SENSITIVITY-BASED STRUCTURAL HEALTH MONITORING OF SKELETAL STRUCTURES

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Abstract

The paper presents an idea of structural health monitoring (SHM) for a class of vibration problems. The approach is based on the Virtual Distortion Method (VDM), in the framework of which sensitivity analysis can be effectively performed. The sensitivities are then employed in a gradient-based optimization algorithm of damage identification. Fundamentals of the method have been briefly explained. Numerical example of a real bridge structure has been presented for the time-domain version of the proposed identification algorithm.

1. Introduction

This paper presents a sensitivity-based approach to structural damage identification within structural health monitoring (SHM). The inverse problem of damage identification is formulated and solved in the framework of the Virtual Distortion Method (VDM) [1]. The method stemming originally from structural mechanics belongs to fast reanalysis methods. It allows for calculating analytical sensitivities and is particularly efficient for skeletal structures. It was previously used to identify stiffness/mass modifications in trusses and frames when subjected to arbitrarily dynamic [2] or harmonic [3] excitations.

The paper presents numerical simulations of a skeletal structure and demonstrate effectiveness of the VDM-based damage identification algorithm. The proposed methodology has been the subject of tests performed on a railway truss bridge since mid 2007 with permission of PKP PLK S.A. Numerical results for the model of the bridge are included in the presentation for the case of reduction of Young's modulus. Other damage types to be identified by the VDM software tools can be considered as well e.g. degradation of stiffness/mass, loosening of bolts, weakening of welds. Non-linear effects such as plastic deformations and connections with sliding friction can be also covered by VDM.

2. Virtual Distortion Method in SHM

The Virtual Distortion Method belongs to fast reanalysis methods [4]. This basically means that it is capable of quick and effective change of a baseline response referring to the initial structure in order to reflect the behaviour of a modified structure at minimum computational effort. The method has been previously applied to various direct problems of structural mechanics e.g. optimal topology design or progressive collapse analysis [5]. Recently, it has also proved useful in SHM thanks to efficient sensitivity analysis enabling the solution of the inverse problem of damage identification. A collection of SHM applications of VDM including structures and systems can be studied in [6].

The essence of modelling in VDM is to introduce pseudo-quantities (virtual distortions) to the system in order to modify its properties without having to change input data for the initial system. This way for instance, the stiffness matrix of an analyzed structure remains the same throughout VDM-based analysis. Basically, there are two types of virtual distortions used – pseudo-strains modelling changes of stiffness and pseudo-forces modelling changes of mass in the system.

The influence matrix is a fundamental concept of VDM, which is widely used in numerical algorithms. The point is to build a map of inter-relations in the system. This is achieved by capturing the global response of the system due to a local perturbation realized by applying unit virtual distortion in a selected structural member. Repetition of the local perturbation in every member eventually produces the full matrix. The structure for which the influence matrix is assembled must be discrete (e.g. truss) or FEM-discretized (e.g. frame). Due to the size of the influence matrix, most powerful VDM-based analysis can be run for skeletal structures with at most 3 virtual distortions per member. Extension of the idea for plates or shells should consider the question of numerical efficiency.

3. Sensitivity-based identification of damage

The idea of sensitivity-based damage identification in truss structures will be now briefly presented. Linear systems are analyzed and structural damage of a truss element is modelled as degradation of stiffness and/or loss of mass.

With the influence matrix storing strain responses D, a superposition of initial strain response ε^L due to external load and residual strain response ε^R due to structural modification can be performed as follows:

$$\varepsilon = \varepsilon^L + \varepsilon^R = \varepsilon^L + D\varepsilon^0 \tag{1}$$

where: ε – total strain, ε^0 – virtual distortion modelling stiffness modifications.

The internal forces in truss members due to real modification of stiffness e.g. of Young's modulus and due to introduction of virtual distortion ε^0 supposed to model this fact are the following:

$$\begin{aligned} P &= E' A\varepsilon \\ P &= EA\left(\varepsilon - \varepsilon^0\right) \end{aligned} \tag{2}$$

where: E – initial Young's modulus, E' – modified Young's modulus, A – cross-sectional area.

The static postulate of VDM says that the forces in eq. (2) be equal, which leads to the relation for the coefficient of modification μ:

$$\mu = \frac{E'}{E} = \frac{\varepsilon - \varepsilon^0}{\varepsilon} \tag{3}$$

With the influence matrix storing displacement responses B, similarly to eq. (1) we can establish relations for displacements or accelerations:

$$u = u^L + u^R = u^L + Bf^0 \tag{4}$$

where: u – total displacement, f^0 – virtual distortion modelling mass modifications.

Equations of motion of an undamped system with real modification of mass and with virtual distortions f^0 supposed to model this fact have the form:

$$\begin{aligned} M' \tfrac{d^2}{dt^2} u + Ku &= f \\ M \tfrac{d^2}{dt^2} u + Ku &= f + f^0 \end{aligned} \tag{5}$$

where: M' – modified mass matrix, M – initial mass matrix, K – stiffness matrix.

The dynamic postulate of VDM requires that the inertia forces and accelerations be equal, yielding:

$$\left(M' - M\right)\tfrac{d^2}{dt^2} u + f^0 = \Delta M \tfrac{d^2}{dt^2} u + f^0 = 0 \tag{6}$$

If both damage degradation and mass loss are considered, eq. (3) and (6) constitute a set of equations which should be solved for the stiffness-modelling distortions ε^0 and mass-modelling distortions f^0.

In the identification process, a minimum of the following objective function is sought:

$$F(\mu) = \sum_{sensor} \frac{\left(\varepsilon - \varepsilon^M\right)^2}{\left(\varepsilon^M\right)^2} \tag{7}$$

where: ε^M – measured strain.

The modification coefficient has to be bounded in the range <0,1> in order to consider structural damage. Gradient of the function (7) with respect to the design variable μ is calculated using the chain rule of differentiation:

$$\frac{\partial F}{\partial \mu} = \frac{\partial F}{\partial \varepsilon}\left(\frac{\partial \varepsilon}{\partial \varepsilon^0}\frac{\partial \varepsilon^0}{\partial \mu} + \frac{\partial \varepsilon}{\partial f^0}\frac{\partial f^0}{\partial \mu}\right) \tag{8}$$

Partial derivatives of ε^0 and f^0 with respect to μ are computed by both side differentiation of the set of equations (3) and (6), which produces the same left-hand side matrix as in the primary set (3) and (6). The design variable μ is updated in iterations by an optimization routine e.g. simple steepest descent [2] or advanced Levenberg-Marquardt [7].

The described model updating process effectively solves the inverse problem of parameter identification relying on strain measurements and gradient-based optimization methods. Depending upon excitation, either static, harmonic or fully dynamic versions of the presented approach may be considered.

4. Numerical results for the model of a real bridge

A railway truss bridge (see Fig. 1) has been investigated in situ in the SHM context. Some results of the conducted measurement sessions are reported in [8].

Fig. 1 Investigated railway truss bridge in Nieporet

Based on technical documentation provided by PKP PLK S.A., a numerical model of the bridge has been built in ADINA. The combined truss-frame model consists of the carrying structure (truss) and the railway track support (frame). For clarity of presentation and minimizing the computational effort, only the truss part highlighted in Fig. 2 is the subject of damage identification in this paper. Four members marked in bold are equipped with sensors (s). An impulse force applied vertically (as initial velocity) to a bottom node (see Fig. 2) is the source of excitation for the structure. Fully dynamic version of the approach has to be used.

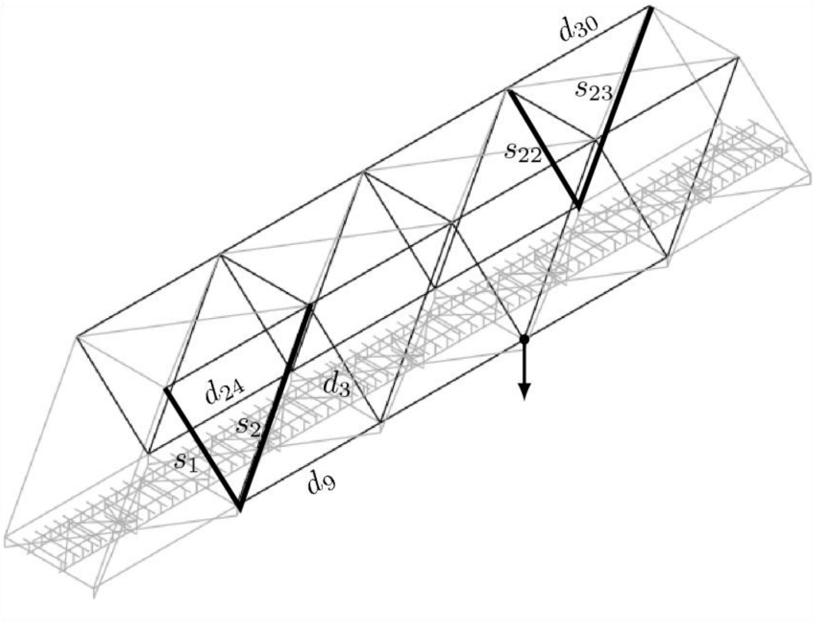

Fig. 2 3D model of the bridge built in ADINA (the analyzed truss part in black)

Four damage scenarios are considered. Damage (d) is modeled as a reduction of Young’s modulus in all simulations. A list of damage scenarios is given in Tab. 1. Results of identification for scenarios I-IV are depicted in Fig. 3-6 respectively.

Scenario	Damaged elements (μ)
I	3(0.7), 30(0.7)
II	9(0.7), 24(0.7)
III	3(0.7), 9(0.7), 24(0.8), 30(0.6)
IV	3(0.7), 30(0.7) incl. 10% noise

Tab. 1 Considered damage scenarios

Four sensors are placed in selected truss elements. The time-domain version of the VDM-based identification algorithm used here produces good results for a small number of sensors (asset), however is quite time-consuming (flaw).

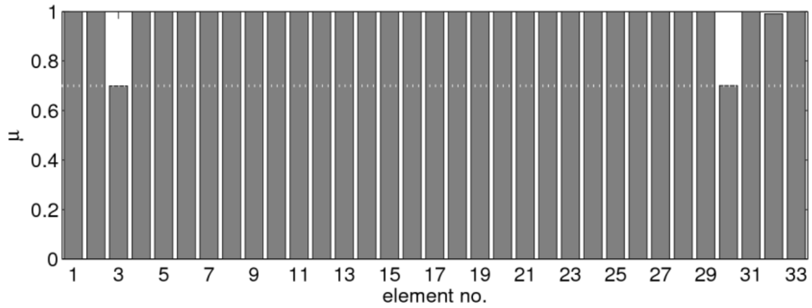

Fig. 3 Identification results for damage scenario I

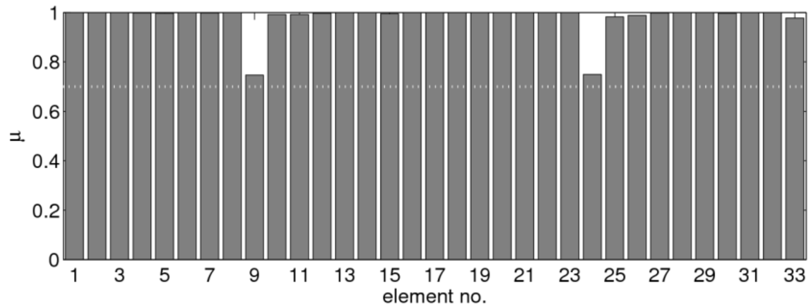

Fig. 4 Identification results for damage scenario II

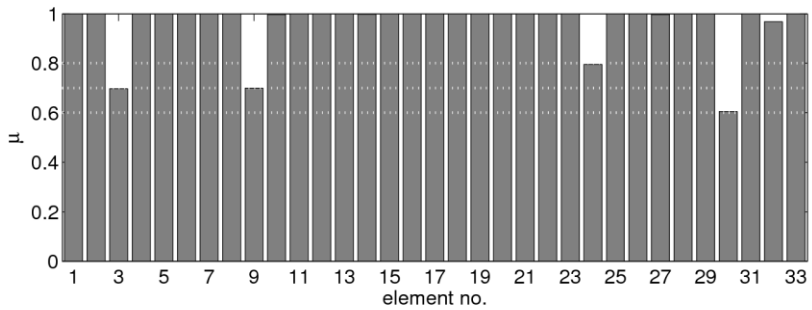

Fig. 5 Identification results for damage scenario III

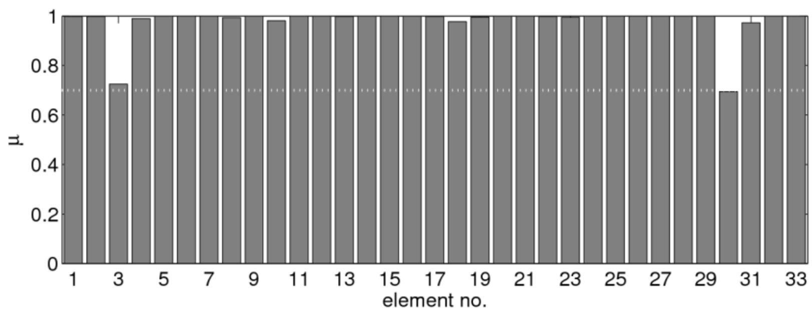

Fig. 6 Identification results for damage scenario IV

It can be observed that the identification result for damage scenario I is better in terms of accuracy than the result for damage scenario II. This comes from the fact that scenario II includes two symmetrically positioned truss members of the bridge. Thus the algorithm encounters difficulties in distinguishing them due to possible similarity of responses. It is interesting that damage scenario III, which is basically a combination of I and II including four members, has been identified with better accuracy than scenario II alone. Damage scenario IV including 10% random noise has been successfully identified as well. Although the drop of the objective function is smaller for scenario IV by two orders of magnitude compared to the corresponding scenario I (without noise), the quality of identification in terms of damage location and intensity is similar.

The impulse way of excitation adopted in this paper proved to produce the most significant differences in time-domain responses of the bridge (see Fig. 7).

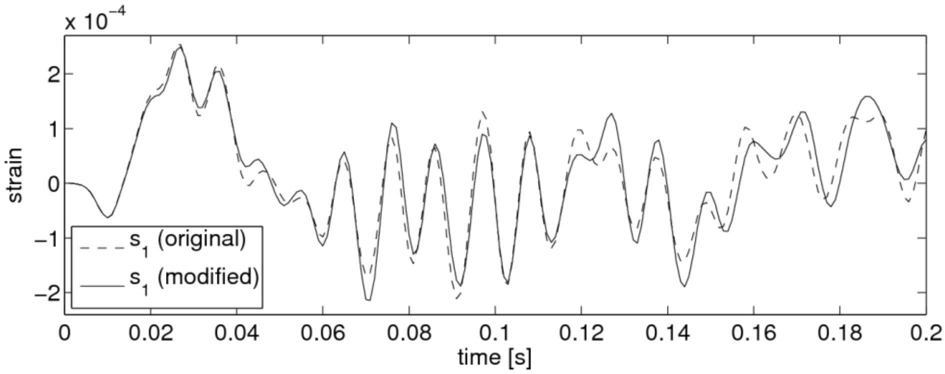

Fig. 7 Time-domain responses of reference and damaged structure for damage scenario I

Three optimization methods were employed in the damage identification algorithm. Performance of the Levenberg-Marquardt (LM), steepest descent with line search (SD-LS) and basic steepest descent (SD) methods can be studied in Fig. 8 for scenario I. The LM and SD-LS methods achieve the termination criterion (drop of the objective function by five orders of magnitude) much faster.

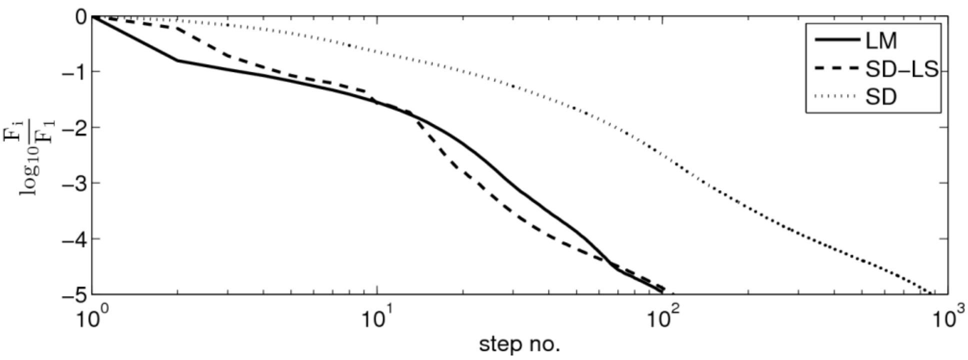

Fig. 8 Performance of optimization methods for damage scenario I

5. Conclusions

The VDM-based approach using sensitivity analysis has been presented. Three options of the approach can be considered when solving the inverse problem of damage identification – static, harmonic and fully dynamic. In this paper the last option has been chosen for demonstration of efficiency of the approach. Identification procedure has been performed using the numerical model of a truss railway bridge. The identification algorithm proved to be satisfactorily accurate even in the presence of 10% random noise.

Further research will move towards verification of the identification algorithm using experimental responses of the bridge. A problem to be considered for practical realization is the way of excitation. Generation of both the impulse and harmonic force require special equipment (drop-weight device and shaker). These two types of excitation seem to be the best for identification quality though.

Acknowledgements

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