PARAMETRIC MODEL OF UNI-DIRECTIONAL FIBER–MATRIX COMPOSITE

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1. Introduction

Uni-directional (UD) composite consists of a huge number of thin, nearly parallel fibers made of one material, immersed in a matrix made of another material. UD tows are formed in wide thin tapes used as laminae in multi-layer (sandwich) laminate structures, as well as in multi-fiber yarns used to produce woven-fabric laminae that again serve to build-up sandwich laminates. An example of such composite is carbon/epoxy composite widely exploited in aircraft structures.

Macroscopic material properties of the composite (treated as homogeneous continuum) obviously depend on properties of its microstructure constituents (fibers and matrix) as well as of their geometric arrangement. Our objective is to establish the parametric relationship between them, i.e. present the constitutive equation in such a form that enables to predict the mechanical properties of a UD tow given values of certain parameters that uniquely describe its microstructure. Consequently, one should be able to determine sensitivity of the macroscopic mechanical properties to variations of the parameters.

In the case of the UD tows, the following microstructure parameter set is of our interest:

$$\mathbf{h} = \{h_k\} = \{\, E^{\mathrm{m}} \,,\, \nu^{\mathrm{m}} \,,\, E^{\mathrm{f}}_p \,,\, E^{\mathrm{f}}_t \,,\, \nu^{\mathrm{f}}_{pt} \,,\, \nu^{\mathrm{f}}_{tt} \,,\, G^{\mathrm{f}}_{pt} \,,\, \varphi \,\} \tag{1}$$

where E^{m}, ν^{m} are isotropic elastic constants of matrix, E^{f}_p, E^{f}_t, ν^{f}_{pt} , ν^{f}_{tt} , G^{f}_{pt} are transversely isotropic elastic constants of fiber (p and t correspond, respectively, to the parallel and transverse directions to the fiber axis) and φ is the fiber volume ratio. The objective is to compute the elastic moduli of the composite as functions of these parameters. It is expected (and will be proven in computations) that the resulting properties are transversely isotropic, thus the following functions will be sought for,

$$E_p(\mathbf{h}), \quad E_t(\mathbf{h}), \quad \nu_{pt}(\mathbf{h}), \quad \nu_{tt}(\mathbf{h}), \quad G_{pt}(\mathbf{h}). \tag{2}$$

There are several analytical solutions to this problem — let us mention approximate formula based on the classical mixture theory, as well as much more advanced models like e.g. those of Hashin–Rosen [1] and of Halpin–Tsai [2]. Unfortunately, they are all incomplete. Even the most sophisticated Hashin–Rosen model allows to only determine 4 of 5 elastic constants and requires assumption of fiber isotropy. Thus, the use of numerical methods seems inevitable to address the problem in a complete manner.

2. Methods

To assess the macroscopic elastic constants for a particular instance of $\mathbf{h}$ (i.e. for particular values of h_k) the well known homogenization method based on the concept of a periodic representative volume element (RVE) is used, see e.g. [3] for details. The original contribution in this formulation is averaging of results over different fiber arrangement patterns, defined by the angle α in Fig. 1a. The real distribution of fibers in the microstructure is assumed an average of patterns shown in Fig. 1a with the angles α and β assumed equally distributed within the limits $60° \le \alpha \le 120°$ and $0 \le \beta \le 180°$. Orthotropic elastic constants (more strictly – elements of the stiffness matrix $\mathbf{C}$) determined with the RVE presented in Fig. 1b at a given fiber ratio value φ are then averaged by consecutive integration over the two angles. As a result, transversely isotropic stiffness matrix is obtained and the engineering constants E_p , E_t , ν_{pt} , ν_{tt} , and G_{pt} are computed.

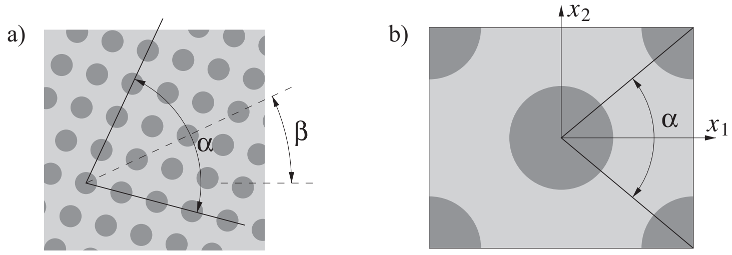

Figure 1. Parameterized pattern of fibers in the UD composite (a); a representative cross-section element (b)

In order to determine the parametric constitutive model, i.e. the relationships (2), one needs to perform the above described homogenization for a sufficiently large set of values of the parameters $\{h_k\}$. This has been done with the use of a multi-processor workstation with eight Intel Xeon processors and with the commercial code Abaqus. A dedicated parametric mesh generator has been created to obtain high quality meshes of the RVE at different values of fiber ratio φ and pattern angle α. Each of the parameters was assigned a set of its 4 to 6 characteristic values which resulted in as much as 72 000 different computational cases to be analysed. Only the parameter E_p^{f} was assumed unit in all computations (with all other stiffness moduli defined as its fraction) — the resulting moduli E_p , E_t , and G_{pt} need thus to be finally scaled by the actual value of the modulus E_p^{f}.

The results have a form of tabularized data base of composite elastic constants for the given 'instances' of microstructure, defined with particular sets of the parameter values $\{h_k\}$. Elastic constants for an arbitrary set of parameter values can be obtained by appropriate interpolation of the data base results. Sensitivity of the composite elastic constants with respect to the microstructure parameters can be also easily obtained with the same interpolation procedure.

3. Conclusion

A complete parametric elastic constitutive model of the UD composite allows to more easily design composite structures, perform sensitivity analysis of its response with respect to the microstructure parameters, as well as to numerically optimize the microstructure properties in view of better global structural performance.

References

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