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MICRO-MACRO DEPENDENCY FOR ELASTIC CONSTANTS IN A NUMERICAL MODEL OF GRANULAR MATERIAL

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1. Introduction

The discrete element method (DEM) is a suitable tool to model granular materials. In the DEM, a material is represented by an assembly of particles interacting among one another with contact forces. Interparticle interaction models can be based on different types of contact laws incorporating different physical effects such as elasticity, viscosity and friction. The contact model in the DEM can be treated as a micromechanical material model. Determination of micromechanical parameters is a key issue in the use of the DEM. Appropriate micromechanical allow us to obtain desired macroscopic behaviour. There is still a lack of full understanding of many micromechanical mechanisms which are inherent in the DEM and influence macroscopic behaviour of DEM models [1]. This paper is devoted to investigation of micro-macro dependency for elastic constants in a DEM model of granular material.

2. Numerical and analytical methodology for determination of micro-macro relationships

Macroscopic behaviour of a granular material has been investigated numerically performing simulation of the triaxial compression test which is a standard laboratory test procedure widely used to measure mechanical properties of soils and other granular materials. The discrete element simulations have been carried out using the DEM program Dempack [2]. A cylindrical specimen of 5,500 particles with confining membrane walls has been generated. Similar to the experimental procedure, after application of the confinig pressure, the axial strain has been increased monotonically while keeping constant the pressure on the lateral walls. The elastic moduli, the Young's modulus and Poisson's ratio, have been determined in a standard way from the initial slopes of the stress–axial strain and volumetric-axial strain curves.

Numerically determined macroscopic properties have been compared with theoretical nalytical predictions according the Voigt and best fit hypotheses described in [3]. The analytical formula are presented in Table 1, where the elastic macroscopic moduli, E and v are expressed in terms of the micromechanical parameters: k_n and k_t – the contact stiffness in the normal and tangential direction, r – the average radius of the particles, N – the total number of inter-particle contacts in the volume V.

Table 1. Analytical estimation of the elastic moduli of the particle assembly.

Parameter	Voigt hypothesis	Best fit hypothesis
Young's modulus	$E = \frac{4Nr^2}{3V} \cdot \frac{2k_n + 3k_t}{4k_n + k_t}$	$E = \frac{20Nr^2}{3V} \cdot \frac{k_t}{2k_n + 3k_t}$
Poisson's ratio	$\nu = \frac{k_n - k_t}{4k_n + k_t}$	$\nu = \frac{k_n - k_t}{2k_n + 3k_t}$

3. Discussion of the results

Numerical simulations have been performed for the confining pressure 100 kPa, the contact stiffness in the normal direction k_n from the interval 9 kN/m – 1.3 MN/m and the k_t/k_n ratio from the interval $(0.1-1.0)$. The results are presented in Figs. 1 and 2 in the form of the curves showing the relationships between the dimensionless parameters: $Er/2k_n$, v and k_t/k_n for different values of k_n. The dependence of the Poisson's ratio on the k_t/k_n ratio for different values of k_n is plotted in Fig. 2. Numerical results in Figs. 1 and 2 are compared with the analytical estimations according to the Voigt and best fit hypotheses. Quite a good agreement can be observed especially for lower values of k_n and k_t/k_n. The dependence of the Young's modulus E on the microscopic stiffness k_n is shown in Fig. 3 in comparison with the results obtained by other authors.

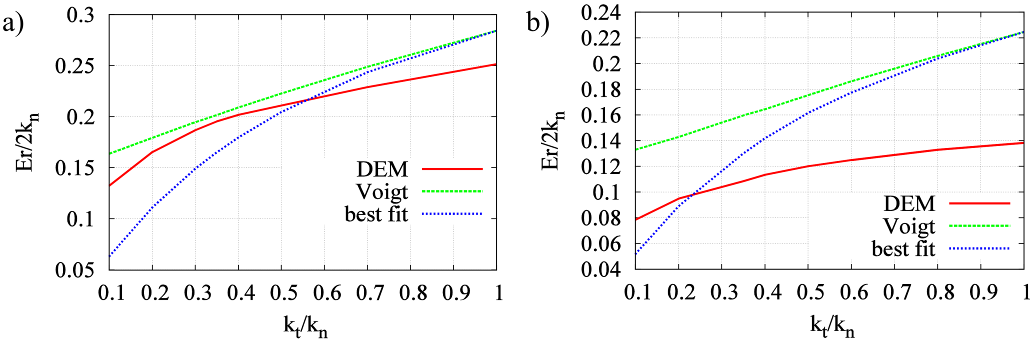

Fig. 1. Dimensionless micro-macro relationship for the Young's modulus for different values of k_n: a) 9 kN/m, b) 34 kN/m.

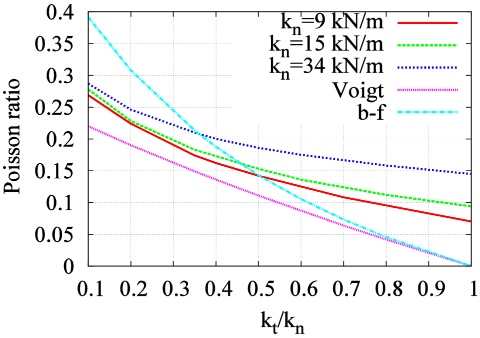

Fig. 2. Micro-macro relationships for the Poisson's ratio.

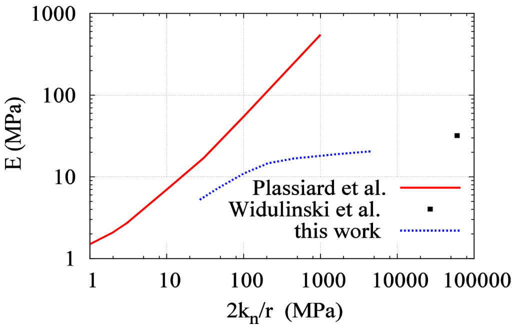

Fig. 3. Comparison of the numerical relationships from different works.

References

1. J. Rojek, C. Labra, O. Su, E. Oñate (2012). Comparative study of different discrete element models and evaluation of equivalent micromechanical parameters, *International Journal of Solids and Structures*, **49**, 1497–1517.
2. CIMNE (2010), *Dempack, explicit nonlinear dynamic analysis by the finite and discrete element method*, web:www.cimne.upc.edu/dempack.
3. C.L. Liao, T.P. Chang, D.H. Young (1997). Stress-strain relationship for granular materials based on the hypothesis of best fit, *Int. J. Solid. Structures*, **34**, 4087–4100.
4. J. Plassiard, N. Belheine, F. Donze (2009). A spherical discrete element model: calibration procedure and incremental response, *Granular Matter*, **11**, 293–306.