.

Hsueh and Becher (1996) analyzed residual thermal stresses in CMC reinforced with ellipsoidal inclusions. The thermal expansion coefficient of the ceramic matrix was assumed isotropic whereas that of the inclusion was assumed transversely isotropic to be closer to a real situation. The Eshelby solution for an inhomogeneous inclusion was the starting point. However, to account for the finite volume fraction of particles, the Mori-Tanaka method was finally employed to compute stress components and the average stress inside the inclusion and the average stress in the matrix. Closed form analytical solutions were derived for specific inclusion shapes of discs, spheres and fibres, and compared with FEM calculations performed for several engineering composites like SiC whisker-reinforced Al_2O_3. A good agreement was noted for the average residual stress between measured and calculated values when the volume fraction of inclusions was less than 0.3. For higher values of f, though, discrepancies were observed that were traced back to the interaction effects neglected in the analysis.

The residual thermal stresses are necessary to compute the interface fracture toughness. In Kolhe *et al.* (1996), the metal ceramic microstructure of a two-phase composite (Ni/Al_2O_3) was studied experimentally and then the model was developed to analyze the effect of the residual thermal stress on the interfacial cracking upon cooling from the processing to the room temperature. The residual stresses were estimated for spherical and cylindrical inclusions as a function of the inclusion volume fraction. Then the fracture parameters SIFs and G were calculated as a function of the phase angle. The critical particle size was obtained from the classical Griffith criterion. In the case of spherical inclusions, a hydrostatic stress developed inside the inclusion and no plastic deformation thus occurred. Once debonding has occurred, the assumption of hydrostatic tensile stress state inside the inclusion ceased to be valid. However, as the plastic deformation is largely constrained, the assumption of small scale yielding remained valid in accordance with experimental observations.

The crack shielding under thermal shock was modelled in Jin and Batra (1999) as bridging of plastically stretched metal particles. A double-edge cracked strip (Fig. 5) subjected to sudden cooling from temperature T_0 to T_a was considered in a plane strain. The thermal stress intensity factors were sought.

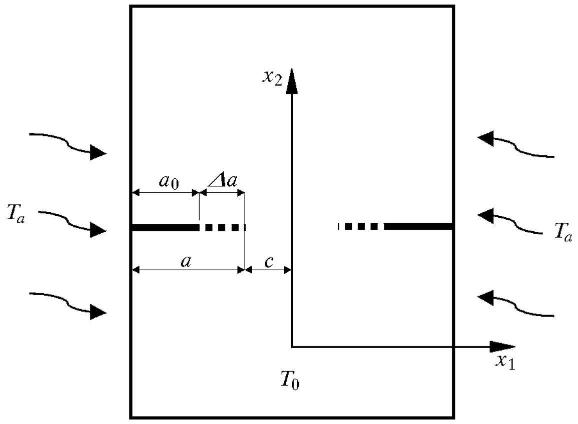

Fig. 5. Edge cracks with bridge zone Δa in infinite strip under thermal shock $\Delta T = T_0 - T_a$

When the cracks are not considered, the problem reduces to a task of the classical nonsteady heat flow in the plane strain. The nonlinear temperature profile within the strip induces the thermal stress in the x_2 direction

$$\sigma_{22}^T = \frac{E_c \alpha_c \Delta T}{1 - \nu_c} \Big[-2 \sum_{n=1}^{\infty} \lambda_n^{-1} (-1)^{n-1} \cos(\lambda_n \widetilde{x}) \exp(-\lambda_n^2 \widetilde{t}) + 2 \sum_{n=1}^{\infty} \lambda_n^{-2} \exp(-\lambda_n^2 \widetilde{t}) \Big] \tag{3.1}$$

where $\widetilde{x} = x_1/b$, $\lambda_n = \pi(n - 1/2)$, $\widetilde{t} = t\kappa_c/b^2$; κ_c, α_c, E_c and ν_c are the conductivity, thermal expansion coefficient, Young's modulus and Poisson's ratio of the composite, respectively. These quantities may be estimated using one of the effective media techniques of micromechanics.

However, for the thermal crack problem with the bridging zone Δa (Fig. 5), in addition to Eq. (3.1) there is also the bridging stress σ in the x_2 direction to be included in the set of boundary conditions. Following previous works of the Santa Barbara group (Mataga, 1989; Sigl *et al.*, 1988), it was assumed in Jin and Batra (1999) that the bridging traction law is of the linear form $\sigma = \sigma_0(1-u/u_0)$, where u is the COD with u_0 being its maximum value when the bridging is lost.

A singular integral equation for the thermal crack problem at hand was derived and solved analytically in Jin and Batra (1999), making it possible to compute the thermal stress intensity factors at the crack tip for different values of the bridge zone size. The toughening effect of particle bridging in the thermal shock problem was clearly demonstrated. Interestingly enough, it was shown that for longer cracks the thermal SIF may be reduced to zero due to the metal particle bridging – a desirable crack arrest effect.

By equating the thermal SIF with the critical value K_{cr}, the crack length increase was determined for the Ni/Al_2O_3 composite vs. that of monolithic Al_2O_3. While the crack growth in monolithic Al_2O_3 turned out to be unstable, the thermal shock damage in Ni/Al_2O_3 proved to be significantly less.

4. Cavitation

Cavitation, or void nucleation and growth, is another fracture mechanism that is encountered in the metal phase of the CMC. It has been a topic of considerable research interest in the mechanics of solids and materials science for more than six decades now (e.g. early papers by *R*. Hill on void instability in the fortieths) due to its role as the initiator of failure mechanisms such as crack growth, crazing and shear band development. There is a vast body of literature on void nucleation and growth in nonlinear elastic solids (e.g. an overview paper by Horgan and Polignone, 1995), rigid-plastic solids (e.g. Tvergaard *et al.*, 1992 and references contained therein), and elastic-plastic solids (e.g. Needleman, 1987; Levy, 1995; Huang *et al.*, 1991). The cavitation has a detrimental effect on ductility and toughness of structural metals and composites, including CMC.

As for the void nucleation, in the case of irregular particles, the mechanism starts with cracking of a particle. For regular equiaxed inclusions, the mechanism of void nucleation is controlled by the inclusion-matrix interface. Strong interfaces separate by growth of initial microdefects located at the interface, while weak interfaces separate by interfacial decohesion. According to Levy (1995), the modelling of cavity formation at particles with spherical symmetry can be grouped according to two nucleation criteria: (1) critical interfacial stress criterion – void nucleation is the critical event at a particle rigidly bonded to a plastically deformed matrix, (2) energy criterion – void nucleates when the total potential energy that would be released in creating the void becomes comparable to the surface energy generated. These two nucleation criteria should be viewed as artificial ones introduced for modelling purposes and are not necessarily sufficient to guarantee the void nucleation. An alternative approach is to treat the cavitation as a process starting with initial separation of an interface and ending with complete decohesion (e.g. Needleman, 1987). In this approach, the void nucleation appears as a natural consequence of the interaction between the linear elastic inclusion, linear elastic matrix and a nonlinear interface. The constitutive relations are specified independently for the matrix, the inclusion, and the interface. As for the interface, the Dugdale-Barenblatt cohesive zone model with account for geometry changes was used within a purely continuum setting in Needleman (1987) and followed later on in Levy (1995). In the constitutive equation for the interface in Needleman (1987), the traction components were related to the interface displacement jumps what necessitated introduction of the characteristic length.

As for the void growth, its volumetric growth rate is proportional to the average strain rate and increases strongly with the increasing stress triaxiality. However, if the void in an elastic-plastic material is subjected to a pure hydrostatic tension exceeding the uniaxial yield stress by several times, then the so-called cavitation instability occurs. According to theoretical predictions, above that critical hydrostatic stress the cavity grows without bound driven by the elastic energy stored in the surrounding material. In the literature (cf. Tvergaard *et al.*, 1992), the cavitation instability has been approached in the framework of nonlinear elasticity either as a bifurcation from a homogeneously stressed material to a material containing a void, or as the growth of a pre-existing void. For the known modelling advantages, the problem of spherical particles and, geometrically analogous, the problem of a circular cylindrical void have been given the most attention by the analysts (Horgan and Polignone, 1995).

As mentioned above, the cavitation instabilities in elastic-plastic materials require stress levels of the order of 5 times and more above the yield limit in the uniaxial case. Such stress levels are not found at sharp notches nor at the tip of blunting cracks. They may be present, though, due to highly constrained plastic flow in ductile particles in brittle matrix composites, as reported by Ashby *et al.* (1989) from a series of experiments on glass reinforced with lead. These experiments have shown several small voids growing inside a ductile particle leading to a dimpled fracture surface (Fig. 2c) or an enormous growth of a single void (Fig. 2a) in favour of the theoretical concept of cavitation instability (Needleman, 1987; Huang *et al.*, 1991). The effect of triaxiality in surrounding materials is such that the high triaxiality promotes multiple voiding while decaying triaxiality reduces the tendency for nucleation of other voids (Tvergaard, 1992).

The axisymmetric cavitation states were thoroughly investigated in Huang *et al.* (1991) for elastic-perfectly plastic solids and for a power hardening elastic-plastic material (Tvergaard *et al.*, 1992) with spherical and cylindrical voids. It was shown in Tvergaard *et al.* (1992) that the critical stress σ_c for the spherical void instability can be evaluated from the following integral

$$\frac{\sigma_c}{\sigma_y} = -\int\limits_0^\infty \frac{1}{\mathrm{e}^{3\varepsilon/2} - 1} f(-\varepsilon)\, d\varepsilon \tag{4.1}$$

In Eq. (4.1), $f(\varepsilon)$ is the uniaxial constitutive law for a power hardening solid specified as follows

$$f(\varepsilon) = \frac{\sigma}{\sigma_y} = \begin{cases} \varepsilon/\varepsilon_y & \text{for elastic range} \\ (\varepsilon/\varepsilon_y)^{1/n} & \text{for plastic range} \end{cases} \tag{4.2}$$

where ε denotes the logarithmic strain, σ the uniaxial true stress, σ_y the initial yield stress, n the hardening coefficient.

As pointed out in Zimmermann *et al.* (2001) and Prielipp *et al.* (1995), the threshold stresses of a sufficient magnitude to trigger the void instability are attained in the metal phase during the processing of CMC at some stage of the cooling process. For example, for aluminium, the void instability critical stress σ_c ranges from $5\sigma_y$ to $10\sigma_y$, while the hardening exponent is 5 or higher depending on the temperature regime (Prielipp *et al.*, 1995). However, void instability in Al_2O_3/Al was not observed, apparently due to different boundary conditions assumed in the theoretical model (Tvergaard *et al.*, 1992), i.e. a sphere embedded in an infinitely extended incompressible elastic-plastic solid under remote stress vs. a sphere embedded in a ceramic matrix under temperature induced stress field (Zimmermann *et al.*, 2001).

5. Fracture in CMC with interpenetrating networks

The Interpenetrating Phase Composites (IPC), also called interpenetrating metal ceramic networks, contain no discrete particles or fibres but consist of completely interconnected networks of solid phases which form almost porosity-free composites. If one of the phases were removed from IPC, the other phase(s) would form an open-celled foam with a non-zero rigidity. For metal ceramic IPC, the metallic phase is usually the one that is filling the cells of a porous ceramic skeleton or preform.

Composites with interpenetrating networks of ceramic and metallic phases constitute a new subclass of CMC which, beginning with the mid nineties, experienced a rapid growth in terms of processing methods and tailoring of mechanical properties. However, a rapid growth was not observed in the modelling of fracture and crack growth in these materials – this particular field seems to be still in an infancy stage.

IPC are designed to benefit from the superior properties of both constituents, i.e. hardness and wear resistance of ceramics as well as fracture toughness of metals. They are produced via different technological methods such as: direct oxidation of metals, metal infiltration into a ceramic preform using external pressure or without external pressure, hot pressing, reactive metal infiltration (cf. Skirl *et al.*, 2001). An example of a low-temperature interpenetrating composite is Al_2O_3/Al whereas that of a high-temperature is Al_2O_3/Ni_3Al. Both composites have been extensively investigated experimentally with regard to the fracture toughness, fracture strength, elastic moduli, and thermal expansion coefficients (e.g. Skirl *et al.*, 2001; Hoffman *et al.*, 1999; Prielipp *et al.*, 1995; Raddatz *et al.*, 1998). For example, at 30% of Ni_3Al content, the Al_2O_3/Ni_3Al composite manifested the strength of 675 MPa and the fracture toughness $K_{Ic} = 9.2\,\mathrm{MPa}\sqrt{\mathrm{m}}$ which exceeds the fracture toughness of monolithic Al_2O_3 by a factor of 4. The gas pressure infiltration of the liquid metallic phase into a porous ceramic preforms is a commonly used fabrication technique. The ceramic preform may form a random porous network of a sintered aluminium oxide or contain hollow parallel channels or regular grids if special processing techniques are applied (Raddatz *et al.*, 1998).

As for toughening processes in the interpenetrating composites, the crack bridging mechanism plays the same central role as for the particulate CMC. However, the very feature of IPC that each phase is spatially continuous has not yet been properly accounted for in the modelling of crack bridging. Usually, the topologically continuous network of ductile ligaments is replaced for modelling purposes by discrete spherical inclusions (e.g. Hoffman *et al.*, 1999) or unidirectional fibres (Raddatz *et al.*, 1998).

Generally speaking, the resistance of a composite to crack growth $R(a)$ can be formally decomposed as in Prielipp *et al.* (1995)

$$R(a) = R_0 + \sum_i R_{\mu i}(a) \tag{5.1}$$

where R_0 is the crack tip resistance, while $R_{\mu i}(a)$ represent the toughness enhancements due to the closure stresses $\sigma_i(u)$ in the ligaments, and are given by

$$R_{\mu i}(a) = f_i \int_0^{u_i^*} \sigma_i(u)\, du \tag{5.2}$$

In Eq. (5.2), u^* is the crack opening in the last active ligament, f_i is the volume fraction of ligaments with the closure stress $\sigma_i(u)$. Assuming the small-scale yielding and neglecting the bridging effect by the ceramic phase, the fracture toughness of the composite, estimated for the crack length when the first ductile ligament fails, can be expressed as

$$K_R = \sqrt{K_0^2 + E_c' R_\mu(a)} \tag{5.3}$$

where K_0 is the fracture toughness at the crack tip and E_c' is the plane strain Young modulus of the composite.

The exact form of the $\sigma_i(u)$ function in Eq. (5.2) is a sensitive point that needs further research. In Prielipp *et al.* (1995), $\sigma_i(u)$ was assumed constant up to the maximum crack opening at which the first ligament fails. The qualitative predictions of fracture toughness (Eqs (5.3), (5.2)) were favourably verified in Prielipp *et al.* (1995) by comparison with the data measured for the Al/Al_2O_3 composite in function of the Al content with the average ligament diameter as a parameter.

In Raddatz *et al.* (1998), aluminium was infiltrated into a ceramic Al_2O_3 preform containing parallel micro-channels, introduced on purpose during processing, so that the ductile phase form a unidirectional family of parallel Al fibres. In terms of modelling, this enabled application of the existing analytical methods (e.g. the weight function method) to compute bridging stresses and crack opening displacements as well as prediction of the R-curves of the composite. However, a question arises whether this is still an interpenetrating metal ceramic composite or rather a particular type of fibre-reinforced CMC?

In conclusion, while reviewing the existing models of the interpenetrating metal ceramic composites, several observations can be made:

- mechanical properties such as the fracture toughness, strength, elastic constants, thermal expansion coefficients depend not only on volume fractions of constituent phases but also on their spatial (anisotropic) distributions
- there is a lack of fracture models for interpenetrating composites accounting for real spatial distributions of constituent phases
- conventional micromechanics methods based on the Eshelby tensor (effective media, effective field techniques) do not seem suitable for determination of effective properties since the interpenetrating phase cannot be extracted as dispersed inclusions (Feng *et al.*, 2003)
- the simple rule of mixture to model thermal expansion coefficients is not applicable (Hoffman *et al.*, 1999)
- image analysis and concept of connectivity could be used for prediction of the effective properties (cf. excellent agreement of predicted effective moduli with test data reported in Feng *et al.*, 2003)

6. Damage in particulate CMC: micromechanical approach

In the modelling of damage processes, the main issue is the derivation of macroscopic constitutive equations and ensuing effective properties of the damaged material. For particulate CMC, the mutual interaction of inclusions and the microcracks is an important effect that should be accounted for in constitutive models. The concept of an RVE is routinely used in such models, and periodic distributions of microcracks are often assumed to simplify the analyses. The microcracks are either assumed as traction-free, or the bridging effect of the reinforcement particles is introduced.

The effective elastic moduli of materials containing defects have been a topic of primary studies for several decades due to their direct relevance to engineering problems. The effective compliance tensor $\overline{\mathbf{S}}$ is necessary to relate the average stress $\overline{\boldsymbol{\sigma}}$ to the average strain $\overline{\boldsymbol{\varepsilon}}$ tensor in constitutive equations for a microcracked material – the very goal of damage models. In particulate CMC, the situation is exacerbated by the fact that reinforcing particles may interact with microcracks and influence the deformation and fracture processes. On the top of that, the microcracks may interact with each other so may the reinforcing particles.

Generally speaking, two approaches to investigate the interactions of different defects can be distinguished in the mechanics of heterogeneous materials:

direct defect-defect interaction methods based on the microscale analysis of local stress fields (e.g. Nemat-Nasser and Hori, 1999; Kachanov, 1993) and approximate effective continuum (or effective field) methods utilizing the Eshelby solution. The effective continua/field techniques comprise the self-consistent method, differential method, three-phase composite model, Mori-Tanaka's theory, and alike (e.g. Mura, 1987; Nemat-Nasser and Hori, 1999; Kachanov, 1993; Aboudi, 1991).

One of the widely spread techniques to estimate the effective properties is the Self-Consistent Method (SCM). There is quite a number of studies on inclusions in matrix materials using the self-consistent mechanics based on the works of R. Hill and B. Budiansky published in the mid sixties. The same can be said on microcracks in matrix materials. However, the approaches to these two categories of problems are quite different. For particulate CMC undergoing matrix microcracking, the two approaches should be combined within the self-consistent framework in order to arrive at the effective moduli. This was done in Huang *et al.* (1993) for spherical inclusions and penny-shaped microcracks. The interactions among the matrix, inclusions and the microcracks were implicitly accounted for via the self-consistency requirement. Random and non-random (parallel) microcrack distributions were considered. Several interesting conclusions were drawn in Huang *et al.* (1993). For example, for stiff inclusions (e.g. MMC) the effect of inclusions and microcracks can be decoupled. It means that the effect of microcracking may be represented through multiplying the accordingly computed effective elastic modulus of the matrix with inclusions by the factor $(1 - 16/9\omega)$, with ω being the Budiansky-O'Connell microcrack density parameter. For compliant inclusions (e.g. CMC with soft particles), this decoupling does not hold. As the volume concentration of inclusions increases, the relation of Young's modulus of the composite $\overline{E}$ versus ω gradually deviates from linearity.

The crack bridging is an important deformation mechanism in CMC with metal particles (cf. Section 2). In the case of multiple microcracks, the bridging effect makes the overall stress-strain relations deviate from linearity (strain hardening), increases the composite fracture toughness and prevents sudden fracture in the very moment of microcracks localisation into a macrocrack. Therefore, it is of primary importance to determine the overall moduli and the nonlinear constitutive equations for brittle matrix composites containing multiple bridged microcracks. This was a theme of investigations in Karihaloo *et al.* (1996) and Wang (2002) using the formalism of pseudo-tractions (cf. Nemat-Nasser and Hori, 1999; Kachanov, 1993). Starting with a simple volume averaging of the strain and stress fields for a damaged body, the equilibrium and stress boundary conditions as well as the Gauss theorem, the overall (average) strain and stress tensors are related by the well-known expression

$$\overline{\varepsilon} = \mathbf{S}^0 : \overline{\boldsymbol{\sigma}} + \frac{1}{2V_{RVE}} \sum_k \int\limits_{A^{(k)}} (\boldsymbol{n} \otimes [\boldsymbol{u}] + [\boldsymbol{u}] \otimes \boldsymbol{n}) \, dA = (\mathbf{S}^0 + \mathbf{H}) : \overline{\boldsymbol{\sigma}} = \overline{\mathbf{S}} : \overline{\boldsymbol{\sigma}} \quad (6.1)$$

where $\mathbf{S}^0$ is the compliance tensor of the virgin matrix material, $[\boldsymbol{u}]$ is the COD vector, $\boldsymbol{n}$ is the unit vector normal to the microcrack surface, $A^{(k)}$ is the surface area of the k-th microcrack. The additional secant compliance tensor due to microcracks $\mathbf{H}$ depends on the microcrack configuration and density. For bridged microcracks, it also depends on the bridging law, and for a nonlinear bridging law it becomes stress dependent $\mathbf{H}(\overline{\boldsymbol{\sigma}})$.

The effectiveness of constitutive equation (6.1) is contingent upon our ability to compute the crack opening displacement $[u_i]$. In Karihaloo *et al.* (1996) and Wang (2002), the method of pseudo-tractions was used to derive the $[u_i]$. For multiple bridged microcracks, the stress-consistency condition on each crack reads

$$\sigma_{ij}^p(\boldsymbol{x}_k) - \sum_{k \neq l} \int\limits_{A^{(k)}} K_{ijmn}(\boldsymbol{x}_l, \boldsymbol{x}_k) \sigma_{mn}^p(\boldsymbol{x}_k) \, d\boldsymbol{x}_k + p_{ij}(\boldsymbol{x}_k) = \sigma_{ij}^0 \quad (6.2)$$

where $\sigma_{ij}^p(\boldsymbol{x}_k)$ is the pseudo-traction on the faces of the k-th microcrack, $p_{ij}(\boldsymbol{x})$ is the bridging stress, σ_{ij}^0 is the homogeneous applied stress, the kernel $K_{ijmn}(\boldsymbol{x}_l, \boldsymbol{x}_k)$ represents the stress at $\boldsymbol{x}_l$ induced by concentrated crack surface tractions of unit intensity at $\boldsymbol{x}_k$. Having calculated the pseudo-tractions from Eq. (6.2), the COD can be obtained as

$$[u_i](\boldsymbol{x}_k) = \int\limits_{A^{(k)}} P_{im}(\boldsymbol{x}_k, \boldsymbol{x}_l) \sigma_{mn}^p(\boldsymbol{x}_l) n_n \, dA_k \quad (6.3)$$

where the kernel $P_{im}(\boldsymbol{x}_k, \boldsymbol{x}_l)$ represents the COD at $\boldsymbol{x}_k$ induced by a set of concentrated unit loads on crack faces at $\boldsymbol{x}_l$ – a standard solution available in fracture mechanics handbooks. As for the bridging law $p_{ij}([u_i])$, some linear and nonlinear forms have been proposed (e.g. Karihaloo *et al.*, 1996; Wang, 2002). Finally, the overall compliances $\mathbf{H}$ are then computed from Eq. (6.1).

The micromechanical modelling of a metal particle reinforced CMC subject to cyclic loading was a topic in Kotoul (2001). The CMC composite consisted of spherical elasto-plastic inclusions and an elastic matrix undergoing microcracking in tensile zones around the inclusions. Using basic constitutive relation (6.1), the concept of the RVE and the modified Mori-Tanaka method, the overall stress-strain curves were obtained for the cyclic loading beyond the elastic range with a non-zero mean stress (ratcheting). The experimentally measured accumulation of the overall strain vs. number of cycles was reproduced with a reasonable accuracy. A pronounced ratcheting was predicted during the first cycles followed by shakedown later on.

An important problem in the modelling of damage in particulate CMC is the interaction between the inclusion and multiple microcracks. An integral equation approach capable of handling closely packed microcracks in the vicinity of an inclusion was developed by Hu and coworkers in a series of papers (Hu and Chandra, 1993; Hu *et al.*, 1993; Hu *et al.*, 1994). A perfect bonding was assumed between the matrix and the inclusion, and crack closure effects were neglected. In particular, the work of Hu *et al.* (1993) furnished a fundamental solution for the crack tip behaviour (SIFs) that can further be used in continuum constitutive modelling of particulate composites with dilute concentration of inclusions and systems of matrix microcracks. The crack-crack and crack-inclusion interactions were considered while inclusion-inclusion interactions were neglected. Typically to many interaction problems, the superposition technique was employed. The original problem was superimposed of a homogeneous subproblem and perturbed subproblems, as shown in Fig. 6.

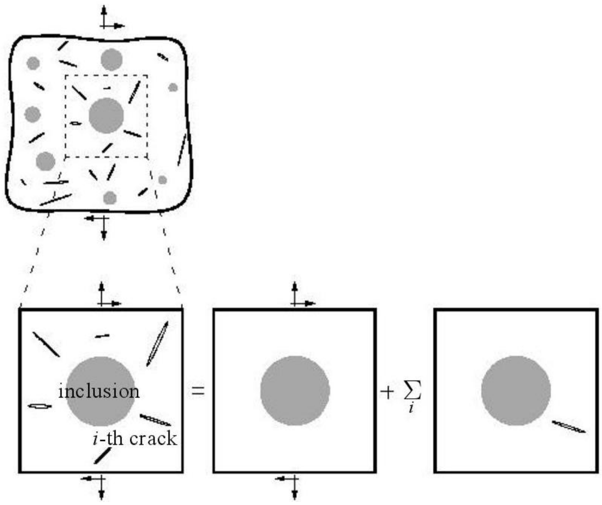

Fig. 6. Inclusion-microcracks interaction: superposition of sub-problems

The microcracks were represented through continuous distributions of dislocations. An elasticity solution for an infinitely extended matrix containing an inclusion and subject to a point dislocation was used to determine the stresses outside the inclusion. Then, the effects of all microcracks on the k-th microcrack were analyzed via corresponding distributions of dislocations. Imposing the stress-free conditions and summing up the effects of all microcracks on the k-th microcrack, governing integral equations were obtained for normal and tangential stress components. These equations when combined with side conditions (crack tip closure condition) imposed on the dislocation density tensor components furnished an integral equation of the original interaction problem (Fig. 6). The dislocation distributions were then found by a numerical scheme employing the Gauss-Chebychev quadrature. Once the dislocation distributions were known, so were the SIFs. It was shown in Hu and Chandra (1993) that if the matrix was subjected to a remote loading, then microcrack-inclusion interactions would generate local stress shielding (retardation) for harder inclusions and stress amplification for weaker inclusions. If, however, a transformation loading was imposed on the inclusion, then stress amplifications were obtained for hard inclusions.

The integral representation of the crack-crack and crack-inclusion problems developed in Hu and Chandra (1993), Hu *et al.* (1993), Hu *et al.* (1994) was devised for infinite bodies, simplified crack geometry, spherical voids and simple loading conditions. Although crucial insight into the behaviour of interacting microdefects was so gained, the simplified geometry and loading conditions made the applicability of the obtained solutions of somewhat limited utility in real-life situations, where complex geometries and loading modes are involved. Therefore, the next logical step in the modelling was to address this issue using powerful computational techniques devised to handle real-life macroscale problems. Such an attempt is found in Jiang *et al.* (1996), where a hybrid micro-macro BEM formulation was developed capable of handling crack-crack interactions, crack-elastic inclusion interactions as well as interactions with boundaries of the system. The fundamental solution, necessary in the BEM, provided an input from the microscale analysis that was incorporated into a macroscale computational technique. This made it possible to investigate the effects of macroscale variations in geometrical parameters and boundary conditions on the microscale evolution of damage (microcrack clusters). Succinctly stated, the microscale phenomena were captured by the fundamental solution, while the standard BEM technique was used to link them to the macroscale problem. This had a positive effect on the proliferation of degrees of freedom – a common difficulty in numerical attempts to relate micro and macro features of a problem. In the present case, a conventional BEM approach would require simultaneous discretization of the external boundary and microscale inclusion and cracks spanning 3-6 orders of magnitude in dimensions – a prohibitive task for any numerical technique. Therefore, a two-step approach was proposed in Jiang *et al.* (1996). The first step dealt with microfeatures and the second step incorporated results from the microscale into a macroscale analysis, the augmented fundamental solution being a bridge for transition between the two scales. In the microscale step, the superposition technique was employed. The solution to the first subproblem of an elastic inclusion embedded in an undamaged infinite elastic matrix subject to a point load was sought using the Airy stress functions. For the second subproblem of multiple interacting microcracks and the inclusion (no point load), the classical representation of a crack as a continuous distribution of dislocations was used. The ensuing integral equations were solved using the Gauss-Chebyshev scheme adopted for singular equations to obtain displacement and stress fields in the second subproblem. As a result, appropriate kernels were computed for use in the macroscale BEM analysis. These kernels captured the microscale crack-crack interactions as well as the crack-inclusion interactions. The proposed hybrid micro-macro BEM approach was developed for radial microcracks in the neighbourhood of an inclusion. It can be extended to handle the debonding of microcracks at the particle-matrix interface. It can also be generalized by introducing the RVE concept or a unit cell to estimate the effective material moduli for damaged CMC.

Apart from the matrix microcracking and particle breakage, the interfacial debonding is one of the frequently encountered damage modes in CMC. When the interfacial strength of matrix-particle bonding is relatively weak and CMC is under a triaxial tension, the debonding will become the dominant damage mechanism. Under a monotonically increasing triaxial tension, the interfacial debonding develops progressively making the composite more compliant. The statistical distribution of interfacial strengths and the volume fraction of particles were found to play major roles in the overall stress-strain behaviour. A micromechanical model was proposed in Baney *et al.*, (1996) to describe the gradual debonding process of a two-phase brittle matrix composite with aligned oblate inclusions under a triaxial tension (spherical inclusions were considered in Tohgo and Weng, 1994). For complete debonding to happen, a triaxial tensile stress state is needed. Unlike the spherical inclusions, a two-phase system with aligned oblate disc inclusions is axisymmetric. Therefore, an analysis was needed to examine at what conditions the complete debonding was attained in the oblate inclusions. As all inclusions exist statistically on equal footing and yet do not debond simultaneously, a statistical approach to the debonding process seemed both appropriate and appealing. In Baney *et al.* (1996), the Weibull function was used to represent probability of interfacial debonding at an oblate inclusion under a triaxial stress state. The Mori-Tanaka method was then employed to predict the overall moduli of the composite system involving two kinds of inclusions of the same shape – the bonded and the debonded ones. The debonded inclusions were either still-bonded or completely debonded (voids). The resulting stress-strain curve of the progressively debonding composite started out with that of the perfectly bonded composite, then deviated from it to finally approach the stress-strain curve of the porous materials (voided). The computed overall moduli revealed loss of stiffness that was anisotropic and strongly dependent on the inclusion thickness-to-diameter ratio.

The effect of debonded interface was studied in many ways by assuming various interfacial conditions, e.g. continuity of the normal displacement and tractions while allowing a jump in the tangential displacement, or by assuming linear relations between the tractions and the displacement jumps at the interface. In Yuan *et al.* (1997) the extent of debonding was simulated by uni-symmetric or bi-symmetric debonding geometries. The goal was to predict the influence of the debonded interface on the effective elastic moduli of a composite. The methodology used for this purpose was that of a two-dimensional FEM. To facilitate the analysis, a periodic rectangular geometry of the composite was assumed which enabled introduction of a unit cell. Parametric studies on the effect of debonding angle, shear moduli ratios of the matrix and reinforcement as well as the reinforcement volume fraction on the composite shear moduli were performed using FEM (Yuan *et al.*, 1997).

The Voronoi Cell Finite Element Method (VCFEM) developed by Ghosh and co-workers was modified in Guo *et al.* (2003) to investigate the matrix-inclusion interfacial debonding for particulate composites. Damage initiation was simulated by partial debonding controlled by the critical normal stress whereas progressive debonding was simulated with an interface remeshing method in which the critical interfacial node at the crack tip was replaced with a pair of nodes along the separated interface. A new hybrid variational principle was derived from the element complementary energy function to account for the interface traction reciprocity on the bonded interface and traction-free conditions on the debonded part of the interface. The numerical calculations in Guo *et al.* (2003) proved the effectiveness of VCFEM in modelling the interfacial damage in two-phase composites. When compared with commercial FEM packages like MARC and ABAQUS, good agreements of results furnished by these packages and VCFEM were obtained. The advantage of VCFEM resides in its capability to discretize complex microstructures. Thus, it can considerably reduce the number of degrees of freedom.

7. Damage in particulate CMC – Continuum Damage Mechanics approach

Micromechanical damage models possess the remarkable ability of relating the heterogeneous microstructure of a material to the macrobehaviour of a specimen. However, for practical engineering applications, rigorous micromechanical analyses of inhomogeneous stress and strain fields for arbitrary 3D microcrack patterns including kinking, forking, crack arrests, crack clustering, etc. may turn out to be a formidable task despite continuously increasing computer power and more efficient numerical algorithms. The presence of the second-phase reinforcement and all sorts of interaction effects exacerbate the situation even more. Consequently, it is not quite surprising that phenomenological damage models were not abandoned. To the contrary, phenomenological and micromechanical models coexist in the open literature or, what is more consequential in terms of modelling, the latter often serves as an inspiration for the former. The research effort in the field of phenomenological damage modelling is typical of any branch that is still under development, i.e. while older and more established models are being implemented numerically to solve particular initial boundary-value problems, new or improved models are still appearing, although at much lower pace than in the seventies or the eighties. More elaborate discussions on phenomenological damage models can be found in Talreja (1994), Krajcinovic (1996), Kachanov (1993), Basista (2001).

The philosophy of the modelling behind the CDM is the following. By its very nature, damage is a discrete and stochastic phenomenon induced by a large number of interacting microdefects. An exact analysis of the behaviour of these microdefects is a formidable task, still beyond our present computing capabilities. Alternatively, the whole problem can be seen as a collective effect of all microdefects on the overall material response. Therefore, the damaged material may be treated as a fictitious continuum with microdefects uniformly smeared within its volume. Consequently – and this constitutes the basic concept of the CDM – it is conceivable that an instantaneous state of material deterioration in every point of the body be represented, in an average sense, by a specific field variable called the *damage variable*. The thus introduced damage variable is nothing else but a macroscopic internal variable. Any CDM model can, thus, be suitably formulated within the well-established framework of irreversible thermodynamics with internal variables (e.g. Lemaitre and Chaboche, 1985). The effect of a large number of microcracks can be described mathematically by a single damage variable provided that the material sample is statistically homogeneous. As for composite materials the

Continuum Damage Mechanics methodology was extensively used by many authors in the past. The details and abundant literature lists are provided in a multi-authored monograph (Talreja, 1994). The chapters by Talreja and Ladeveze therein are particularly recommended for more in-depth studies on CDM modelling in composites.

An interesting combination of micromechanics, homogenisation and continuum damage modelling was proposed in Costanzo *et al.* (1996). In this work, macroscopic thermodynamic and dissipation potentials were derived for microscopically inhomogeneous materials such as CMC with growing sharp matrix microcracks and debond microcracks. Even though the periodic boundary conditions were the authors' first choice to simplify the analysis, it was shown that the growing damage violates the periodicity requirement and the uniform strain boundary conditions were eventually used. It was further assumed that each constituent of the composite RVE behaved as a generalized standard material. Consequently, the microscopic constitutive behaviour could be characterised by convex thermodynamic and dissipation potentials. It was shown that the evolution of the RVE was also governed by a convex dissipation macropotential. As a consequence, the rates of internal variables could be derived as derivatives of the macropotential with respect to the generalized thermodynamic forces conjugate to those variables. For growing microcracks, the existence of the macro dissipation potential and the ensuing normality rule is assured whenever the microcrack growth $(\dot{a})$ depends on its own thermodynamic force – the elastic energy release rate (G). This is a well-known result derived earlier by Rice in his fundamental paper (Rice, 1971) on the internal variable thermodynamic framework for inelastic constitutive relations.

8. Damage in interpenetrating phase composites

Similarly as for fracture processes in IPC, no specific models seem to have targeted damage problems in these materials. After a detailed literature search, it seems justified to say that, so far, the modelling has primarily been focused on effective elastic properties of IPC in relation to the content and morphology of constituent phases. To this end, finite element simulations combined with Voigt and Reuss bounds were used to reproduce overall stress-strain curves for a particular IPC strained in uniaxial tension and compression (Daehn *et al.*, 1996).

Apart from applied mechanicians, statistical physicists are also interested in the modelling of IPC behaviour (in their nomenclature – random composites). To this end, they often use the analogy between mechanical (Hooke's law) and electrical system (Ohm's law). In Moukarzel and Duxbury (1994), a random resistor network on a cubic lattice was used to investigate the strength of IPC. Simulations on a random network were compared with data for a particulate composite noting an improved strength and damage tolerance for the former. As for the bridging effect, naturally occurring in IPC, the issues involved in fracture and damage modelling of bridged cracks and microcracks do not differ from other ceramic-metal composites except for the fact that non-discrete geometry of IPC makes the solutions based on the Eshelby tensor not quite suitable (Feng *et al.*, 2003).

Having realized that available analytical models are of limited utility in the modelling of mechanical behaviour of IPC, some authors turned to FEM modelling as a viable alternative (Wegner and Gibson, 2000a; Wegner and Gibson, 2000b). Two important observations have been made: (i) IPC morphologies make it impossible to use 2D modelling and 3D modelling is required, (ii) generic interpenetrating microstructure – whose properties are representative of IPC in general – does not exist. As pointed out in Wegner and Gibson (2000a), a realistic though approximate approach is to resort to a unit cell with appropriate boundary conditions to represent a periodically repeating 3D structure.

In FEM studies presented by Wegner and Gibson (2000a,b) an IPC was modelled with a hexagonal array of intersecting uniformly sized spheres with interstices filled with another phase. Both the spheres and the interstices were elastic and individually formed interconnected networks. This particular choice of regular network geometry was, on one hand, aimed at resembling an IPC processed by the 3D printing technology for which a body of experimental data existed. On the other hand, it was particularly suited for automatic generation of FE meshes for an IPC with different volume fractions of the phases. A sensitive point in this type of modelling is the selection of a unit cell. Periodic boundary conditions were imposed on the unit cell and perfect bond was assumed between the spheres and the other phase (matrix). In order for the chosen microstructures to be interpenetrating, the volume fraction of the intersecting spheres f_s should be above the percolation threshold 0.74 and less than 0.95. The FE model was implemented with the purpose to determine the elastic modulus, the nonlinear response under uniaxial loading and the thermal expansion. For non-interpenetrating microstructures (f_s much lower than the percolation threshold), an interesting result was obtained that the elastic modulus of an IPC with flexible spheres was very close to the Hashin-Shtrikman upper bound, whereas for a stiff non-interpenetrating network of spheres it was close to the lower bound. Simulations were also performed for elastic-perfectly plastic interpenetrating phases.

A number of conclusions were drawn in Wegner and Gibson (2000a). Firstly, the strength and thermal expansion of IPC were enhanced when compared with those for non-IPC. Secondly, the enhancement of thermomechanical properties was achieved not so much by the dual phase contiguity but it was actually the contiguity of the phase with more desirable properties which imparted the improved properties of the composite. Consequently, the most important merit of IPC lies in their ability to combine several desirable properties at once, by bringing together a number of contiguous phases, each of which exhibits one of the properties desired.

9. Suggestions for future modelling topics

Fracture

Even though a lot has been done so far in the modelling of discrete cracks and fracture in CMC with ductile inclusions, there are still some topics that require further research. To this end, the bridging traction law relating the stress and the displacement in stretched ligaments requires more scrutiny. Also, other toughening mechanisms should be looked into as the predictions of composite toughness based on the crack bridging model are not always satisfactory (e.g. Budiansky *et al.*, 1988).

As for interpenetrating metal/ceramic composites, due to the fact that the reinforcing phase can effectively transfer stresses in all directions, one of the key issues would be the appropriate modelling of the stress transfer relation of constituent phases with specific microstructures. Statistical correlation functions and image analysis might be of some help in this regard.

Damage.

For particulate CMC, damage models seem to be well developed both on the micromechanical and the phenomenological level. The room for the modelling is in the introducing of real-life geometries, complex loading conditions and finite dimensions of considered bodies. Also, less modelling effort has been invested so far into the damage modelling of cavitation effects in metal particles – this gap might be worth filling up in future research.

As for interpenetrating metal ceramic composites, there seems to exist an acute lack of all kinds of models for fracture and damage processes mainly due to relative immaturity of this class of composites.

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Modelowanie uszkodzenia i pękania w kompozytach o matrycy ceramicznej – przegląd stanu wiedzy

Streszczenie

Niniejsza praca stanowi przegląd istniejących modeli pękania i uszkodzenia w kompozytach o matrycy ceramicznej (CMC) pod działaniem obciążeń mechanicznych lub termicznych. Nacisk położono na CMC z inkluzjami metalicznymi oraz na CMC typu przenikających się faz metaliczno ceramicznych. Sytuacje, gdy druga faz ma postać inkluzji ceramicznych, nie były analizowane. Rozdziały 2-5 dotyczą problemów pękania (wzrostu makroszczeliny), podczas gdy Rozdziały 6-9 – problemów uszkodzenia (wpływu mikroszczelin na zachowanie się CMC). Na zakończenie pracy zaproponowano listę nierozwiązanych problemów oraz tematów przyszłych badań pożądanych w zakresie pękania i uszkodzenia CMC.

Manuscript received December 9, 2005; accepted for print February 3, 2006