

NUMERICAL ANALYZE AND EXPERIMENTAL RESULTS OF ULTRASONIC WAVES SCATTERING ON A MODEL OF THE ARTERY

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The aim of this paper is to compare the results of the mathematical modeling and experimental results of the ultrasonic waves scattering in the inhomogeneous dissipative medium. The research was carried out for an artery model (a pipe made of a latex), with internal diameter of 5 mm and wall thickness of 1.25 mm. The numerical solver was created for calculation of the fields of ultrasonic beams and scattered fields under different boundary conditions, different angles and transversal displacement of ultrasonic beams with respect to the position of the arterial wall. The investigations employed the VED ultrasonic apparatus. The good agreement between the numerical calculation and experimental results was obtained.

1. INTRODUCTION

The pathological process of atherosclerosis development and its connection with alterations that occur in walls of blood vessels present a matter of interest for numerous scientific and clinical centers worldwide. In case of non-invasive investigations, the ultrasonic measurements for momentary diameters of arteries over the entire cardiac cycle serve as the basis enabling to determine elasticity of arterial walls. Maximum and minimum values for the vessel diameter are associated with respective systolic and diastolic blood pressures measured by a sphygmomanometer. Based on the above measurements, the elasticity factors of the arterial wall are determined [1,2]. In case of non-invasive ultrasonic measurement, reproducibility of the obtained results is an extremely important parameter, since it is used to define sensitivity of the diagnostic tool [3]. The major objective of the thesis was to develop a mathematical model that would be capable of describing spatial and time-dependent distribution of an ultrasonic beam that is emitted by a piezoelectric ring transducer and then backscattered on cylindrical surfaces of the walls in artery models. The developed model was

tested for results of experiments when an elastic pipe was immersed in water. The investigations were carried out using the VED equipment, designed and constructed in the Ultrasonic Department of the Institute of Fundamental Technological Research of the Polish Academy of Sciences, purposefully dedicated for elasticity examination of arterial walls in human body.

2. PHYSICAL MODEL

With use of non-dimensional variables, the equation that defines the propagation of sonic waves in a homogenous (with undisturbed parameters of the material) non-linear and absorbing medium, can be expressed by the following equation [4]:

$$\Delta P - \partial_{tt}P - 2\partial_t \mathbf{A}P + q\beta \partial_{tt}(P)^2 = 0 \quad (1)$$

where $\mathbf{A}P \equiv A(t) \otimes P(\mathbf{x}, t)$, $A(t) = F^{-1}[a(n)]$, $P(\mathbf{x}, t)$ is the pressure in the 3D coordinate system \mathbf{x} at the moment of time t ; \mathbf{A} is a convolution-type operator that defines absorption; q is the Mach number (in our case the Mach number is calculated for velocities on the surface of the disturbance); $\beta \equiv (\gamma + 1)/2$; $\gamma \equiv (B/A) + 1$ or γ - adiabatic exponent, $n \equiv f/f_0$ - non-dimensional frequency; f, f_0 - respectively: frequency and characteristic frequency; $a(n)$ - the small signal coefficient of absorption, $\mathbf{A} = F^{-1}[a(n)]$, $F[\cdot]$ - Fourier transform.

For the medium with disturbed materials parameters the equation of the scattered field P^{sc} can be developed from the formula 1 (more details see [5-8]) and takes the form:

$$\Delta P^{sc} - \partial_{tt}P^{sc} - 2\partial_t \mathbf{A}P^{sc} = -\Pi \partial_{tt}(P^{sc} + P^{in}) \quad (2)$$

where P^{in} - incident field which fulfils the equation (1), $\Pi(\mathbf{x}) \equiv 1 - 1/c_r^2$ - scattering potential, c_r^2 - disturbed dimensionless sound velocity.

3. SOLVER

Construction of a solver for backscattered fields is the fundamental issue for setting up a numerical model of an experiment that is aimed to reflect real situations that occur in ultrasonography practice. The solver that we constructed is composed of three parts:

1. Solver for the incident field. It is the solver that bases on codes JWNUT2D and JWNUT3D [9], which we have been using for many years. The first code solves the equation in the axially symmetrical cases, the second one is applicable to whichever one-sided boundary conditions.
2. Solver for the backscattered field. It is the tool that is able to calculate parameters of back-scattered fields and their pressures on the detector surface, whereas the tool uses

numerically determined incident field and information on geometrical and material parameters of the target as the basis for calculations.

3. Simulator of the electronic receiver channel that is used for calculation of pulse responds $h(t)$ of this unit. Distribution of pressure on the surface of the probe is averaged over the entire probe surface (the theory of piezoelectric phenomena says that electric signals at probe output are proportional to the aforementioned average value).

$$P_E(t) = \frac{1}{S} \int_{S(\mathbf{x})} P^{sc}(S(\mathbf{x}), t) Ap(S(\mathbf{x})) dS \quad (3)$$

where $S(\mathbf{x})$ denotes a point on the transducer surface, S stands for the transducer surface area and $Ap(S(\mathbf{x}))$ is the apodization function for the transducer surface. In this study $P_E(t)$ is referred to as the echo. The RF signal $P_{RF}(t)$ represents a single line of scanning and is calculated as follows:

$$P_{RF}(t) = h(t) \otimes P_E(t) , h = F^{-1}[H(n)] \quad (4)$$

where $H(n)$ is the system transmittance.

4. RESULTS

Experimental setup is shown on Fig. 1. The research was carried out for a pipe made of latex, with internal diameter of 5 mm and wall thickness of 1.25 mm. The investigations employed the VED ultrasonic apparatus. The frequency of the transmitted ultrasound was 6.75 MHz. The pulses were measured by means of the hydrophone of the type: Sonic Technologies Model 800 Bilaminar Hydrophone. During the performed research the front surface of the pipe wall was positioned in the focus of the ultrasonic probe. The RF electric signal, $P_{RF}(t)$, corresponding to echoes reflected by the pipe walls, was recorded at the output of the RF receiver by means of the digital oscilloscope AGILENT 54641D. To highlight relationships between target (pipe) dimensions and wavelength of the echoed signals (both acoustic waves and corresponding electric waveforms) the scales were converted all the time into 3D ones and expressed in millimetres. Signal amplitudes were presented as relative values. Results for calculations and measurements are presented in Fig. 2, 3 and 4.

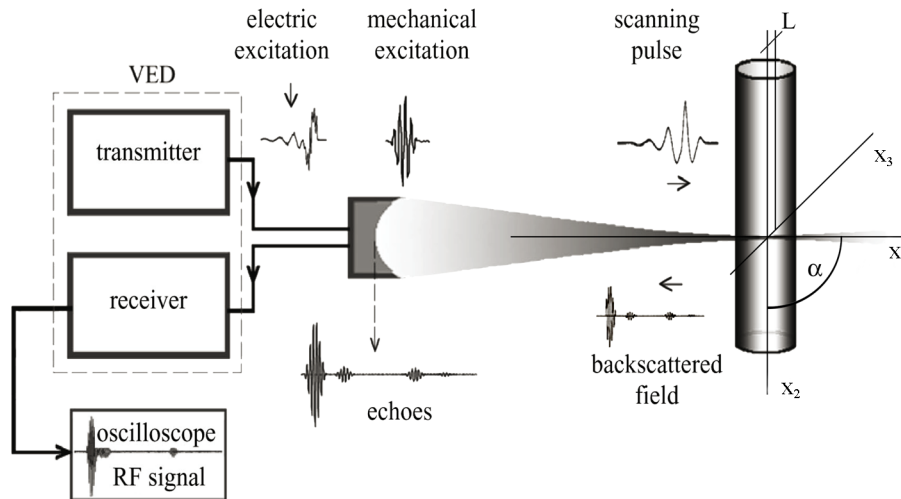


Fig. 1. Experimental setup.

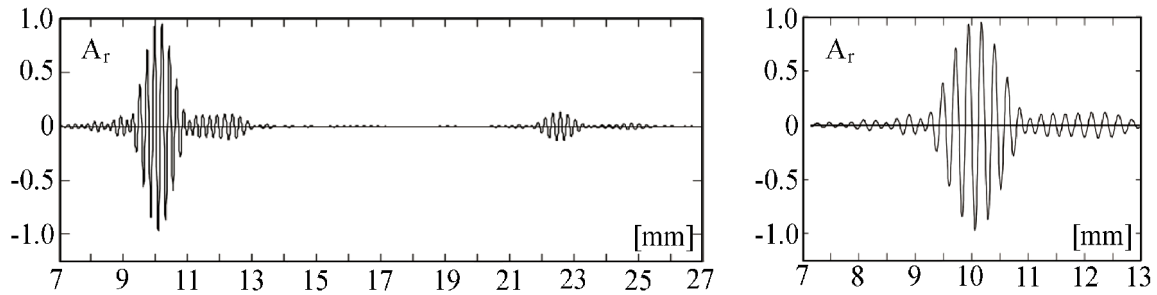


Fig. 2. a) The RF signal $P_{RF}(t)$ calculated from the numerical model by means of the formula 2; b) the expanded RF signal, from the first pipe wall. A_r – the relative amplitude (with respect to the maximum value of the RF signal amplitude).

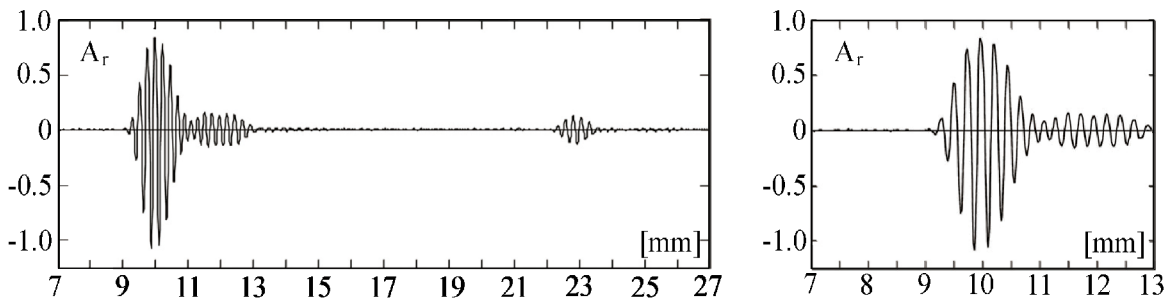


Fig. 3. a) The RF signal $P_{RF}(t)$ measured by means of the VED apparatus; b) the expanded RF signal from the first pipe wall, A_r – the relative amplitude (with respect to the maximum value of the RF signal amplitude).

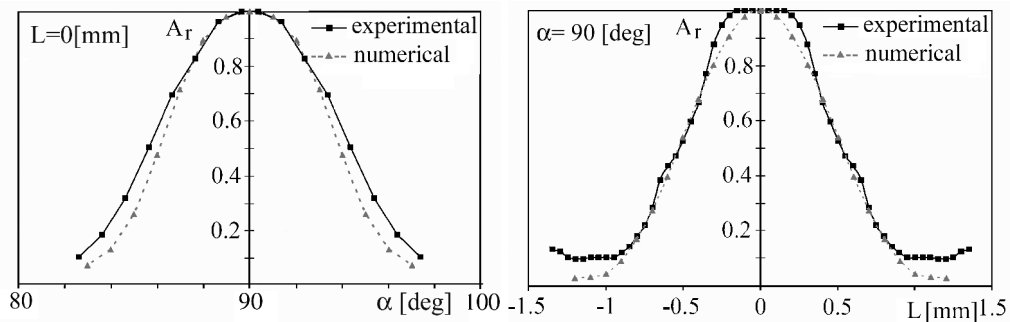


Fig. 4. Changes of the maximal RF-signal amplitude as a function of angle α and the transversal displacement L of the ultrasonic beam axis x_1 , with respect to the arterial model axis x_2 (see Fig.1). A_r - maximal RF-signal amplitude with respect to the maximal RF – signal amplitude for the angle $\alpha = 90\text{deg}$ and transversal displacement $L = 0\text{mm}$.

5. CONCLUSIONS

Comparison between the results that were obtained from numerical calculations and from measurements (Fig. 2 – 4) serves as a proof that the numerical model that was developed by our own enables simulation of the experiments with a good coherence, which was the actual objective of the study. It is the matter of high importance when the designing process of measurement equipment is to be optimized.

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