ON BEHAVIOR OF NONLINEAR NINE-NODE SHELL ELEMENTS IN THIN LIMIT

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1. Introduction

The paper concerns nine-node quadrilateral shell elements derived for the Reissner's kinematics. They are based on the Green strain and the potential energy, and are applicable to large (unrestricted) rotations. Drilling rotation is included via the drill rotation constraint (RC) imposed by the penalty method. Hence, the elements have 6 dofs per node, i.e. 3 displacements and 3 rotational parameters, including the drilling rotation.

The basic nine-node isoparametric Lagrangian shell element suffers from locking in unidirectional bending, caused by approximations of the transverse shear strains and of the membrane strains. The too stiff response of the element becomes more visible when the shell thickness decreases.

Several methods of avoiding locking were proposed in the literature. The Uniform Reduced Integration (URI) results in a rank-deficient stiffness matrix, requiring a stabilization. Another method is the Selective Reduced Integration (SRI), where different integration rules are applied to the membrane, bending and transverse shear strain energies. Successful is also the technique based on two-level approximations of strains called the Assumed Strain (AS) method; our nine-node element based on it is described in [1].

2. Formulation of nine-node shell element

In the present work, we consider an *extended configuration space*, defined in terms of the deformation function $\boldsymbol{\chi}$ and rotations $\mathbf{Q} \in SO(3)$. The rotations are constrained by the Rotation Constraint (RC) equation,

$$\text{skew}(\mathbf{Q}^T \mathbf{F}) = \mathbf{0}, \tag{1}$$

where $\mathbf{F} \doteq \nabla \boldsymbol{\chi}$. The two-field 3D functional with rotations is defined as follows:

$$F_2(\boldsymbol{\chi}, \mathbf{Q}) \doteq \int_B \mathcal{W}(\mathbf{F}^T \mathbf{F}) \, \mathrm{d}V + F_{RC} + F_{\text{ext}}, \tag{2}$$

where the RC term has the penalty form,

$$F_{RC} \doteq \int_B \frac{\gamma}{2} \text{skew}(\mathbf{Q}_0^T \mathbf{F}_0) \cdot \text{skew}(\mathbf{Q}_0^T \mathbf{F}_0) \, \mathrm{d}V, \tag{3}$$

and $\gamma \in (0, \infty)$ is the regularization parameter. For shells, only the drilling rotation part is left in the RC, and then $F_{RC} = F_{drill}$.

The initial (reference) configuration of the shell is parameterized in terms of $\boldsymbol{\xi} = \{\xi^\alpha, 2\zeta/h\}$, $\alpha = 1, 2$, where $\xi^\alpha \in [-1, +1]$ are the natural coordinates parameterizing the reference (middle) surface, and $\zeta \in [-h/2, +h/2]$ is the coordinate used in the direction normal to this surface. h denotes the initial shell thickness.

In the current configuration, the position vector is expressed by the Reissner kinematical hypothesis,

$$\mathbf{x}(\xi^\alpha, \zeta) = \mathbf{x}_0(\xi^\alpha) + \zeta \, \mathbf{Q}_0(\xi^\alpha) \, \mathbf{t}_3(\xi^\alpha), \tag{4}$$

where $\mathbf{x}_0$ is the position of the reference surface, $\mathbf{t}_3$ is the shell director, and $\mathbf{Q}_0 \in SO(3)$ is a rotation tensor, which is constant over ζ. For the rotations, we assume that $\mathbf{Q}(\xi^\alpha, \zeta) \approx \mathbf{Q}_0(\xi^\alpha)$, where $\mathbf{Q}_0(\xi^\alpha)$ is the rotation at the reference surface, for details see [2].

For the Reissner's kinematics of Equation (4), the Green strain $\mathbf{E} \doteq \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{I})$ can be approximated linearly over the thickness, i.e. $\mathbf{E}(\zeta) \approx \boldsymbol{\varepsilon} + \zeta\,\boldsymbol{\kappa}$, and the analytical integration of the strain energy over the thickness, yields the shell strain energy density (per unit area of the reference surface) in the additive form, $\mathcal{W}_{\rm sh} = \mathcal{W}_0 + \mathcal{W}_1$, where

$$\mathcal{W}_0 = h\,\left[\tfrac{1}{2}\lambda\,(\mathrm{tr}\boldsymbol{\varepsilon})^2 + G\,\mathrm{tr}\boldsymbol{\varepsilon}^2\right], \tag{5}$$

$$\mathcal{W}_1 = \frac{h^3}{12}\,\left[\tfrac{1}{2}\lambda\,(\mathrm{tr}\boldsymbol{\kappa})^2 + G\,\mathrm{tr}\boldsymbol{\kappa}^2\right]. \tag{6}$$

Note that $\mathcal{W}_0$ consists of the membrane and transverse shear energy, while $\mathcal{W}_1$ of the bending and twisting energy.

3. Numerical tests

We will present a behavior of various our nine-node shell elements for a set of benchmark tests involving very thin structures. One of these tests is the analysis of an initially twisted beam shown in Fig.1. Our results will be compared with the results obtained by the MITC9 element of ADINA and the S9R5 element of ABAQUS.

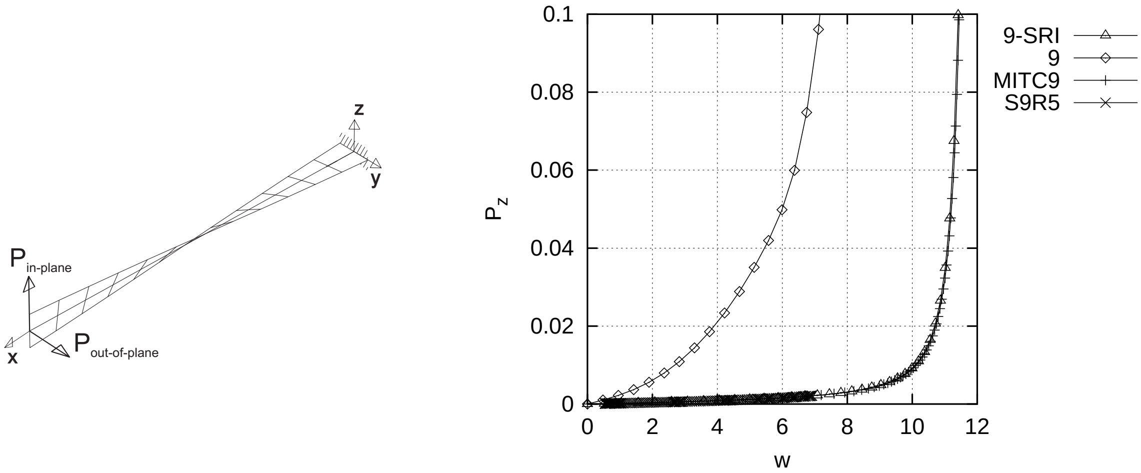

Figure 1. Twisted beam. Nonlinear solution for very thin beam $h = 0.0032$.

Acknowledgements

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References

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[2] Wisniewski K., Turska E.: *Warping and in-plane twist parameter in kinematics of finite rotation shells,* Comput. Methods Appl. Mech. EngngVol.190, No.43–44, 5739–5758 (2001)