

## DISCRETE ELEMENT SIMULATION OF ROCK CUTTING PROCESSES

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**Abstract.** This paper presents numerical modelling and simulation of rock cutting processes. The model considers a tool–rock system. The rock is modelled using spherical discrete elements. Formulation of the discrete element method has been briefly reviewed. The model has been calibrated by simulation of the UCS and Brazilian tests. Simulation of rock cutting with a single point attack pick of a roadheader has been carried out. The 3D analysis allowed us to predict three components of cutting forces. The numerical model of rock cutting has been validated using the results of laboratory cutting tests. A good qualitative and quantitative agreement of numerical results with experimental measurements has been found out.

**Keywords:** DEM, rocks, rock cutting, simulation.

### Introduction

Variety of rock-cutting technologies is used in civil as well as in mining engineering. Fig 1 shows a roadheader, a machine used in rock excavation works.



**Fig 1.** Rock excavation with a roadheader

The basic physical phenomenon occurring during cutting is rock desintegration under mechanical action of a cutting tool. Design of cutting tools and setting parameters of cutting operations requires knowledge about the cutting process.

Cutting force is one of the main factors characterizing a cutting process. Theoretical evaluation of the cut-

ting force is not an easy task. Simple analytical models, like those developed by Evans (1965) or by Nishimatsu (1972), can provide a very approximate estimation of cutting forces only. Numerical methods based on the continuum models, like finite element methods, have serious problems in modelling discontinuities of the material occurring during rock cutting (Jonak and Podgórski 2001).

The present paper presents possibilities of modelling rock cutting using a discrete element model. The discrete element method takes into account all kinds of discontinuities and material failure characterized with fracture and is commonly regarded as a suitable tool to study rock cutting (Huang 1999; Stavropoulou 2006; Su *et al.* 2009).

### Basic assumptions of the rock cutting model

A numerical model of rock cutting has been developed within the authors' own implementation of the discrete element method (Rojek *et al.* 2001; Oñate and Rojek 2004). The system consisting of a tool and rock sample is considered in the model. The rock material is represented as a collection of rigid spherical (in 3D) or cylindrical (in 2D) particles interacting among themselves with contact forces. The tool is treated as a rigid body with a surface discretized with triangular facets. The tool-rock interaction is modelled assuming Coulomb friction model.

## Discrete element method formulation

The translational and rotational motion of the  $i$ -th discrete element is governed by the standard equations of the rigid body dynamics:

$$m_i \ddot{\mathbf{u}}_i = \mathbf{F}_i \quad (1)$$

$$J_i \dot{\boldsymbol{\omega}}_i = \mathbf{M}_i \quad (2)$$

where  $\mathbf{u}$  is the element centroid displacement in a fixed (inertial) coordinate frame  $\mathbf{X}$ ,  $\boldsymbol{\omega}$ – the angular velocity,  $m$ – the element mass,  $J$ – the moment of inertia,  $\mathbf{F}$ – the resultant force, and  $\mathbf{M}$ – the resultant moment about the central axes. Vectors  $\mathbf{F}$  and  $\mathbf{T}$  are sums of all forces and moments applied to the  $i$ -th element due to external load, contact interactions with neighbouring spheres and other obstacles, as well as forces resulting from damping in the system. The form of the rotational equation (2) is valid for spheres and cylinders (in 2D) and is simplified with respect to a general form for an arbitrary rigid body.

Equations of motion (1) and (2) are integrated in time using the central difference scheme. The time integration operator for the translational motion at the  $n$ -th time step is as follows:

$$\ddot{\mathbf{u}}_i^n = \frac{\mathbf{F}_i^n}{m_i} \quad (3)$$

$$\dot{\mathbf{u}}_i^{n+1/2} = \dot{\mathbf{u}}_i^{n-1/2} + \ddot{\mathbf{u}}_i^n \Delta t \quad (4)$$

$$\mathbf{u}_i^{n+1} = \mathbf{u}_i^{n-1} + \dot{\mathbf{u}}_i^{n+1/2} \Delta t \quad (5)$$

The first two steps in the integration scheme for the rotational motion are identical to those given by equations (3) and (4):

$$\dot{\boldsymbol{\omega}}_i^n = \frac{\mathbf{M}_i^n}{J_i} \quad (6)$$

$$\boldsymbol{\omega}_i^{n+1/2} = \boldsymbol{\omega}_i^{n-1/2} + \dot{\boldsymbol{\omega}}_i^n \Delta t \quad (7)$$

The vector of incremental rotation  $\Delta\boldsymbol{\theta}_i^{n+1}$  is calculated as:

$$\Delta\boldsymbol{\theta}_i^{n+1} = \boldsymbol{\omega}_i^{n+1/2} \Delta t \quad (8)$$

If necessary it is also possible to track the total change of rotational position of particles.

Explicit integration in time yields high computational efficiency of the solution for a single step. The disadvantage of the explicit integration scheme is its conditional numerical stability imposing the limitation on the time step  $\Delta t$ . The time step  $\Delta t$  must not be larger than a critical time step  $\Delta t_{cr}$

$$\Delta t \leq \Delta t_{cr} \quad (9)$$

determined by the highest natural frequency of the system  $\omega_{max}$

$$\Delta t_{cr} = \frac{2}{\omega_{max}} \quad (10)$$

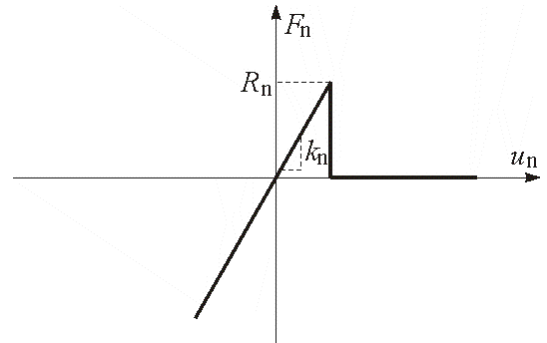
Exact determination of the highest frequency  $\omega_{max}$  would require solution of the eigenvalue problem defined for the whole system of connected rigid particles. In an approximate solution procedure adopted, the maximum frequency is estimated as the maximum of natural frequencies of the mass–spring systems defined for the contact pairs of particles.

The overall behaviour of the system is determined by the cohesive/frictional contact models assumed for the interaction between contacting spheres. The contact force between two particles  $\mathbf{F}$  can be decomposed into the normal and tangential components,  $\mathbf{F}_n$  and  $\mathbf{F}_T$ , respectively

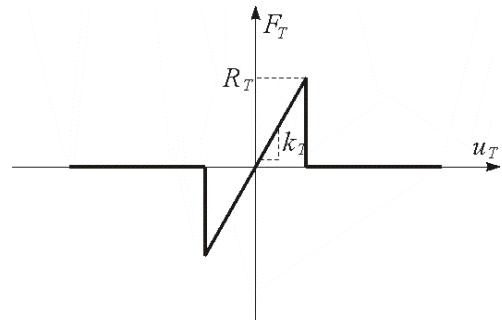
$$\mathbf{F} = \mathbf{F}_n + \mathbf{F}_T = F_n \mathbf{n} + \mathbf{F}_T \quad (11)$$

where  $\mathbf{n}$  is the unit vector normal to the particle surface at the contact point.

The contact forces  $F_n$  and  $\mathbf{F}_T$  are obtained using a constitutive model formulated for the contact between two rigid spheres. In the present formulation rock materials are modelled using elastic perfectly brittle model of contact interaction, in which we assume initial bonding for the neighbouring particles. These bonds can be broken under load allowing us to simulate initiation and propagation of material fracture. Contact laws for the normal and tangential direction for the elastic perfectly brittle model are shown in Figs 2 and 3.



**Fig 2.** Force–displacement relationship for the elastic perfectly brittle model in the normal direction



**Fig 3.** Force–displacement relationship for the elastic perfectly brittle model in the tangential direction

When two particles are bonded the contact forces in both normal and tangential directions are calculated from the linear constitutive relationships:

$$F_n = k_n u_n \quad (12)$$

$$\|\mathbf{F}_T\| = k_T \|\mathbf{u}_T\| \quad (13)$$

where  $k_n$  and  $k_T$  are the interface stiffness in the normal and tangential directions, and  $u_n$  and  $\mathbf{u}_T$ — the normal and tangential relative displacements, respectively.

The tensile and shear contact forces are limited by the tensile and shear interface strengths,  $R_n$  and  $R_T$ , respectively:

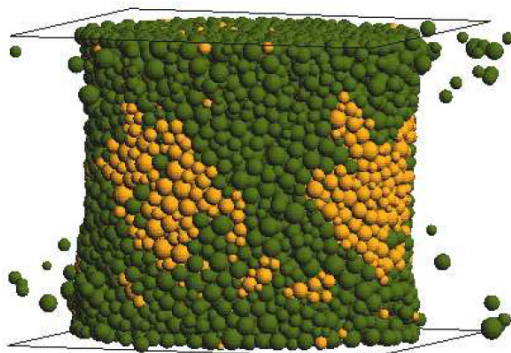
$$F_n \leq R_n \quad (14)$$

$$\|\mathbf{F}_T\| \leq R_T \quad (15)$$

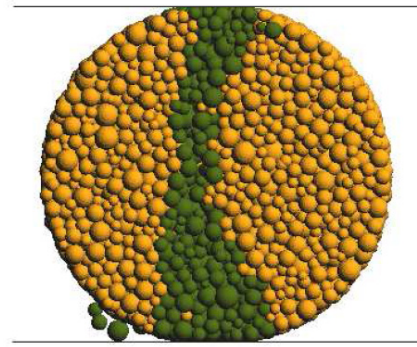
Cohesive bonds are broken instantaneously when the interface strength is exceeded either by the tangential contact force or by the tensile contact force. After debonding the elements can interact according to the frictional contact conditions without cohesion. Similarly the frictional contact is assumed for the tool-rock interaction

### Determination of rock model parameters

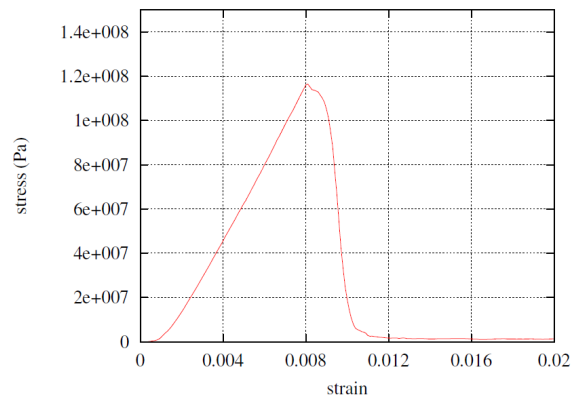
Determination of the model parameters is the first step in the discrete element simulation of rock cutting process. A set of micromechanical parameters yielding required macroscopic rock properties has been determined using the methodology developed by Huang (1999) based on the combination of the dimensional analysis with numerical simulation of the standard laboratory tests for rocks, unconfined compression test (Fig 4) and Brazilian test (Fig 5). The stress-strain relationship obtained in the numerical simulation of the unconfined compression test is shown in Fig 6. It is similar to the stress-strain curves obtained in laboratory. The force-time curve obtained in the simulation of the Brazilian test is plotted in Fig 7. These curves allow us to determine macroscopic properties of the material modelled with the discrete element method.



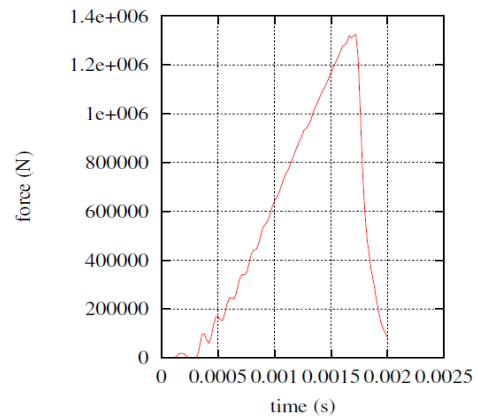
**Fig 4.** Results of the numerical simulation of the unconfined compression test



**Fig 5.** Results of the numerical simulation of the Brazilian test



**Fig 6.** Simulation of the unconfined compression test: stress–strain curve



**Fig 7.** Simulation of the Brazilian test: force–time curve

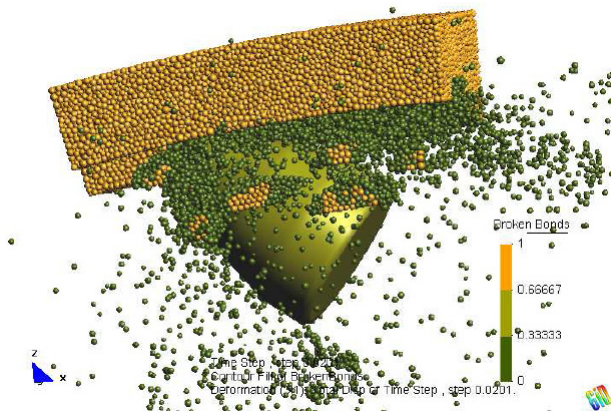
### Simulation of rock cutting with a roadheader pick

Validation of the rock cutting model has been carried using experimental results obtained in a laboratory test performed in the laboratory of Sandvik Mining and Construction (Fig 8). The tests consisted in cutting of a sandstone block with a rotating roadheader pick. Mechanical properties of the rock have been determined experimentally and are the following: Young modulus  $E = 18690$  MPa, Poisson ratio  $\nu = 0.23$ , compressive strength  $\sigma_c = 127$  MPa and tensile strength  $\sigma_t = 12$  MPa.

The test has been analysed using a three dimensional discrete element model. Rock sample has been discretized using 71,200 spherical particles. For the rock considered the following set of micromechanical parameters has been found for the DEM model: contact stiffness in the normal direction  $k_n = 5.4 \cdot 10^6$  N/m, contact stiffness in the tangential direction  $k_T = 2.16 \cdot 10^6$  N/m, cohesive bond strengths in the normal and tangential direction,  $R_n = R_T = 86$  N. The results of numerical simulation are shown in Fig 9. Splitting of chips typical for brittle rock cutting can be seen.

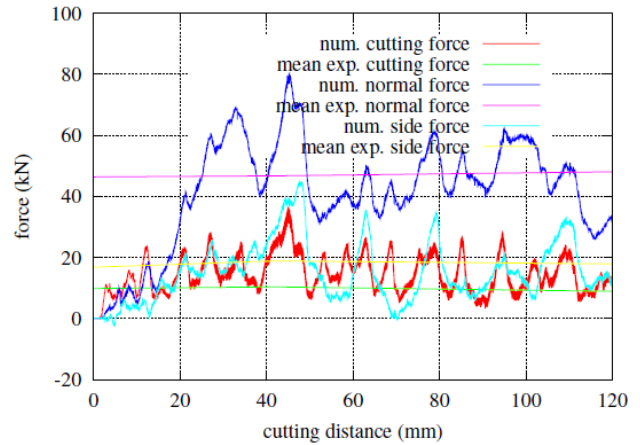


**Fig 8.** Laboratory rock cutting test (courtesy of Sandvik Mining and Construction GmbH, Zeltweg, Austria)



**Fig 9.** Numerical simulation of the rock cutting test

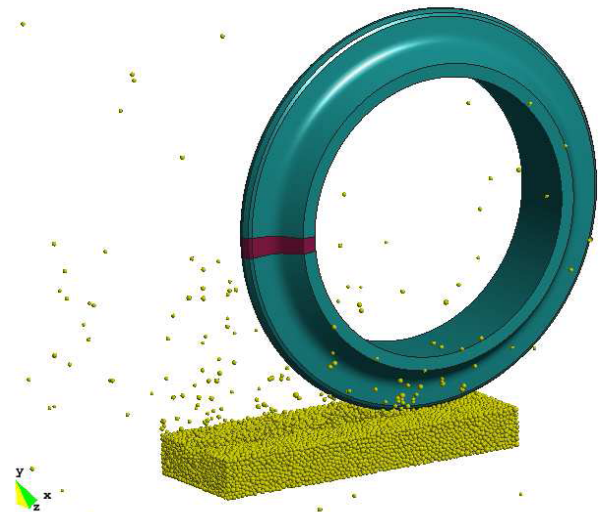
The three components of cutting forces obtained in simulation are plotted in Fig 10. Numerical forces are compared with experimental average measurements. Quite a good agreement can be observed.



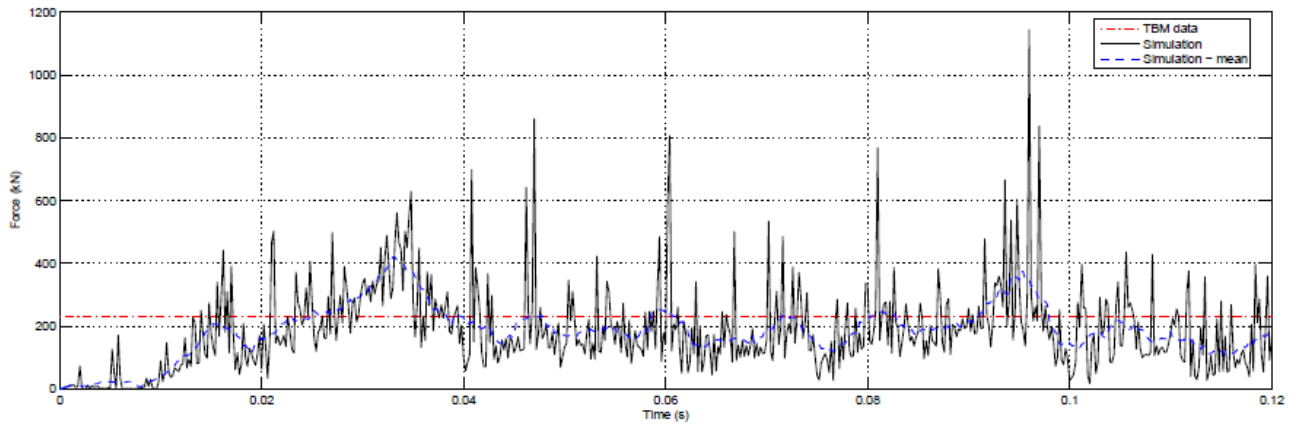
**Fig 10.** Rock cutting forces– comparison of numerical results with experimental average values

### Simulation of rock cutting with a TBM disc cutter

Tunnel Boring Machine (TBM) is used in tunnel excavation. TBMs perform rock cutting with disc cutters mounted on a cutterhead. The linear cutting test has been established to predict TBM performance. The linear cutting test has been simulated. Fig 11 shows the simulation results. The granite properties are assumed in the simulation, appropriate DEM parameters being evaluated. Fig 12 shows the normal cutting force history. Numerical results have been compared with experimental ones provided by Herrenknecht AG. A good agreement between the numerical and average experimental values is clearly seen.



**Fig 11.** Simulation of rock cutting with a TBM disc cutter



**Fig 12.** Simulation of rock cutting with a TBM disc cutter– comparison of numerical cutting forces with experimental average values

### Concluding remarks

The three-dimensional discrete element model of rock cutting is capable to represent correctly complexity of a rock cutting process. A good qualitative and quantitative agreement of numerical results with experimental measurements has been found out in the validation of the model developed in the present work. The 3D model developed can be employed in the design of rock cutting tools and processes.

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