EXTENSION OF THE RYCHLEWSKI LIMIT STATE CRITERION ACCOUNTING FOR THE ASYMMETRY OF ELASTIC RANGE

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1. The subject of research

Recently the application of modern materials exhibiting some unconventional features becomes still more common. Among those new materials one should distinguish various composites, materials with internal micro- or nanostructure, modern alloys, heavily rolled metal sheets etc. It is clearly visible those materials distinguish themselves with certain uncommon mechanical properties which for a long time were considered unnecessary or negligible in the analysis of the problem of reaching a limit state. Among those features one should mention: low elastic symmetry, pressure dependence, Lode angle dependence, strength-differential effect etc. Many of the classical propositions of the limit state criteria cannot involve all or at least majority of those effects. The aim of authors is to provide possibly general yet precise limit state condition of simple mathematical form derived from physical energy-based considerations.

2. The Rychlewski limit state criterion

J. Rychlewski performed in 1984 the spectral analysis of elasticity tensors [1] describing precisely the mathematical form of constitutive relations of anisotropic linear elasticity in terms of eigenvalues (Kelvin moduli) and eigenstates of stiffness and compliance tensors. He introduced also the concept of energetically orthogonal decompositions of elastic energy density [2] and suggested that a linear combination of the terms of such a decomposition might be considered as a measure of material effort. The limit state criterion derived from the hypothesis or Rychlewski is of the following form:

$$\frac{\Phi_1}{h_1} + \frac{\Phi_2}{h_2} + ... + \frac{\Phi_\mu}{h_\mu} = 1, \qquad \mu \leqslant 6 \tag{1}$$

3. Extension of the Rychlewski criterion using the concept of influence functions

Due to strictly energetic formulation of the limit state criterion of Rychlewski, which is expressed as a quadratic from in space of stress states, it cannot account for the strength differential effect. The authors suggest to extend the original proposition of Rychlewski in the same manner as Burzyński [3] developed the classical hypothesis of Huber so that it involved asymmetry of the elastic range, namely by multiplying the terms of the energy decomposition by certain stress state dependent functions. Depending on their character they should be termed *influence functions* or *stress mode indicators*. The modified limit state criterion takes the form

$$\eta_1 \cdot \Phi_1 + \eta_2 \cdot \Phi_2 + ... + \eta_\mu \Phi_\mu = 1, \qquad \mu \leqslant 6 \tag{2}$$

There are certain assumptions made on the properties of the parameters η :

- It is assumed that the parameters η depend only on the stress state corresponding with the term of energy decomposition which is respective for that parameter.

- Parameters η are assumed to be isotropic in their domain – i.e. they can be expressed as a scalar functions of the invariants of their argument.
- Parameters η respective for the deviatoric (shear) stress states are assumed to be an even (symmetric) functions – in particular they will be often assumed to be independent of the norm of their arguments. They will be termed then the *shear mode indicators*.
- Parameter η is called the influence function if it depends on the norm of its argument (it changes its value as the magnitude of the corresponding stress state changes) – otherwise it is called the stress mode indicator.

Further properties of the influence functions and stress mode indicators can be derived basing on those basic assumptions and additional analysis taking into account the dimension of the corresponding stress state subspace

4. Specification of the new limit state criterion for chosen elastic symmetries

The new limit state criterion was specified for all plane elastic symmetries and most of spatial elastic symmetries after assuming that the considered energetically orthogonal decomposition of the elastic energy density was the – so called by Rychlewski – main energy decomposition, namely the decomposition into the terms corresponding with different eigensubspaces of the elasticity tensors. The criterion for each symmetry involves in general different number of influence functions and stress mode indicators depending on the stress states uniquely determined by the elastic properties of the considered material. For certain cases a direct formula was given which enables finding the value of the unknown influence functions for certain values of their arguments basing on simple strength tests.

5. Summary

A new proposition of an energy-based limit state criterion for anisotropic materials exhibiting asymmetry of the elastic range is presented. Certain assumption on the criterion parameters are made. Specification of the limit state criterion for a large number of elastic symmetries is given as well as the general methodology of determination of the form of unknown parameters of the criterion. Detailed discussion on the introduced criterion was already published in [4] [5] [6].

6. References

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