PARTITIONED THERMO-MECHANICAL COUPLING OF SMA CONSTITUTIVE MODEL

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1. General

For many kinds of materials (e.q. metals, shape memory alloys, etc.), thermal changes coupled with the deformation process can have significant effect on the mechanical response and also on the accuracy of numerical analysis. In order to take them into account, monolithic coupling algorithms have been used, however a partitioned approach can also be applied. An advantage of the partitioned thermo-mechanical coupling is the possibility to re-use already developed software to simulate various kinds of multiphysics scenarios, i.e. various coupled fields: structure, heat transfer, fluids, magnetic, etc. To realize the partitioned approach, communication protocols for exchange of information between the used codes are needed [1]. To this end, Component Template Library (CTL) [2], developed at the Institute of Scientific Computing, TU Braunschweig, Germany, is used as a middleware between the FEM programs for the structural analysis (PAKS) and the heat transfer analysis (PAKT).

2. Methods

The CTL is a library which provides an exchange of information between the program codes through communication protocols. The PAKT and PAKS programs, developed at the Faculty of Engineering, University of Kragujevac, are transformed into the components with the communication interface. The SMA constitutive model based on the Lagoudas theory [3] is modified and adopted to depend only on scalar values of effective stress, strain and martensitic volume fraction and is implemented into the PAKS program. The martensitic phase transformation is related to a dissipation of internal energy and a change of the material temperature what violates the thermodynamic equilibrium in the structure. The PAKS program has been extended to compute the energy released during the phase transformation which behaves as an internal heat source in the heat transfer component PAKT. The partitioned coupling approach, based on assumptions of Block-Gauss-Seidel numerical procedure [1, 4] is employed to re-establish the balance.

3. Partitioned coupling algorithm

In order to use the partitioned approach, a thermo-mechanical problem has to be defined as a nonlinear system of equations $f(\approxeq, T) = 0$ and $g(\approxeq, T) = 0$ where f, g are thermal and structural subproblems and $\approxeq$ and T are displacement and temperatures, respectively [4]. The first equation is solved by PAKT, while the second one is solved by PAKS. To calculate the influence of a run of the phase transformation on the temperature field, the dissipative energy [3, 5] is calculated in eachi ntegration point as follows:

$$W_i = (\psi_i - \rho \Delta s_0 T_i) \Delta \xi_i \,,$$

where i is the current number of integration point, ψ is the thermodynamic force, $\rho \Delta s_0$ is the stress influence coefficient, T is the temperature in integration point and $\Delta \xi$ is the increment of martensitic volume fraction in the time step. The dissipative energy of the whole construction and the global convergence criterion can be defined as:

$$W^k = \sum_{i=1}^{P} W_i^k \,, \qquad \frac{W^k - W^{k-1}}{W^{k-1}} < tol_w \,,$$

where P is the number of integration points, k is the iteration counter and tol_w is the minimal desired tolerance.

4. Numerical results

This coupling algorithm has been verified using the experimental results of TiNi SMA subjected to tension with various strain rates carried out at the IPPT PAN [5]. The stress (black line) and temperature changes (red line) obtained at the strain rate 10^{-3} s^{-1} are presented by solid line, whereas the numerical results of the stress and the temperature changes by dotted lines, respectively (Fig. 1). A good agreement between the experimental and numerical data has been obtained.

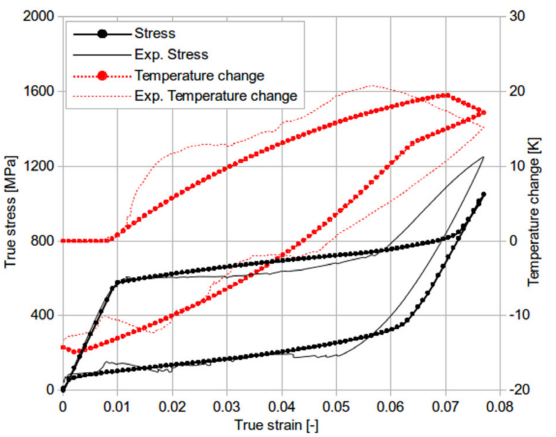

Fig. 1. Comparison of numerical and experimental results for TiNi SMA at strain rate 10^{-3} s^{-1}.

5. Conclusions

The partitioned coupling algorithm, based on the CTL, PAKS and PAKT programs, proved to be a good approach for the accurate numerical analysis of the stress-induced martensitic transformation developing in TiNi SMA subjected to tension. A similar approach can be used for advances thermo-mechanical analysis in the case of other thermo-sensitive materials.

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