Multiobjective shape optimization of selected coupled problems by using evolutionary algorithms

Adam Długosz[1] **and Tadeusz Burczyński**[1,2]

[1]*Department for Strength of Materials and Computational Mechanics, Silesian University of Technology, ul. Konarskiego 18a 44-100, Gliwice, Poland*
adam.dlugosz@polsl.pl; tadeusz.burczynski@polsl.pl

[2]*Institute of Computer Science, Cracow University of Technology, ul. Warszawska 24, 31-155, Cracow, Poland*
tburczyn@pk.edu.pl

Abstract

In this paper an improved multiobjective evolutionary algorithm is used for Pareto optimization of selected coupled problems. The proposed algorithm is compared with the Non-Dominated Sorting Genetic Algorithm (NSGA-II) for several test benchmark problems of unconstrained and constrained optimization. The results of the tests show its usefulness as an optimization tool, which is used for shape optimization of different structures modelled as coupled field problems. Coupling of the mechanical, electrical and thermal fields is considered in the paper. Boundary-value problems of the thermo-elasticity piezoelectricity and electro-thermo-elasticity are solved by means of finite element method. Different types of functionals are formulated on the basis of results obtained from coupled field analysis. Numerical examples for exemplary two- or three-objective optimization are presented.

Keywords: multiobjective optimization, evolutionary algorithms, multiphysics, coupled field problems, finite element method

1. Introduction

Shape optimization of structures is an important phase in engineering design. Real-world problems often have multiple conflicting objectives. It requires the application of efficient multiobjective optimization tools, especially for complex problems. In multi-objective optimization problems, there are several objectives or cost functions to be minimized or maximized simultaneously. Conflicting objectives cause that one objective function improves and another deteriorates. Obviously, in these problems there is no single solution that is the best with respect to all objectives. The designer has to chose a solution from a set of solutions which is called optimal in the sense of Pareto. For a Pareto optimal solution there exists no other feasible solution which would decrease some objectives (suppose a minimization problem) without causing simultaneous increase in at least one other objective. With this definition of optimality after optimization several trade-off solutions are achieved (*Pareto optimal set*). In the present paper multiobjective shape optimization is performed for selected multiphysics tasks. Three different coupled filed problems are considered: thermoelesticity, piezoelectricity and coupling of electrical, thermal and mechanical fields. Boundary-value problems are solved by means of Finite Element Method (FEM) [1,5]. Different types of functionals are formulated on the basis of results obtained from coupled field analysis.

2. MOOPTIM tool

Among many different types of multiobjective genetic and evolutionary algorithms, Strength Pareto Evolutionary Algorithm [4] and Non-Dominated Sorting Genetic Algorithm [2] are the most popular multiobjective optimization tools. Consecutive versions of such algorithms SPEA2 and NSGAII represent the state-of-the-art in evolutionary MOPs and have many practical applications in different engineering disciplines. In this work, own implementation of the MultiObjective OPTIMization tool based on evolutionary algorithm (MOOPTIM) is used for optimization. Some specific methods implemented in NSGAII are applied in MOOPTIM. Compared to the NSGAII, the proposed implementation has more evolutionary operators. An other difference between these algorithms is related to the formation of parent population, there is no binary tournament selection operator and some other modifications. The algorithm was tested on several benchmark problems and engineering problems. The results obtained by using MOOPTIM in most cases are better in comparison with the results obtained by using NSGAII [3]. Figure 1 presents a example of the effectiveness of proposed algorithm compared to the NSGAII. Such problem (ZDT-4) has a large number of local Pareto fronts. Computations has been performed for the same number of fitness evaluations. For 30 independent runs MOOPTIM founds a set of Pareto solutions much closer to the true Pareto front than to the NSGAII.

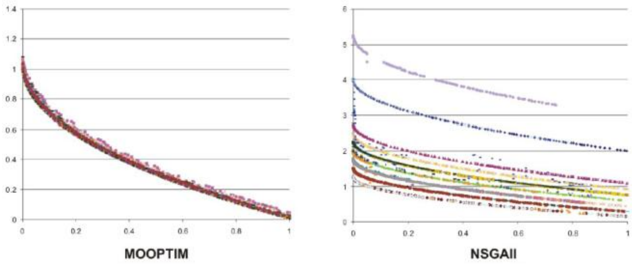

Figure 1: MOOPTIM – NSGAII comparison on ZDT-4 problem

3. Formulation of the problem

MOOPTIM is applied to the shape optimization of different structures by the minimization or maximization of appropriate functions. In the present work following types of boundary value problems are considered: thermoeleasticity, piezoelectricity and electrical-thermal-mechanical analysis. These problems are described by the appropriate partial differential equations. The equations with arbitrary geometries and boundary conditions are usually solved by numerical methods. FEM is used to solve boundary-value problems. Thermoelesticity and electro-thermoelasticity is weakly coupled, which requires solving electrical, thermal and mechanical analysis separately. Coupling is carried out by transferring loads between the considered analysis and by using staggered procedures. Matrix equations of electrical, thermal and mechanical problem can be expressed as follows:

$$\mathbf{K}_E \mathbf{V} = \mathbf{I} \tag{1}$$

$$\mathbf{K}_T \mathbf{T} = \mathbf{Q} + \mathbf{Q}_E \tag{2}$$

$$\mathbf{K}_M \mathbf{u} = \mathbf{F} + \mathbf{F}_T \tag{3}$$

where: where $\mathbf{K_E}$ is the electrical conductivity matrix, $\mathbf{K_T}$ is the thermal conductivity matrix, $\mathbf{K_M}$ is the stiffness matrix, $\mathbf{Q_E}$ is the heat generation vector due to current flow, $\mathbf{F_T}$ is the force due to thermal strain vector, **V**, **T**, **u**, are the nodal vector of voltage, temperature, displacements, respectively, **I**, **Q**, **F**, are the nodal vector of current, heat fluxes and applied forces, respectively. The thermal and mechanical problems are coupled through thermal strain loads $\mathbf{F_T}$. Coupling between the electrical and thermal problems is done by the heat generation due to electrical flow $\mathbf{Q_E}$. Piezoelectricity couples electrical and mechanical fields. This problem is solved by using strong coupling method. It requires the usage of coupled finite elements, which have all mechanical and electric degrees of freedom (displacements and electric potential). Matrix equations of static piezoelectricity can be expressed as follows:

$$\begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\varphi} \\ \mathbf{K}_{\varphi u} & \mathbf{K}_{\varphi\varphi} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{\Phi} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_u \\ \boldsymbol{\rho}_\varphi \end{bmatrix} \tag{4}$$

where $\mathbf{K}_{uu}$ is mechanical stiffness matrix, $\mathbf{K}_{u\varphi}$, $\mathbf{K}_{\varphi u}$ are piezoelectric stiffness matrices, $\mathbf{K}_{\varphi\varphi}$ is dielectric stiffness matrix, $\mathbf{F}_u$ is force vector and $\boldsymbol{\rho}_\varphi$ is charge flux vector.

FEM software Patran/Nastran, Mentat/ Marc and Ansys Multiphisics are used to solve all multiphisics problems.

Different functionals based on the results derived from coupled field analyses are formulated. For the considered problems functionals can be calculated on the basis of nodal results of electrical, thermal and mechanical quantities. Objective function values are also calculated on the basis of area and volume of the structure.

4. A numerical example

An example of multiobjective shape optimization of MEMS structures is presented. The array of the three thermal actuators fabricated from polycrystalline silicon is considered (Figure 2). The device is subjected to the electrical, thermal and mechanical boundary conditions. The multiobjective problem concerns determining the specified dimension of the actuators, which minimizes the volume of the structure, minimize the maximal value of the equivalent stress and maximize deflection of the arms. 6 design variables are assumed for the optimization task.

Figure 3 presents a set of Pareto optimal solution for the minimization of the volume and maximization of deflection, whereas Figure 4 presents the results of minimization of the volume and the minimization of the maximal value of equivalent stress.

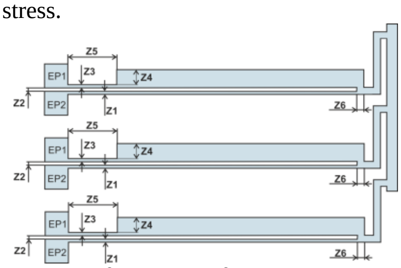

Figure 2: Geometry of the array of the actuators

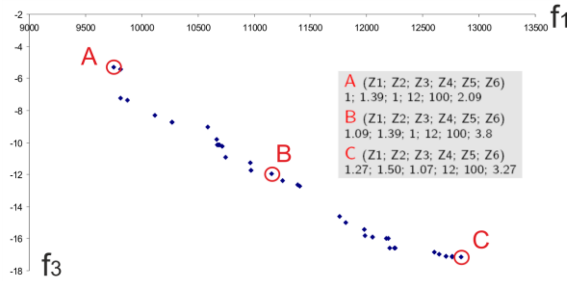

Figure 3: The set of Pareto optimal solutions for minimization volume and maximization of the deflection of the actuator

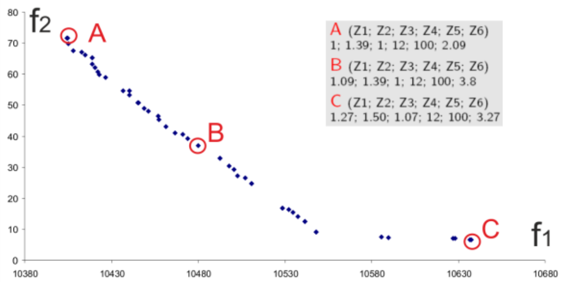

Figure 4: The set of Pareto optimal solutions for minimization volume and Von Mises stress

5. Final remarks

In the present work the MOOPTIM algorithm has been used for multiobjective shape optimization of structures. The direct problems concern coupling beetween mechanical, thermal and electrical field. The application of the FEM software requires evaluation in several steps for each single solution (the modification of the geometry, creating finite element mesh, etc.). It can be a very-time consuming task, especially for more complicated geometry. Moreover the solution of the coupled problems is more time-consuming when compared to the single-field problem. To reduce the time of the optimization, parallel computation of can be used.

Acknowledgment

The research is financed by the Polish science budget resources in the years 2008-2011 as a research project.

6. References

[1] Beer, G., Finite element, boundary element and coupled analysis of unbounded problems in elastostastics, *Int. Meth. Eng.*, vol. 19, pp. 567-580, 1983.

[2] Deb, K., Pratap, A, Agarwal, S., and Meyarivan, T, A fast and elitist multi-objective genetic algorithm: NSGA-II, *IEEE Transaction on Evolutionary Computation*, 6(2), pp. 181-197, 2002.

[3] Długosz, A., Multiobjective Evolutionary Optimization of MEMS Structures, *Computer Assisted Mechanics and Engineering Sciences*,vol. 17, no.1, pp.41-50, 2010.

[4] Zitzler, E., Laumanns, M., Thiele, L., SPEA2: Improving the Strength Pareto Evolutionary Algorithm, *TIK-Report 103*, 2001.

[5] Zienkiewicz, O.C., Taylor, R.L., *The Finite Element Method*, Butterworth Heinemann, Oxford 2000.