Sensitivity analysis of deep drawing process with rigid-viscoplastic material model *

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Abstract

The objective of the paper is formulation of sensitivity analysis for flow approach simulations of deep drawing problems with respect to arbitrary design parameters. First, finite element formulation of the primary problem is presented. Its important feature is the full algorithmic tangent viscosity matrix which, as it will be shown, is a necessary tool in sensitivity calculations. The algorithmic (consistent) tangent matrix is unfortunately asymmetric, which is addressed in our considerations. The semi-analytical formulation of sensitivity is used, which means that some complex design derivatives in the sensitivity equations are estimated by finite difference method. The finite difference method will be used as a reference method for verification of the sensitivity results.

Keywords: FEM, finite element method, flow approach, sensitivity analysis

1. Introduction

The flow approach to modeling of large plastic deformation processes [1, 7] is a numerically efficient alternative to traditional displacement-based finite element formulations, both implicit quasi static and explicit dynamic simulations. It is based on the rigid-viscoplastic constitutive equation in which stress is a function of strain rate and the form of the equation is analogous to nonlinear elasticity, with strain replaced by strain rate. The formulation is suitable, in particular, to model deep drawing of metal sheets [8] or other plastic metal forming processes, like rolling or extrusion.

The most efficient known optimization algorithms are the gradient methods. To effectively use them, one needs information on design sensitivity of the structural response. Design sensitivity analysis has been widely discussed regarding traditional displacement FE formulations [3], including also large elasto-plastic deformations [5,6]. Although issues of sensitivity analysis in flow approach formulations of sheet metal forming have also been addressed in the literature [2, 4, 9], complete formulation of the problem including geometric nonlinearities and full form of the consistent tangent matrix has not yet been developed.

The present paper is an attempt to fill this gap in the state of the art. The results of sensitivity analysis may allow to fully integrate the deep drawing algorithm with a gradient-based optimization system and thus accelerate the process of optimization of tool geometry and stamping control parameters.

2. Formulation of rigid-viscoplasticity problem

2.1. Finite element formulation

In sheet metal forming analysis, shell formulation with plane-stress constraints is adopted for sheet. Under this assumption, the constitutive equation of a rigid-viscoplastic material assumes the form

$$\boldsymbol{\sigma} = \boldsymbol{D}(\dot{\boldsymbol{\varepsilon}})\dot{\boldsymbol{\varepsilon}} \tag{1}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are stress and strain, respectively, and $\boldsymbol{D}$ denotes strain-rate-dependent viscosity matrix. Formal analogy between equation (1) and a formulation of nonlinear elasticity makes it easy to solve the plastic flow problem in a similar manner as nonlinear elasticity problems, with strains replaced by strain rates and nodal displacements with velocities.

The finite element equilibrium equation assumes the vector-matrix form,

$$\bar{\boldsymbol{K}}(\boldsymbol{q}, \dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} = \boldsymbol{F}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \tag{2}$$

in which $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ and $\boldsymbol{F}$ denote generalized nodal displacement, velocity and external load vectors, respectively, while $\bar{\boldsymbol{K}}$ is the secant viscosity matrix. $\boldsymbol{F}$ is a sum of prescribed loads applied to sheet, reactions due to kinematic boundary conditions (prescribed velocities) and contact reactions, all of them dependent on $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$ (contact is modelled with penalty approach).

Since $\bar{\boldsymbol{K}}$ is actually also dependent on history of deformation, the problem is solved in subsequent time steps, with the following implicit integration scheme assumed for generalized nodal displacements at $t_{n+1} = t_n + \Delta t$,

$$\boldsymbol{q}^{n+1} = \boldsymbol{q}^{n} + \left[(1-\vartheta)\dot{\boldsymbol{q}}^{n} + \vartheta\dot{\boldsymbol{q}}^{n+1}\right]\Delta t \tag{3}$$

where the solution $\boldsymbol{q}^n$ and $\dot{\boldsymbol{q}}^n$ is known from the previous time step and $\vartheta \in (0, 1]$ is the implicit integration parameter.

2.2. Iterative computation scheme

Having obtained the solution for the time instant t_n, we can solve the nonlinear equation system (2) for t_{n+1} with the use of the Newton iteration scheme. The solution procedure consists in calculation of the secant viscosity matrix $\bar{\boldsymbol{K}}$ and the force vector $\boldsymbol{F}$ for consecutive approximations of $\dot{\boldsymbol{q}}$ and determination of its correctors $\delta\dot{\boldsymbol{q}}$, according to

$$\frac{\mathrm{d}}{\mathrm{d}\dot{\boldsymbol{q}}}\left(\bar{\boldsymbol{K}}\dot{\boldsymbol{q}} - \boldsymbol{F}\right)\delta\dot{\boldsymbol{q}} = \boldsymbol{F} - \bar{\boldsymbol{K}}\dot{\boldsymbol{q}}\,, \qquad \dot{\boldsymbol{q}} := \dot{\boldsymbol{q}} + \delta\dot{\boldsymbol{q}} \tag{4}$$

The above scheme is repeated until the convergence criterion is fulfilled. It can be written in the compact form as

$$\boldsymbol{K}\,\delta\dot{\boldsymbol{q}} = \boldsymbol{r} \tag{5}$$

where

$$\boldsymbol{K} = \bar{\boldsymbol{K}} + \frac{\mathrm{d}\bar{\boldsymbol{K}}}{\mathrm{d}\dot{\boldsymbol{q}}}\dot{\boldsymbol{q}} - \frac{\mathrm{d}\boldsymbol{F}}{\mathrm{d}\dot{\boldsymbol{q}}}\,, \qquad \boldsymbol{r} = \boldsymbol{F} - \bar{\boldsymbol{K}}\dot{\boldsymbol{q}}, \tag{6}$$

are called, respectively, the algorithmic tangent viscosity matrix and the vector of unbalanced (residual) nodal forces corresponding to the current approximate solution $\dot{\boldsymbol{q}}$. Note that, in view of

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equation (3), the following relation holds at the current time instant t_{n+1} for any variable dependent on both $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$:

$$\frac{\mathrm{d}(\cdot)}{\mathrm{d}\dot{\boldsymbol{q}}} = \frac{\partial(\cdot)}{\partial\dot{\boldsymbol{q}}} + \vartheta\Delta t\,\frac{\partial(\cdot)}{\partial\boldsymbol{q}}\,. \tag{7}$$

2.3. Algorithmic tangent matrix

The full expression of the tangent viscosity matrix $\boldsymbol{K}$ is quite elaborated, as it includes derivatives of numerous geometrically nonlinear variables, expressions for contact forces etc. Most of the components appear to be symmetric, however, some are not. Fortunately, the asymmetric terms are only those related to dependence of $\bar{\boldsymbol{K}}$ and $\boldsymbol{F}$ on nodal displacements $\boldsymbol{q}$ and thus, according to (7), they contribute to $\boldsymbol{K}$ mutiplied by $\vartheta\Delta t$. In other words,

$$\boldsymbol{K} = \boldsymbol{K}_{\mathrm{symm}} + \boldsymbol{K}_{\mathrm{asymm}}\vartheta\Delta t. \tag{8}$$

This allows to estimate the asymmetric terms as small and neglect them in the iteration procedure without significant loss in convergence speed.

3. Sensitivity analysis

Differentiating the main system of equations with respect to a design parameter h, one can obtain

$$\frac{\mathrm{d}\bar{\boldsymbol{K}}}{\mathrm{d}h}\dot{\boldsymbol{q}} + \bar{\boldsymbol{K}}\frac{\mathrm{d}\dot{\boldsymbol{q}}}{\mathrm{d}h} = \frac{\mathrm{d}\boldsymbol{F}}{\mathrm{d}h}\,. \tag{9}$$

Note that all terms depend on h either explicitly, or through other variables that are design-dependent. Among those variables, some are known at the beginning of the time step (so their design derivatives are also known) and some — and here we are talking about $\dot{\boldsymbol{q}}$ — are not, and the derivative $\frac{\mathrm{d}\dot{\boldsymbol{q}}}{\mathrm{d}h}$ is unknown. For simplicity of notation, let us define the 'explicit design derivative' of any variable $a(\dot{\boldsymbol{q}}(h),\ldots,h)$ as

$$\frac{\tilde{\mathrm{d}}a}{\mathrm{d}h} = \left.\frac{\mathrm{d}a}{\mathrm{d}h}\right|_{\dot{\boldsymbol{q}}=\mathrm{const}}, \qquad \text{i.e.} \quad \frac{\mathrm{d}a}{\mathrm{d}h} = \frac{\mathrm{d}a}{\mathrm{d}\dot{\boldsymbol{q}}}\frac{\mathrm{d}\dot{\boldsymbol{q}}}{\mathrm{d}h} + \frac{\tilde{\mathrm{d}}a}{\mathrm{d}h}\,. \tag{10}$$

The so defined explicit derivative includes all the design derivatives of arguments of a that are known at the beginning of the time step computations.

With this notation, equation (9) can be rewritten after transformations as

$$\boldsymbol{K}\frac{\mathrm{d}\dot{\boldsymbol{q}}}{\mathrm{d}h} = \frac{\tilde{\mathrm{d}}\boldsymbol{r}}{\mathrm{d}h}\,. \tag{11}$$

This means that one has to solve a linear system of equations with the same tangent coefficient matrix as was used in the last equilibrium iteration. This makes the sensitivity computations very time-efficient, especially that the matrix $\boldsymbol{K}$ has already been decomposed at the moment. All one has to do is to build the right hand side vector in (11).

It must be admitted that the matrix we actually use in the computations is in fact only the symmetric approximate of the full tangent matrix. This does not usually introduce significant error in the results obtained. However, to make the analysis more strict, one can consider solving the system (11) in iterations, with moving the asymmetric terms to the right-hand side. This will slightly extend the computation time.

Computation of the right-hand side vector given in equation (11) for an arbitrary design parameter may appear a very difficult task. In order to ensure the highest possible level of generality of the program we assume that the sensitivity will be determined by the semi-analytical differentiation method. In this approach, the explicit design derivative of $\boldsymbol{r}$ is estimated using the following formula:

$$\left.\frac{\tilde{\mathrm{d}}\boldsymbol{r}(\dot{\boldsymbol{q}}(h),\ldots,h)}{\mathrm{d}h}\right|_{h=h_0} \approx$$
$$\approx \frac{\boldsymbol{r}(\dot{\boldsymbol{q}}(h_0)\,,\,\ldots\,,\,h_0{+}\Delta h) - \boldsymbol{r}(\dot{\boldsymbol{q}}(h_0)\,,\,\ldots\,,\,h_0)}{\Delta h}\,, \tag{12}$$

where Δh is perturbation of design parameter h around its primary value h_0.

4. Final remarks

As it appears from the presented formulation, the design sensitivity problem is linear and thus time-efficient. The consistent tangent viscosity matrix is necessary in the formulation. Unfortunately the matrix is not symmetric but negligence of the asymmetric terms does not appear to introduce significant errors in the results.

The fully analytical implementation of design sensitivity is very difficult. The presented semi-analytical formulation employs finite difference quotient to compute the explicit design derivative of nodal residual forces.

Numerical examples will be presented at the conference presentation.

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