Differentiation of random structure properties using wavelet analysis of backscattered ultrasound

Barbara GAMBIN[1], Janusz WOJCIK[1], Olga DOUBROVINA [2]

[1]Institute of Fundamental Technological Research, Polish Academy of Sciences, Pawinskiego 5B, 02-106 Warsaw, Poland, bgambin@ippt.pan.pl, jwojcik@ippt.pan.pl
[2] Belarussian State University, Kurchatova, 5, 220045 Minsk, Belarus, dubrovinaov@tut.by

The aim of this work was to find the differences between random media by analyzing the properties of the ultrasound signals backscattered from the inhomogeneities. A numerical model is used to generate two types of random media. The first has the randomness in scatterers' positions and the second has the randomness in the size and acoustical properties of scatterers. The numerical model of wave scattering has been used to simulate the RF (radio frequency) signals caused by the incident pulse traveling as a plane wave. The markers of randomness type differences between the scattering media were obtained with the help of the spectral and wavelet analysis. The effect of differences in randomness type is more spectacular when the wavelet analysis is performed.

Keywords: spectrogram, scalogram, wavelets, random scattering structure.

1. Introduction

Ultrasound differentiation of soft-tissue benign lesions from the malignant lesions is a scientific challenge, and what is more important, is crucial for the future progress of diagnostic methods. Scattering phenomena of ultrasound in living soft tissues are much more complicated than the scattering taking place in soft tissue phantoms. The living tissue inhomogeneities exist on many different levels of magnitude, e.g. internal cell structures, cells walls, aggregates of cells, collagen fibers, the walls of blood vessels, etc, see [1] and exhaustive references therein. The simplest and the most popular tissue scattering model is simplified to describe the scattering of acoustic waves by particles, "small" inclusions, imbedded in a continuous, acoustically and geometrically homogeneous medium. The model belongs to the group of point scattering models. Particularly, this model can describe soft tissues having comparatively stiff cell nucleolus imbedded in a soft matrix of homogeneous surrounding medium (cancer tissue), or a gelatin based phantom containing glass particles. There is no answer for the question what constituent of the living soft tissue microstructure is responsible for the scattering of ultrasound. Additionally, the precision knowledge about the structure of the scattering medium which is contained in the collected backscattered signals, is limited by the method of measurement, i.e. the transmitter/receiver properties, see recently published paper [2]. Using the point scattering models, different parametric imaging methods were applied, one of which analyses the statistical properties of the backscattered signals envelope. The method needs many evaluations by phantom experiments, and what is more important, the theoretical considerations are based on the assumptions that the sizes of scatterers are infinitely small and the number of scatterers is infinite. The different statistical distributions of amplitude values are approximated by empirical histograms, and the discussion of links between the distribution type and the simple scattering models are performed, cf. [1,3]. These methods result in drawing the parametric maps, in which regions of the lesions from healthy tissue, and the types of the lesions can be differentiated. The feasibility studies of the parametric map are performed for every special soft tissue type independently, and they can not quantitatively describe the real geometry and physics of the scattering process. It is well documented in literature that RF data (RF radio frequency) has become more readily available to the research community, such that its potential has not fully unveiled yet. From a data processing standpoint using RF data suggests many advantages. First of all, it is generally much richer in information due to the comparably higher resolution. In this paper a wavelet method is investigated, of how to take advantage of this rich pool of information, which we will generate from the numerical model. The special focus is devoted to the demonstration of how knowledge of the scatterers' randomness can be seen in RF signals backscattered from the medium. Our idea concerns the following approach. We will perform numerical simulations of ultrasound backscattered RF signals from different media with well defined structure. Three types of the scattering media are considered below, and they are not here identified with particular tissue structures, the problem studied here is rather a purely theoretical one. We focus on the problem, if we are able to detect the type of the medium randomness by the backscattering signal analysis using the numerically generated medium model? It seems to be important to find new tools which can be used in further study, of more practical meaning in tissue differentiation methods. The paper is organized as follows. At first, we construct the three types of media. The randomness of the media is realized not only by assuming the random space configuration of scatterers' locations, but also by assuming the random sizes and physical properties of scatterers. Then, we simulate the incident pulse generated by a transducer, and the acoustical field scattered from the structure is calculated. The backscattered signals are extracted, and the envelope of analytical signals is calculated, to perform the spectral and wavelet analysis. At the end conclusions are stated.

2. Numerical models of random media

In general, the medium is assumed to be composed of scatterers embedded in a homogeneous matrix, called the host medium. First, the periodic distribution of points in three-dimensional space as the centers of cubic cells is generated. The cells form a parallelogram in the Cartesian coordinate system (X, Y, Z), consisting of 41 cells in Z direction and 11 cells in each X and Y direction. Each cube of dimension 0.005 mm contains a spherical scatterer, which is situated in the center and has the diameter of $a1 = 0.00005$ mm. Physical properties of the two-phase medium such as, the density and sound velocity of the host medium and scatterers, are fixed. The example of the medium is shown in Fig. 1 below. Media with random components suppose adding random fluctuations of the scatterers' position in the periodic cell, scatterers' size, density and sound velocity. We define three types of the medium: 1 — without random components, i.e. the periodic structure; 2 — with random geometry component; 3 —

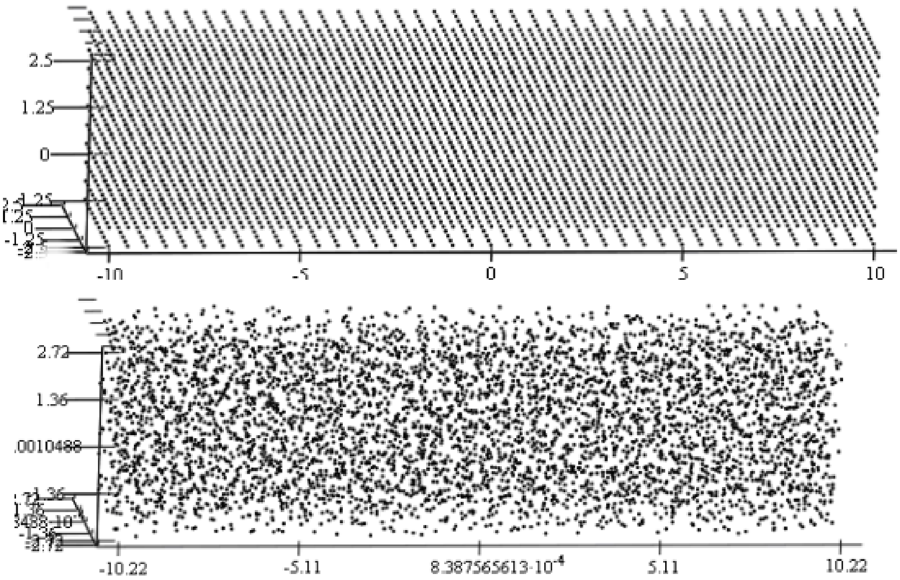

Fig. 1. Examples of constructed mediums.

with random physics component.

The numerical model of media having random geometry component, the type 2 of the medium, is generated by moving the scatterer from the center of the periodic cell by the random vector $\delta X_j = \begin{pmatrix} \delta z_j \\ \delta x_j \\ \delta y_j \end{pmatrix}$, where δz_j, δx_j, δy_j are independent random variables uniformly distributed in the intervals $[-0.45z_j, 0.45z_j]$, $[-0.45x_j, 0.45x_j]$, $[-0.45y_j, 0.45y_j]$ correspondingly and $j = 0, \cdots, 4960$ is index of scatterers numbering. One realization of such a medium is presented in Fig. 1 below.

The type 3 of the medium, having a random physics component is generated by the scatterers possessing the random radius $ar_j + \delta ar_j$, the random density $qr_j + \delta qr_j$ and the random speed of sound $cr_j + \delta cr_j$, where $j = 0, \cdots, 4960$. The random fluctuations are normally distributed with mean value $\mu_{qr_j} = \mu_{cr_j} = \mu_{ar_j} = 0$ and standard deviation $\sigma_{qr_j} = 0.05qr_j$, $\sigma_{cr_j} = 0.05cr_j$, $\sigma_{ar_j} = 0.025ar_j$.

3. Backscattered signals from the random media

Numerical model of the scattering process is based on the theoretical model of the ultrasound linear wave propagation in heterogeneous media. Under the single scattering approximation together with the model of transmitter/receiver characteristics echoes coming from the medium heterogeneities are calculated as a solution of a heterogeneous Helmholtz equation. The medium heterogeneities are described by variations of sound velocities and densities. The next step is to simulate numerically the transmission of ultrasounds waves coming from the transducer, scattering and propagating through the inhomogeneous structure. The linear model of wave propagation is assumed, calculations were performed for a transducer with radius 15 mm for the plane wave transmission and point detection; see for details [4].

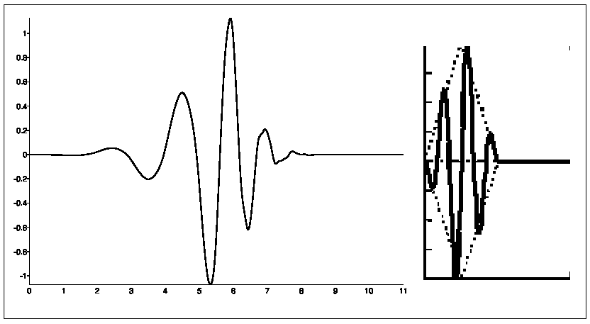

Fig. 2. The initial impulse (left) and Daubechies 6 wavelet function (right).

Initial impulse is formed from three sines and triangular envelope, as follows:

$$Fe(t, w, dur, n) = Sign \cdot Env(t, dur, n) \sin(w(t - 0.5dur)),$$

where $Env(t, dur, n) = 1-2\left(\frac{t-0.5dur}{dur}\right)^n$, n denotes a polynomial power, dur duration time and t time-variable, respectively, $Sign$ is equal 1 or -1, parameter w generally linearly depends on the carrier frequency N_{car}. In this case the following values are taken: $dur = \frac{LC \cdot 2\pi}{N_{car}}$, $LC = 3$ is number of cycles of the impulse and $w = N_{car} = 330$. Let's notice that N_{car} is a dimensionless frequency. The obtained impulse and its envelope are visualized in Fig. 2 on the right.

The incident wave is considered as a plane wave, backscattered signal is formed under the assumption of the physical quantities having the values as follows: the speed of sound in host medium $c_0 = 1500$ m/s, the speed of sound in the scatterers $c_1 = 1600$ m/s, the density of the host medium $q_0 = 1000$ kg/m^3 and in the scatters $q_1 = 1200$ kg/m^3, the repetition in space $\lambda_0 = 0.099$, the dimensionless wave vector is $K_0 = \frac{\pi}{\lambda_0} = 31.3$.

Examples of backscattered signals from the periodic medium and the two randomly inhomogeneous media, are presented in Fig. 3. The blue line in each picture denotes the Hilbert envelope of the signal that will be considered further as data for the spectral and wavelet analysis. We will denote the type I signal corresponding the the 1st medium, type II — to the 2nd medium and type III — to the 3rd medium. The image corresponds to I, II, III types of media from the top to the bottom.

4. The spectral and wavelet analysis

The wavelet transform provides both time/spatial and frequency (or scale) resolution information about the signal, and the details of signal in different frequency bands i.e. Multi-Resolution Analysis (MRA), see [5]. The decimation operation provides the next scale level. The frequency bands are thus octaves. Descrete Wavelet Technique (DWT) has various applications in ultrasound image processing: used for de-noising, segmentation, and feature extraction. Wavelet transform based features are able to capture the subtle variations of texture in spatial and frequency domain and also details about the multiple frequency bands. It is the main reason we use this technique. The Daubechies 6 wavelet family [6] was chosen to perform the analysis in the method analogously to the one described in papers [7–9]. The motivation for the choice

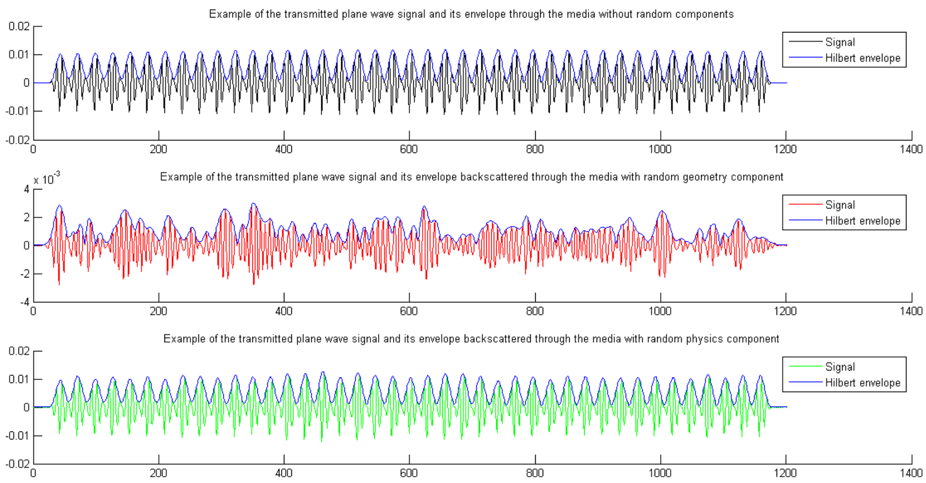

Fig. 3. Examples of signals for transmitted plane wave backscattered by randomly inhomogeneous media (type I, type II, type III).

of wavelet family is based on the similarity of the wavelet shape to the calculated impulse. The Daubechies 6 wavelet function is presented in Fig. 2, left side and the numerically calculated impulse is situated in this figure on the right.

First the spectral analysis of three type of signals was performed. The spectrogram is a two-variable function $Sp(t,\omega) = |FFT(f(t))|^2$ where $f(x)$ is original signal, FFT is the fast Fourier transform, variables t and ω correspond to time and frequency. The spectrograms corresponding to the I, II, III types of signals are presented in Fig. 4 below, from left to right. The spectrogram is the visualization of the amplitude values in the frequency/time plane. The horizontal axis denotes time, the vertical axis frequency range, and the color shows the quantity of the Fourier coefficients. The time has been divided into 7 equal intervals in which the Fourier spectrum is calculated, and presented in the color scale for the considered frequency band.

Then the signals were decomposed by the wavelet analysis. The graphs which represent the distribution of signals' energy for each coefficient (scalograms) are depicten in Fig. 5. The graphs present an other method of computing frequency ranges. A scalogram is the equivalent of a spectrogram for wavelets. Let us remember the general formulae of wavelet decomposition. The discrete wavelets transform may be written in a form $\psi_{k,i}(x) = \frac{1}{\sqrt{a_k}}\psi\left(\frac{x-b_{k,i}}{a_k}\right)$, where $\psi(x,a,b)$ is a wavelet function, the scale coefficient a is the diadic discretization scale $a_k = 2^{-k}$, where $k \in \mathbb{Z}$ is a scale index and the coefficient b is discretized linearly so that $b_{k,i} = i2^{-k}$, $i \in \mathbb{N}$. The corresponding discrete reconstruction formula is $f(x) = C \sum_{k=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} w_{k,i}\psi_{k,i}(x) + R(x)$. Here $w_{ki} = \int_{-\infty}^{\infty} f(x)\overline{\psi_{k,i}(x)}dx$, C is a constant and $R(x)$ is a residual which depends on the choice of wavelet. In the case of Daubechies 6 wavelets $R = 0$.

On the horizontal axis in Fig. 4, there are values of the wavelet transform shift coefficient b_k that mean time, on the vertical axes are values of the scale coefficient a_k, the colors represent the percentage of energy from dark color to light.

Diffusion of the frequencies in signal of type II, means that less energy is transported by

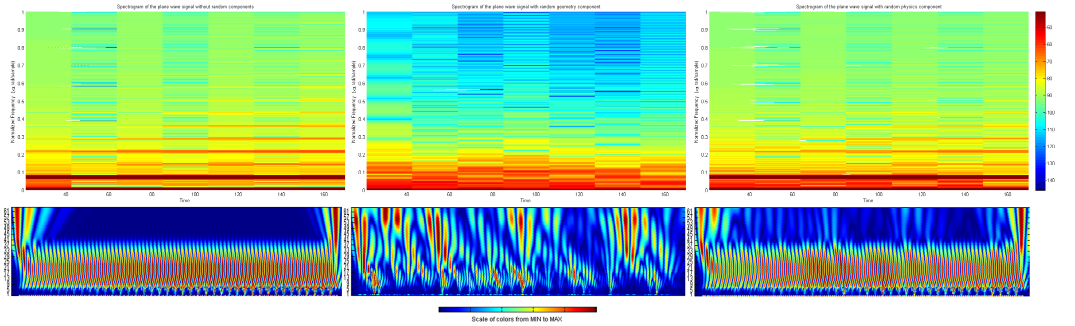

Fig. 4. Spectrogram (above) and scalograms (below), from left to right: type I signal, type II signal, type III signal

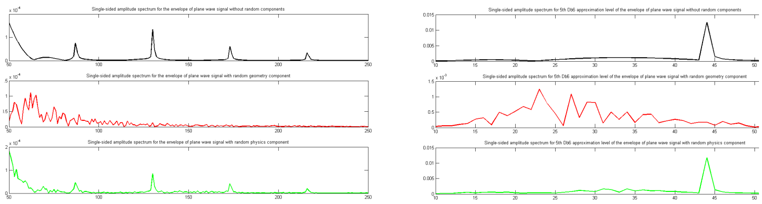

Fig. 5. One-sided spectrum of the signals (left) and the part one-sided spectrum of their 5th level of wavelet approximation, black — type I signal, red — type II signal, green — type III signal

the periodic signals compared to the regular structure of the medium 1.

It is evident that the periodic structure of the scatterers of the medium without random geometry component is preserved in all levels of the wavelet coefficient distribution.

The fast Fourier transforms of three types of signals (left) and their Daubechies 6 5th level wavelet approximation (right) is presented in Fig. 5. The black lines correspond to the type I signal, red line to the type II signal, and green to the type III signal. The fluctuations appearing in the signal with random geometry component, medium 2, differentiate this signal from the regular, periodic geometry distribution of scatterers of medium 1. We can conclude that the randomness of the medium 2, the random distribution of scatterers centers, affect the total blur of peaks, which characterize the periodic medium 1. The blurring takes place in the whole frequency range. The blur stays at all levels of the wavelet approximation, it is also visible on the 5th level, however, it appears on the frequency range shifted to smaller values, see Fig. 5. The finding of any marker of the medium 3 randomness is much more complex. We selected the 5th level of the wavelet approximation from all others up to 7th approximations, as the most representative to demonstrate the randomness of the medium 3. The randomness of medium 3 appears in fluctuations, they are comparatively small to the fluctuations in the medium 2, of energy contributions in scalograms in the higher level of wavelet approximation then the 5th level and in the 5th level itself. The differences between the spectrograms of the medium I and medium II are invisible. While scalograms vary at levels higher than 5th level wavelet approximations. In Fig. 4, above the 5th level of wavelet approximation fluctuations appear, and the color intensities' variations are easy to register. In the image in Fig. 5, these fluctuations are reflected as the green line variations on the left of a maximum value.

5. Conclusions

Having the numerically generated backscattered signals from the random medium. we studied the differences caused by the randomness types. The spectral analysis as well as the wavelet analysis of the three types of backscattered signals corresponding to the non-random medium, and to the two random media were applied. Comparing the spectra of signal amplitudes and calculated FFT of the successive levels of wavelet approximations, we demonstrated that visible differences of randomness types are linked to the frequency range and the degree of the frequency blurring. It is demonstrated that the random location of scatterers results in a blur the peaks in the whole frequency range, in the opposite to the case of randomness caused by fluctuations of the physical properties and scatterers sizes. The FFT of the 5th level of wavelet approximation for medium 3, shows the small variations in the energy spread in the spectral range from 25 to 40, while the signal obtained from the medium 1, in which periodic structure leads not only to the concentrated peaks of harmonics but for uniform energy distribution in the frequency ranges in between the peaks. The comparison of spectrograms and scalograms proves the thesis that the differences between the characteristics of medium I and medium III are better demonstrated on its scalogram then on its spectrogram. The latter is practically undistinguished from the medium I spectrogram. These differences started to be visible from the 5th level of wavelet approximations. We shall notice, that before we decided to look in the envelope properties for the randomness differences, we had looked at first for these differences in the signals spectrum and FFTs of succesive levels of signal wavelet approximations. The results were less informative than the envelope analysis. The paper present the very new idea of seeking the markers for the two randomness types. We use the numerical model of the medium, with different random characteristics, to obtain a backscattered ultrasound signal. The scatterers were small enough that they can be considered as sub-resolution structures. We can conclude that the randomness of geometrical structure of the media has a stronger influence on the randomness of backscattered signal; whereas the variations in size, density and speed velocity of scatterers, i.e. their reflectivities, have a very weak impact on the analyzed signals.

Future studies shall be done for analyzing signals backscattered from the media with the different kinds of the random scatterers distribution, different mean size of scatterers, at least two different shape of scatterers with different mean diameter. Having the experiences of how the wavelet technique can be used to differentiate types of all medium randomness desribed above we can think how to use it for tissue classification. The review paper [10], which is dedicated to the application of wavelet-based techniques in ultrasound image analysis for cancer detection, provides rich references in the field. Two-dimensional wavelet transformation (Gabor transform) which are used in image analysis, should be applied to the numerically generated FR signals backscattered from the heterogeneous medium containing two or more regions with different randomness types. We hope that such a kind of analysis can be an efficient tool for a new tissue differentiation technique.

ACKNOWLEDGMENT
This work was partially supported by the National Science Centre (2011/03/B/ST7/03347).

References

[1] J. Mamou, M. L. Oelze, *Quantitative Ultrasound in Soft Tissues*, Springer Science & Business Media, 2013.

[2] J. Wójcik, N. Żołek and M. Lewandowski, Influence of transmission-reception characteristics of ultrasound transducers on statistics of echoes from nonhomogeneous media, In: *Postępy Akustyki 2016*, Polskie Towarzystwo Akustyczne, Oddział Warszawski, Warszawa, Poland, Vol.1, pp. 627–633, 2016.

[3] F. Destrempes, G. Clouttier, A critical review and uniformized representation of statistical distributions modeling the ultrasound echo envelope, Ultrasound in Med. & Biol., vol. 36, pp. 1037—1051, 2010.

[4] J. Wójcik, J. Litniewski and A. Nowicki, Modeling and analysis of multiple scattering of acoustic waves in complex media: Application to the trabe cular bone, J. Acoust. Soc. Am. vol. 130 (4), pp. 1908—1918, 2011.

[5] P. Addison, *The Illustrated Wavelet Transform Handbook*, IoP, Bristol-Philadelphia, 2002.

[6] I. Daubechies, *Ten Lectures on Wavelets*, SIAM, Philadelphia 1993.

[7] B. Gambin, O. Doubrovina Statistical properties of wavelet transform coefficients of backscattering signal from soft tissues and their phantoms, Hydroacoustics, V. 16, pp. 59–66, 2013.

[8] O. Doubrovina, B. Gambin, E. Kruglenko, Temperature level and properties of wavelet approximations of backscattered ultrasound. Hydroacoustics, vol. 17, pp. 37–46, 2014.

[9] B. Gambin, O. Doubrovina, Wavelet analysis for temperature increase detection from acoustic backscattered signal. In: Complex Analysis and Potential Theory with Applications (T. Aliev Azerogly, A. Golberg, S.V. Rogosin Eds.). Cambridge Scientific Publishers, pp. 63–76.

[10] V. K. Sudarshan, M. R. Krishnan Mookiah, U. Rajendra Acharya, V. Chandran, F. Molinari, H. Fujita, K. Hoong Ng, Application of wavelet techniques for cancer diagnosis using ultrasound images: A Review, Computers in Biology and Medicine vol. 69, pp. 97–111, 2016.