Impact-Load-Based Damage Identification in Joints of Skeletal Structures

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ABSTRACT

The nodal connections in the standard analysis of frame structures are idealized assuming either pinned or fully rigid joints. State of such structural connections is highly important for safe operation of skeletal structures. In this work, the authors propose modeling, detection and identification of semi-rigid joints, i.e. nodal connections covering the range between pinned and fully rigid joints. The presented approach is based on the Virtual Distortion Method (VDM) and dedicated to statically or dynamically loaded two-dimensional frame structures.

INTRODUCTION

Steel frames with semi-rigid joints can be analyzed using the finite element method by application of a designed finite element [1]. In such approach, dedicated element stiffness matrices are computed and then global stiffness matrix is aggregated for further analyses. However, in this article an adaptation of the Virtual Distortion Method (VDM) for modeling, detection and identification of semi-rigid joints is presented. The VDM is a fast reanalysis tool, which can be successfully applied in the field of Structural Health Monitoring (SHM). This method is highly effective for linear systems, cf. [2], [3]. Necessary field measurements can be performed using wireless data transmission systems as presented in [4]. Other examples of VDM-based applications are: damage identification in electrical circuits [5], identification of delamination between laminates [6] or leakages in water distribution networks [7].

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BASICS OF THE VIRTUAL DISTORTION METHOD

The VDM exploits the Finite Element Method (FEM) and operates on finite elements. In general, some structural modifications of a system are modeled by the so-called virtual distortions (pseudo strains), which are imposed on original elements. States of a system caused by introduced virtual distortions without any external load are called distorted states. Updated response of a modified structure is a superposition of the response calculated for the loaded original structure and the response obtained for the distorted state supposed to model structural modifications. The distorted state is a linear combination of components of the ***influence matrix*** and virtual distortions. The influence matrix contains strain responses caused by introduction of unit virtual distortions to finite elements.

For a finite beam element (Bernoulli's theory), there are three basic deformation states: longitudinal, symmetric and asymmetric corresponding to tension, pure bending, bending and shear, respectively. In this work, these deformation states are basic strain components and can be determined by solving the eigenvalue problem for the element stiffness matrix. The three non-rigid eigenstates are presented in Fig. 1. Any other deformation state can be linearly combined from the three basic ones.

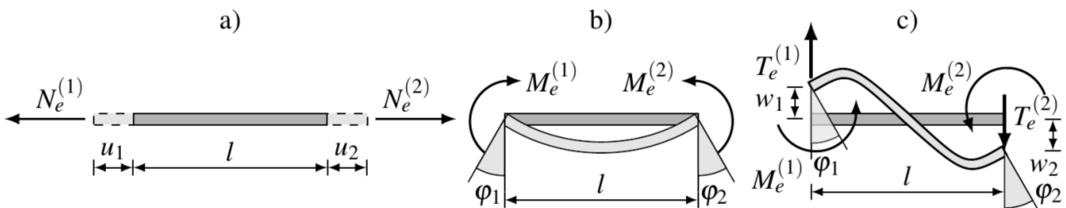

Figure 1. Basic strain components with corresponding compensating loads for 2D beam finite element: a) longitudinal, b) symmetric, c) asymmetric.

For the finite element *e*, the strain components: longitudinal – ϵ_e, symmetric – $\varkappa_e$ and asymmetric – χ_e can be calculated using the following formula:

$$\varepsilon_e = \left[\frac{u_e^{(2)} - u_e^{(1)}}{l_e}, \frac{\varphi_e^{(2)} - \varphi_e^{(1)}}{l_e}, \frac{6}{l_e}\left(\frac{\varphi_e^{(1)} + \varphi_e^{(2)}}{2} - \frac{w_e^{(2)} - w_e^{(1)}}{l_e}\right)\right] = [\epsilon_e, \varkappa_e, \chi_e] \quad (1)$$

where $u_e^{(i)}$, $\varphi_e^{(i)}$, $w_e^{(i)}$ ($i = 1,2$ – number of the local node) are the generalized nodal displacements (as denoted in Fig. 1) and l_e is the length of the finite element. The virtual distortions ε_e^0 are initial strains introduced to the structural finite element *e*. The columns of the influence matrix contain strain responses generated by action of the unit virtual distortions on the system. An introduction of the unit virtual distortion is equivalent to application of the corresponding load to the finite element using self-equilibrated compensating forces (cf. Fig. 1):

$$\begin{aligned} Q(\epsilon_e^0 = 1) &= EA\,[-1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]^T \\ Q(\varkappa_e^0 = 1) &= EJ\,[0 \quad 0 \quad -1 \quad 0 \quad 0 \quad 1]^T \\ Q(\chi_e^0 = 1) &= EJ\,[0 \quad 2/l_e \quad 1 \quad 0 \quad -2/l_e \quad 1]^T \end{aligned} \quad (2)$$

For a structure under load, a semi-rigid joint is modeled by introduction of a non-deformable wedge to the element local node *i*. This wedge is characterized by the angle of $\varphi_e^{0(i)}$, which is called ***angular virtual distortion***. Based on formula (1), the relation between the angular virtual distortion $\varphi_e^{0(i)}$ and the strain virtual distortion ε_e^0 can be written in the following form:

$$\varepsilon_e^0 = \overline{v}_e^{(i)} \varphi_e^{0(i)} \tag{3}$$

where $\overline{v}_e^{(i)} = \frac{1}{l_e}[0 \quad \xi^{(i)} \quad 3]^T$ is the transformation vector for the local node i. The coefficient $\xi^{(i)}$ is equal to -1 for the local node $i = 1$, whereas $\xi^{(i)}$ is equal to $+1$ for the local node $i = 2$. We can see from Eq. (3) that the longitudinal component of the element virtual distortion is equal to zero, i.e. $\epsilon_e = 0$, therefore we can disregard it:

$$\kappa_e^0 = v_e^{(i)} \varphi_e^{0(i)} \tag{4}$$

where $v_e^{(i)} = \frac{1}{l_e}[\xi^{(i)} \quad 3]^T$ and $\varepsilon_e^0 = [0 \quad \kappa_e^0]^T$.

On the other hand, the bending moment $M_e^{(i)}$ for the element *e* in the local node *i* can be expressed by strain components in the following way:

$$M_e^{(i)} = EJw_e^{(i)}\kappa_e \tag{5}$$

where $w_e^{(i)} = [1 \quad \xi^{(i)}]$ is the nodal transformation vector and $\kappa_e = [\varkappa_e \quad \chi_e]^T$ is the subvector of bending strain.

MODELING OF THE SEMI-RIGID JOINTS BY THE VIRTUAL DISTORTIONS

Statics

For now, the introduced Greek indices run over elements with modeled nodal connections and there is no summation over underlined indices. Thus, the updated structural strain responses can be expressed by the equations:

$$\varepsilon_\alpha = \varepsilon_\alpha^L + \left(\overline{D}_{\alpha\beta} - \overline{\delta}_{\alpha\beta}\right)\varepsilon_\beta^0 \quad \text{or} \quad \kappa_\alpha = \kappa_\alpha^L + \left(D_{\alpha\beta} - \delta_{\alpha\beta}\right)\kappa_\beta^0 \tag{6}$$

where reduced matrices $D_{\alpha\beta}$ and $\delta_{\alpha\beta}$ do not contain columns and rows corresponding to axial strain components. Unlike VDM-modeled structural modifications corresponding to axial deformations (e.g. in [2]), we have now the identity matrix $\delta_{\alpha\beta}$ in Eq. (6) because we account for introduction of a non-deformable wedge for calculation of strain components in the distorted structure. This wedge modifies nodal rotation in the element in which it is introduced, cf. Fig. 2.

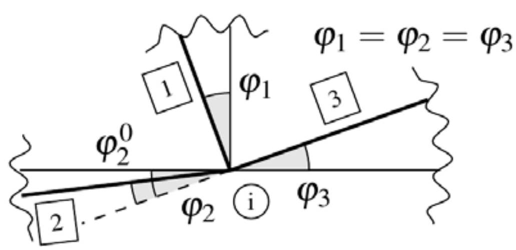

Figure 2. The nodal rotations of elements for non-rigid connection in element no. 2 in a frame-like substructure.

Analogously, in the semi-rigid joints of the loaded frame structure such rotational discontinuities occur at nodes. This means that element cross section adjacent to the node rotate unlike the other element cross sections. Those differences are modeled by the angular virtual distortions $\varphi_\alpha^{0(i)}$. The relationship between the nodal bending moment $M_\alpha^{(i)}$ and distortion $\varphi_\alpha^{0(i)}$ can be postulated to be linear or bi-linear as shown in Fig. 3. For simplicity, in the further consideration the linear model is assumed, which can be written in the following form:

$$M_\alpha^{(i)} = k_{\underline{\alpha}}^{(i)} \varphi_{\underline{\alpha}}^{0(i)} \tag{7}$$

where the nodal rotational stiffness k_α of the nodal connection is defined as: $k_\alpha = E_{\underline{\alpha}} J_{\underline{\alpha}} h_{\underline{\alpha}}^{(i)}$. Here, the parameter $h_\alpha^{(i)}$ denotes rotational stiffness of the nodal connection in the element α for the local node i. Alternatively, Eq. (7) can be rewritten using two diagonal matrices $S_{\alpha\beta} = diag\{E_{\underline{\alpha}} J_{\underline{\alpha}}\}$ and $H_{\beta\gamma}^{(i)} = diag\{\, h_\beta^{(i)}\}$:

$$M_\alpha^{(i)} = S_{\alpha\beta} H_{\beta\gamma}^{(i)} \varphi_\gamma^{0(i)} \tag{8}$$

Eq. (5) may be rewritten similarly to Eq. (8) for a set of modeled nodal joints:

$$M_\alpha^{(i)} = S_{\alpha\beta} W_{\beta\gamma}^{(i)} \kappa_\gamma \tag{9}$$

where $W_{\beta\gamma}^{(i)} = diag\{w_\beta^{(i)}\}$. Analogously to Eq. (4), we have:

$$\kappa_\alpha^0 = V_{\alpha\beta}^{(i)}\, \varphi_\beta^{0(i)} \tag{10}$$

where $V_{\alpha\beta}^{(i)} = diag\{v_\alpha^{(i)}\}$. The matrices $W_{\beta\gamma}^{(i)}$ and $V_{\alpha\beta}^{(i)}$ are of rectangular shape.

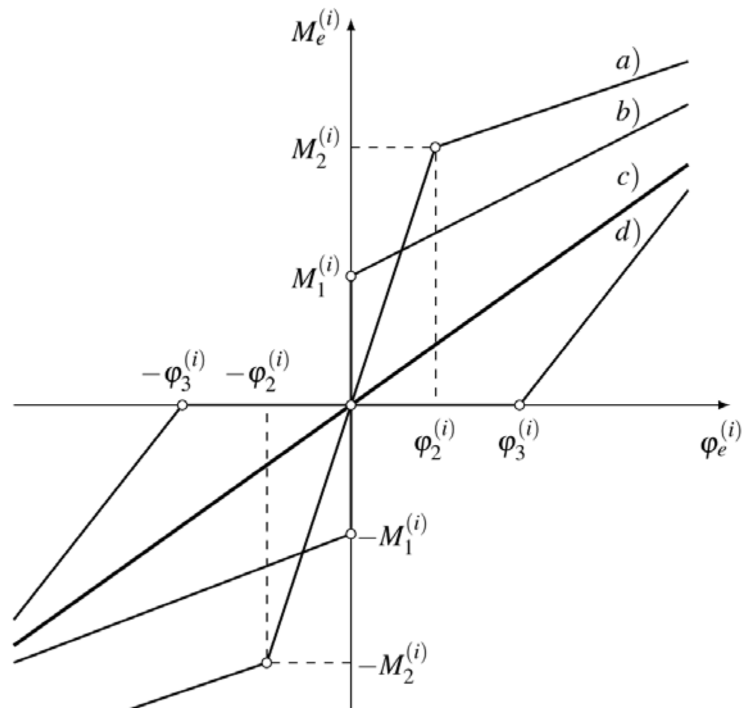

Figure 3. Linear and bi-linear models of the nodal semi-rigid connections.

By comparing relationships (8) and (9) as well as using (6) and (10), the formula for determining the angular virtual distortions $\varphi_\beta^{0(i)}$ has the following form:

$$\left[H_{\alpha\beta}^{(i)} - W_{\alpha\gamma}^{(i)}\left(D_{\gamma\vartheta} - \delta_{\gamma\vartheta}\right)V_{\vartheta\beta}^{(i)}\,\right]\varphi_\beta^{0(i)} = W_{\alpha\gamma}^{(i)} \kappa_\gamma^L \tag{11}$$

Let us note, for the fully rigid connection in the local node i in the finite element α, if $H_{\alpha\alpha}^{(i)} = h_\alpha^{(i)} \to +\infty$, the distortion $\varphi_\alpha^{0(i)} \to 0$. On the other hand, if $H_{\alpha\alpha}^{(i)} = h_\alpha^{(i)} \to 0$, then $\varphi_\alpha^{0(i)} \neq 0$.

Dynamics

For the time domain analysis, the impulse influence matrix $D_{\alpha\beta}(t)$ has to be determined. It is computed by applying impulse excitations i.e. self-equilibrated compensation loadings only in the first time step of the analysis. The strain virtual distortions are time-dependent, thus the updated bending strain components can be expressed by the equation:

$$\kappa_\alpha(t) = \kappa_\alpha^L(t) + \sum_{\tau=0}^{t} D_{\alpha\beta}(t-\tau)\kappa_\beta^0(\tau) - \kappa_\alpha^0(t) \tag{12}$$

Analogously to Eq. (9) and (10), for the time-domain analysis the bending moment $M_\alpha^{(i)}(t)$ can be expressed by the following relationship:

$$M_\alpha^{(i)}(t) = S_{\alpha\beta} W_{\beta\gamma}^{(i)} \kappa_\gamma(t) = S_{\alpha\beta} H_{\beta\gamma}^{(i)} \varphi_\gamma^{0(i)}(t) \tag{13}$$

which by using Eq. (12) leads to the equation for determining the angular virtual distortion $\varphi_\gamma^{0(i)}(t)$:

$$A_{\alpha\gamma}^{(i)} \varphi_\gamma^{0(i)}(t) = b_\alpha^{(i)}(t) \tag{14}$$

where

$$A_{\alpha\gamma}^{(i)} = H_{\alpha\gamma}^{(i)} - W_{\alpha\beta}^{(i)}\left(D_{\beta\vartheta}(0) - \delta_{\beta\vartheta}\right)V_{\vartheta\gamma}^{(i)} \tag{15}$$

and

$$b_\alpha^{(i)}(t) = \begin{cases} W_{\alpha\beta}^{(i)} \kappa_\beta^L(0) & \text{for } t = 0 \\ W_{\alpha\beta}^{(i)} \left[\kappa_\beta^L(t) + \sum_{\tau=0}^{t-1} D_{\beta\gamma}(t-\tau) V_{\gamma\vartheta}^{(i)} \varphi_\vartheta^{0(i)}(\tau)\right] & \text{for } t > 0 \end{cases} \tag{16}$$

Eq. (14) has to be iteratively solved for each time step *t*. Let us note, the matrix $A_{\alpha\gamma}^{(i)}$ is constant for each time step and only the right-hand side of Eq. (14) has to be recalculated. Using the computed virtual distortions $\varphi_\gamma^{0(i)}$ and next using formula (10), the bending strain components $\kappa_\alpha^0(t)$ can be determined. Furthermore, the generalized structural responses $f_A(t)$ (e.g. selected displacement) can be also updated according to the relation:

$$f_A(t) = f_A^L(t) + \sum_{\tau=0}^{t} \breve{D}_{A\alpha}(t-\tau)\kappa_\alpha^0(\tau) \tag{17}$$

where the vector $f_A^L(t)$ contains the pre-computed generalized responses for original structure and $\breve{D}_{A\alpha}(t)$ is the ***generalized influence matrix***. The columns of the matrix $\breve{D}_{A\alpha}(t)$ contain generalized responses corresponding to the vector $f_A(t)$. They are obtained due to introduction of unit, bending-like virtual distortions to the system.

IDENTIFICATION OF SEMI-RIGID JOINTS

For an inverse problem i.e. identification of nodal connection parameters $h_\alpha^{(i)}$, the VDM approach can be effectively applied thanks to its ability of calculation of fast updated responses (cf. Eq. (12), (17)). The identification task can be defined as a gradient-based minimization problem with an assumed objective function. The objective function given below reflects the normalized difference between the known bending strain responses of the modified structure $\kappa_\alpha^M(t)$ (which should be taken from measurements) and the corresponding updated responses $\kappa_\alpha(t)$ (modeled by the virtual distortions $\varphi_\alpha^{0(i)}(t)$):

$$F\left(h_\alpha^{(i)}\right) = \sum_\alpha \frac{\sum_t(\kappa_\alpha(t) - \kappa_\alpha^M(t))^2}{\sum_t(\kappa_\alpha^M(t))^2} \tag{18}$$

Using Eq. (10) and (12), the gradient of the objective function (18) with respect to the optimization variable can be written in the following form:

$$\nabla F_\mu = \frac{\partial F}{\partial \kappa_\alpha}\frac{\partial \kappa_\alpha}{\partial \kappa_\beta^0}\frac{\partial \kappa_\beta^0}{\partial \varphi_\gamma^{0(i)}}\frac{\partial \varphi_\gamma^{0(i)}}{\partial h_\mu^{(i)}}$$
$$= 2\sum_\alpha \frac{\sum_t(\kappa_\alpha(t) - \kappa_\alpha^M(t))}{\sum_t(\kappa_\alpha^M(t))^2}\left[\sum_\tau D_{\alpha\beta}(t-\tau)V_{\beta\gamma}^{(i)}\frac{\partial \varphi_\gamma^{0(i)}(\tau)}{\partial h_\mu^{(i)}} - V_{\alpha\gamma}^{(i)}\frac{\partial \varphi_\gamma^{0(i)}(t)}{\partial h_\mu^{(i)}}\right] \tag{19}$$

where the partial derivatives $\frac{\partial \kappa_\alpha}{\partial \kappa_\beta^0}$ and $\frac{\partial \kappa_\beta^0}{\partial \varphi_\gamma^{0(i)}}$ were obtained using Eq. (12) and (10), respectively. The derivatives $\frac{\partial \varphi_\gamma^{0(i)}}{\partial h_\mu^{(i)}}$ can be calculated by differentiating Eq. (14), which leads to the following relation:

$$A_{\alpha\gamma}^{(i)}\frac{\partial \varphi_\gamma^{0(i)}(t)}{\partial h_\mu^{(i)}}$$
$$= \begin{cases} -\overline{H}_{\alpha\gamma\mu}^{(i)}\varphi_\gamma^{0(i)}(0) & \text{for } t = 0 \\ -\overline{H}_{\alpha\gamma\mu}^{(i)}\varphi_\gamma^{0(i)}(t) + W_{\alpha\beta}^{(i)}\sum_{\tau=0}^{t-1} D_{\beta\gamma}(t-\tau)V_{\gamma\vartheta}^{(i)}\frac{\partial \varphi_\vartheta^{0(i)}(\tau)}{\partial h_\mu^{(i)}} & \text{for } t > 0 \end{cases} \tag{20}$$

In Eq. (20), we have a three dimensional matrix $\overline{H}_{\alpha\gamma\mu}^{(i)} = \frac{\partial H_{\alpha\gamma}^{(i)}}{\partial h_\mu^{(i)}}$. This matrix for μ slice has a two dimensional, square submatrix with zero-values except for $\mu = \alpha = \beta$, i.e. $\overline{H}_{\mu\mu\mu}^{(i)} = 1$. Let us note, the matrix $A_{\alpha\gamma}^{(i)}$ has been calculated previously (cf. Eq. (15)). Eq. (20) has to be iteratively solved for each time step.

The optimization variable can be determined using the steepest descent method:

$${}^{(k+1)}h_\alpha^{(i)} = {}^{(k)}h_\alpha^{(i)} - \Delta^{(k)}F\frac{\nabla_\alpha{}^{(k)}F}{\|\nabla_\alpha{}^{(k)}F\|^2} \tag{21}$$

where the right upper superscript $(k+1)$ denotes the current iteration, and (k) denotes the previous iteration. Δ is a constant in the range of $(0.1 \div 0.3)$.

NUMERICAL EXAMPLE

Let us consider a simple 2D frame structure loaded at node 4 as shown in Fig. 4. The structure is excited by a single Hanning-like impulse function with the time duration of 50ms. The amplitudes of the excitations are equal to: 500N and 750N for horizontal and vertical component of the force P, respectively and 250 Nm for the bending moment M. The structural sections are based on square grid with modular length of 0.51m. All elements have the same physical and geometrical properties: Young's modulus: 210GPa, density: 7850 kg/m^3, cross-section 0.8cm by 8cm. Some analyses using the Newmark method with 500 time steps (analyzed time 500x0.5ms=250ms) were performed for different values of the parameter $h_\alpha^{(i)}$. For element no. 7 in node no. 7, the following values were assumed: 0, 2, 10 $\left[\frac{1}{m\,rad}\right]$. For those cases, the virtual distortions $\varphi_\alpha^{0(i)}$ have been computed using Eq. (14). The corresponding results are shown in Fig. 5. We can see, the greater the value of the parameter $h_\alpha^{(i)}$, the smaller the angular distortion $\varphi_\alpha^{0(i)}$. The updated vertical displacements for node no. 3, presented in Fig. 6 , were calculated using Eq. (17) and compared to the displacements obtained for the original structure (all rigid nodal connections). The differences between the original and current responses tend to be smaller if values of the parameter $h_\alpha^{(i)}$ increase.

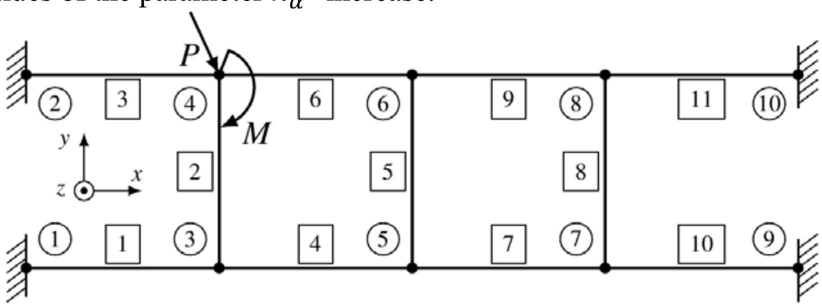

Figure 4. Tested frame structure.

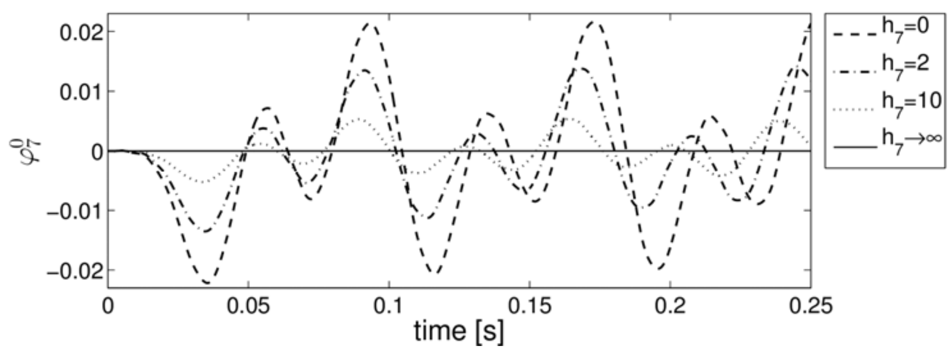

Figure 5. The angular virtual distortions applied to node 7 in element 7 depending on nodal connection.

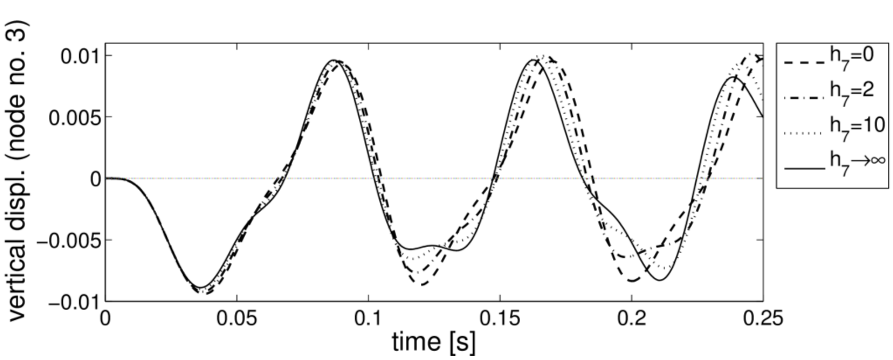

Figure 6. Updated vertical displacement for node 3 at various connection variants for element 7 in node 7.

CONCLUSIONS

In this work, the modeling of semi-rigid joints using the VDM is presented. The model of the nodal connection is assumed to be linear. It is determined by the relation between the nodal bending moment and the perturbation of the angle of rotation called the angular virtual distortion, which is related to previously used strain virtual distortion. Thus, this approach allows for application of the pre-computed influence matrix in various modeling tasks (e.g. modification of axial and bending stiffness, modification of mass).

Purely numerical results of modeling of semi-rigid joints are presented. Gradient-based identification procedure for identification of nodal parameters is proposed. The objective function is defined by the normalized difference between the modeled and measured responses. The measurements are numerically simulated in this paper. In future work, an experimental verification of the proposed theoretical approach is planned.

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