STRUCTURAL OPTIMIZATION OF A FIVE-UNIT SINGLE-BRANCH TRUSS-Z MODULAR STRUCTURE

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Abstract. Truss-Z (TZ) is an Extremely Modular System (EMS). Such systems allow for creation of structurally sound free-form structures, are comprised of as few types of modules as possible, and are not constrained by a regular tessellation of space. Their objective is to create spatial structures in given environments connecting given terminals without self-intersections and obstacle-intersections. In an EMS, the assembly, reconfiguration and deployment difficulty is moved towards the module, which is relatively complex and whose assembly is not intuitive. As a result, an EMS requires intensive computation for assembling its desired free-form geometrical configuration, while its advantage is the economization of construction and reconfiguration by extreme modularization and mass prefabrication. TZ is a skeletal modular system for creating free-form pedestrian ramps and ramp networks among any number of terminals in space. TZ structures are composed of four variations of a single basic module (Truss-Z module, TZM) subjected to affine transformations (mirror reflection and rotation). The previous research on TZ focused on global discrete optimization of the spatial configuration of modules. This contribution reports on the first attempts at structural optimization of the TZM for a single-branch TZ. Namely, the internal topology of a TZM and sizing of its elements are subject to optimization. An important challenge is due the fact that TZM is to be *universal*, i.e., it must be designed for the worst case scenario. There are four variations of each module, and due to symmetries there are thus $4^4 = 256$ unique 5-unit configurations. The structural performance of all of them needs to be evaluated in terms of a typical structural criterion (the maximum von Mises effective stress), and used for structural optimization at the level of a single TZM.

1 INTRODUCTION

A stairway is the most common means of pedestrian vertical transportation used in the built environment. Elevators and escalators are relatively expensive to install and maintain, and their traffic flow capacity is much lower than that of stairs. Moreover, it is not always possible to install an elevator or escalator due to limited space. However, most people occasionally or temporarily cannot use stairs, as when riding a bicycle, pushing a baby stroller or carrying heavy luggage. For elders and people in wheelchairs, stairs form a permanent and impassable barrier.

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This is an important social issue, especially since the proportion of elderly people in society is higher than in the past, and some predict that this tendency will continue [1]. A comprehensive literature review for elderly pedestrians is carried out in [2].

Truss-Z (TZ) is a modular skeletal system for creating free-form ramps and ramp networks among any number of terminals in space. The concept has been introduced in [3]. The motivation for TZ in the context of human mobility, and in particular the mobility problems of elders is discussed in [4]. The underlying idea of this system is to create structurally sound provisional or permanent structures using the minimal number of types of modular elements. Further discussion on modularity vs. free-form can be found in [4].

TZ is comprised of Truss-Z modules, TZMs for short. In principle, TZ connects two points in space, called terminals. Constructing an efficient TZ can be formally expressed as a constrained discrete multicriterial optimization problem. In the previous research only the geometrical properties, such as the total number of modules (n), "geometrical simplicity" (GS) and "number of turns" (NT), have been minimized. The criteria GS and NT measure how many units do not follow a straight line, and how many continuous turns there are in the path, respectively. Common constraints have been the locations of terminals, positions and shapes of obstacles in the environment, etc. This paper reports on the first attempts at the structural optimization of TZM for a single-branch TZ, which focuses on structural performance of the resulting TZs. The following assumptions have been made:

1. The maximal span of TZ is five modules;

2. The TZM mass is constant;

3. The outer shape of TZM is fixed.

Subject to optimization is the internal topology of a TZM and sizing of its elements.

2   THE CONCEPT OF TRUSS-Z

In geometrical terms, all TZ structures are composed of only four variations of a single basic unit or module (R). Figure 1 shows the geometrical properties of R which have been set arbitrarily and used in theretofore research. Unit L is a mirror reflection of the unit R. By rotation, they can be assembled in two additional ways (R_2 is the rotated R, and L_2 is the rotated L), effectively giving four types of units. Some examples are shown in Figure 2. Figure 3 shows an example of a TZ underpass installation.

The structural rigidity of the TZ module has been demonstrated in [5], along with other topological properties such as *nullity*, degree of static indeterminacy (DSI = 0), etc. Due to the modularity of this system, it is natural to apply discrete optimization methods for creating TZ connectors and networks. Such structures can be optimized for various criteria: the minimal number of modules, the minimal number of changes in direction, and in a case of multiple branches, the minimal network distance, etc. Various deterministic and meta-heuristic methods have been successfully implemented for single TZ paths, including backtracking [6], evolution strategy [3], and evolutionary algorithms [7]. These methods produced usually good, but not ideal, solutions. A graph-theoretical exhaustive search method, which produces the best allowable, that is ideal solutions, has been described in [8].
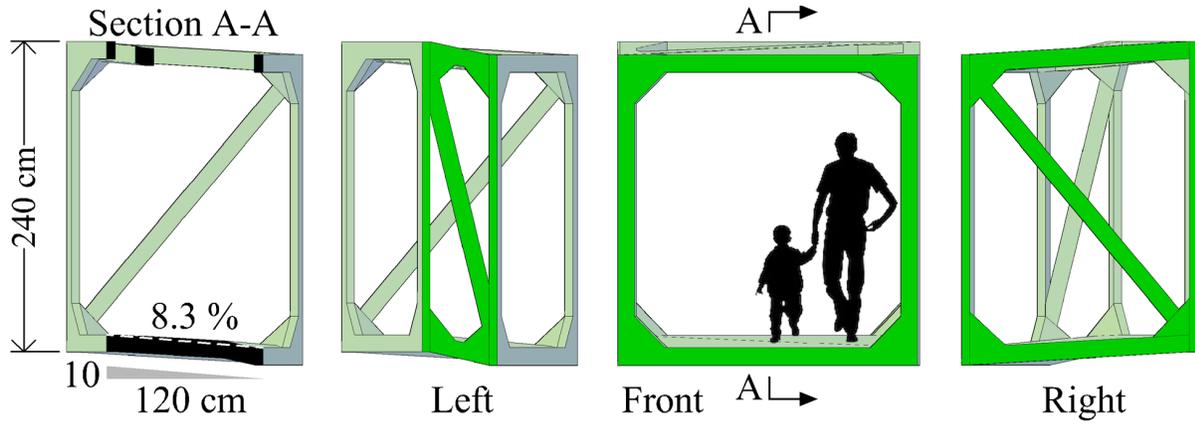
Figure 1: The original TZ basic unit (R). From the left: section A-A showing the slope, and three orthographic views.
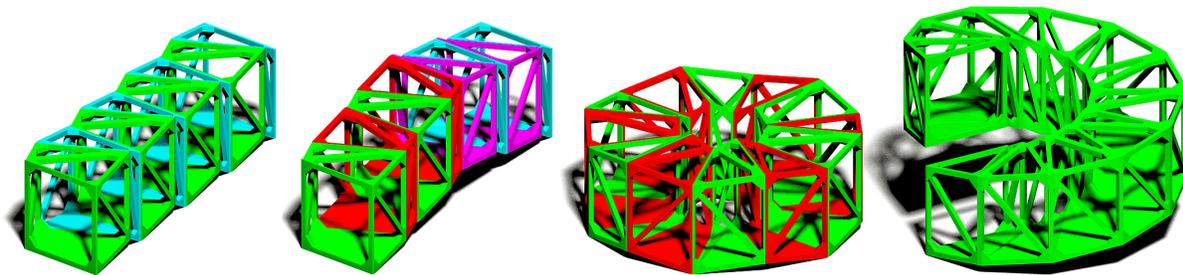
Figure 2: Four basic examples of various sequences of TZMs. From the left: flat and pitched ramps, circle and spiral. The color convention is: R, R_2, L and L_2 are shown in: green, cyan, red and magenta, respectively.

Figure 3: Visualization of retrofitting of a cycling/pedestrian path which continuity has been interrupted by a railway. The lightweight TZ could be partially suspended from the existing bridge. Without the necessity of heavy equipment, TZ is a feasible solution for countless situations.

3 TZ MODULE GEOMETRY

The geometry of the module is determined by the parameters: planar angle θ, width r, "slenderness" s, vertical displacement δ_Z, and height h, as shown in Figure 4. The slenderness s is the ratio between the offset from the apex d to the width r. For the three cases of $s = 0$, $0 < s < \infty$ and $s = \inf$, the corresponding projections of TZM form a triangle, a trapezoid, and a rectangle, respectively.

In this paper, the outer shape of TZM is based on functional considerations, and it the same as presented in theretofore investigations. It is defined by the following parameter values: $\theta_0 = 30°$, $r = 2.4$ m, $s = 0.5$, $\delta_Z = 0.1$ m, and $h = 2.4$ m.

4 STRUCTURAL OPTIMIZATION OF A TRUSS-Z MODULE

4.1 Optimization variables

The outer geometry of a Truss-Z module has been proven in earlier research to be flexible enough to create free-form shaped ramps in complex environments. Thus, the geometry of a module is treated as fixed, and the structural optimization process is limited to two factors:

- *Placement of the diagonals of a module* (*d*). Since there are four module faces with a diagonal (floor, ceiling and two side walls), and since each diagonal can be placed in two configurations, there are a total of 16 different possible configurations of the diagonals.
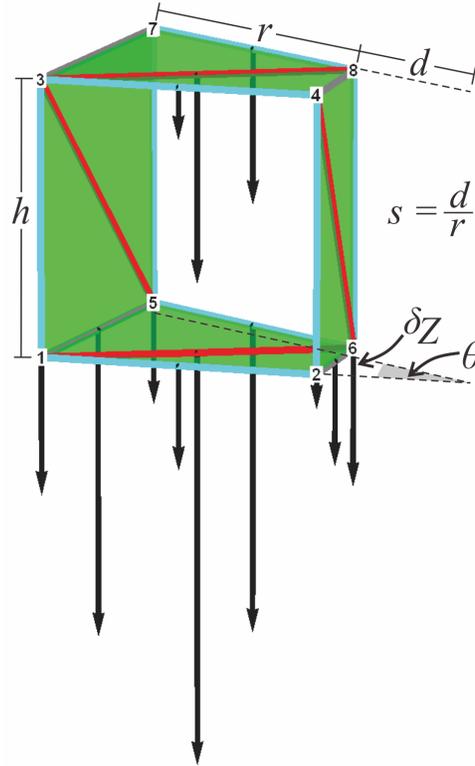
Figure 4: The topological and geometrical parameters of the original TZM. The main frame and the diagonal members are shown in cyan and red, respectively. Black arrows indicate the distribution of the static load (5000 N/m^2) used in the optimization process. The total load is distributed in proportion 4:1 between the ceiling and floor levels, and allocated to the end- and midpoints of the respective horizontal beams in proportion to the area of the neighboring floor/ceiling triangles.

They are denoted by a sequence d of four binary digits (or alternatively, the corresponding decimal number) that denote the configuration of the diagonal within the floor, the right-hand side wall (the longer wall), the left-hand side wall (the shorter wall), and the ceiling, respectively. For example, the specific configuration shown in Figure 4 is denoted by the binary sequence $d = 0101$, that is the decimal number 5.

- *Cross-sections of the module beams* (**x**). Each TZM is modeled as a 3D frame and consists of 16 beams. The material parameters are assumed to correspond to steel with the density of 7850 kg/m^3, Young's modulus 210 GPa, and the shear modulus of 81 GPa. For optimization purposes, it is assumed here that each beam is a circular hollow section with the width of the wall equal to 5 mm, and that only the outer diameters of the sections are subject to optimization. The corresponding 16-element vector is denoted by **x**,

$$\mathbf{x} = (\phi_1, \phi_2, \ldots, \phi_{16}). \tag{1}$$

There is an obvious non-negativity constraint, as well as a single linear constraint that

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fixes the total mass m of the module at the level of 150 kg. The successive parameters in the vector $\mathbf{x}$ describe respectively the sections that join the following pairs of nodes of the module, as numbered in Figure 1: (1, 2), (5, 6), (1, 3), (2, 4), (5, 7), (6, 8), (3, 4), (7, 8), (1, 5), (2, 6), (3, 7), (4, 8), (1, 6) or (2, 5), (4, 6) or (2, 8), (3, 5) or (1, 7), and (3, 8) or (4, 7), where the last four pairs depend on the specific configuration d of the diagonals.

4.2 Static load scheme

The optimization is performed in the static case, in which the entire Truss-Z structure is subjected to a certain static load $\mathbf{F}(d)$. The load is assumed to correspond to the static vertical load of 5000 N/m^2, which is distributed in proportion 4:1 between the ceiling and floor levels, and then allocated to the end- and midpoints of the respective horizontal beams proportionally to the area of the neighboring triangles, as shown in Figure 4. Given a TZ structure, each its module is subjected separately to such a load, which are then all assembled into the global load of the entire structure.

4.3 Assessment criterion

The structural quality of a TZ structure constructed using modules of given cross-sections $\mathbf{x}$ and with diagonal configuration d is quantified here in terms of the maximum von Mises effective stress $\sigma_s^{\max}(\mathbf{x}, d)$ of the TZM sections that occurs in the entire TZ structure under the assumed load vector $\mathbf{F}(d)$. The subscript s denotes a specific configuration of modules that uniquely determines the entire Truss-Z, for example configurations see Figure 2.

The TZ system has a modular character, so that the ultimate configuration s used in specific conditions is not known in advance. As a result, one needs to assess structurally not a single specific configuration s of the modules, but rather the worst case scenario of a certain set S of configurations that are expected to be used in real environments [9]. In practice, a TZ ramp can be assumed to be supported not sparser than every five modules, thus the set S considered here contains all possible configurations of five TZMs with fixed supports in all degrees of freedom of the entrance and exit square plane frames. In general, the total number of such configurations is $5^4 = 1024$, but the left–right and entrance–exit structural symmetries yield 256 essentially different 5-module configurations. The assessment criterion for a TZ module is thus defined based on the worst case among all 256 possible scenarios: it is the maximum effective stress of the beams in all modules in all possible configurations $s \in S$ subjected to the assumed static load $\mathbf{F}(d)$,

$$\sigma_S^{\max}(\mathbf{x}, d) = \max_{s \in S} \sigma_s(\mathbf{x}, d). \tag{2}$$

4.4 Optimization process

During the optimization, the maximum effective stress $\sigma_S^{\max}(\mathbf{x}, d)$ is minimized with respect to the variables $\mathbf{x}$ and d, of which only $\mathbf{x}$ has a continuous character. Due to the discrete nature of the variable d, the optimization process with respect to the vector $\mathbf{x}$ is repeated 16 times,

that is separately for each possible configuration d of diagonals, to yield 16 optimized values:

$$\bar{\mathbf{x}}_d = \arg\max_{\mathbf{x}} \ \sigma_S^{\max}(\mathbf{x}, d), \tag{3}$$

$$\sigma_S^{\max}(d) = \max_{\mathbf{x}} \ \sigma_S^{\max}(\mathbf{x}, d) = \sigma_S^{\max}(\bar{\mathbf{x}}_d), \tag{4}$$

which are then used to find the best configuration of diagonals and the ultimate minimum,

$$\bar{d} = \arg\max_d \ \sigma_S^{\max}(d), \tag{5}$$

$$\sigma_S^{\max} = \max_d \ \sigma_S^{\max}(d) = \sigma_S^{\max}(\bar{d}). \tag{6}$$

Additionally, each of the 16 diagonal configurations d is assessed in terms of the maximum vertical displacement $h(d)$ of the TZ structure composed of the stress-optimized modules (as defined by $\mathbf{x}_d$) under the considered load $\mathbf{F}(d)$.

5 OPTIMIZATION RESULTS

5.1 Configuration of diagonals

The results of the optimization with respect to $\mathbf{x}$, see Eqs. (3) and (4), are shown Figure 5. The horizontal axis represents the maximum effective stress $\sigma_S^{\max}(\bar{\mathbf{x}}_d)$ under the test load, while the vertical axis represent the maximum vertical displacement under the same load, which might possibly occur in a TZ of another configuration s. Each point corresponds to a specific configuration d of the diagonals, which is printed in the point label. It should be emphasized that the objective function is multimodal and has several local minima, and consequently some of the points shown in Figure 5 might represent only local and not global minima.

With respect to the criterion of the maximum effective stress, the best diagonal configuration is found for $d = 15$ (the left-most point in Figure 5). It corresponds to a specific configuration of the diagonals used in the TZs shown in Figure 7. The diagonals join the nodes (2, 5), (4, 6), (1, 7) and (3, 8), as numbered in Figure 1. Notice that each node joins the same number of four beams.

5.2 Diameters of the beams

Figure 6 shows the optimized diameters of the beams $\bar{\mathbf{x}}_{15}$, as computed for the optimum configuration of the diagonals $d = 15$. The starting point of the optimization was the design with all diameters approximately equal to 40 mm, which resulted in the maximum effective stress at the critical level of 519 MPa. In the course of optimization, the maximum effective stress has been reduced to approximately 104 MPa, which is safely below the yield strength of steel and which indicates that the total mass of the module can be significantly decreased. The optimized values of the diameters $\bar{\mathbf{x}}_{15}$ of the beams are shown in Figure 6.

For each beam, the maximum effective stress occurs under the same load $\mathbf{F}(d)$, but it might occur in a different 5-module configuration s of the TZ structure. Figure 7 shows these 16 worst-case configurations for the successive beams. Each beam is drawn using its scaled optimized diameter.
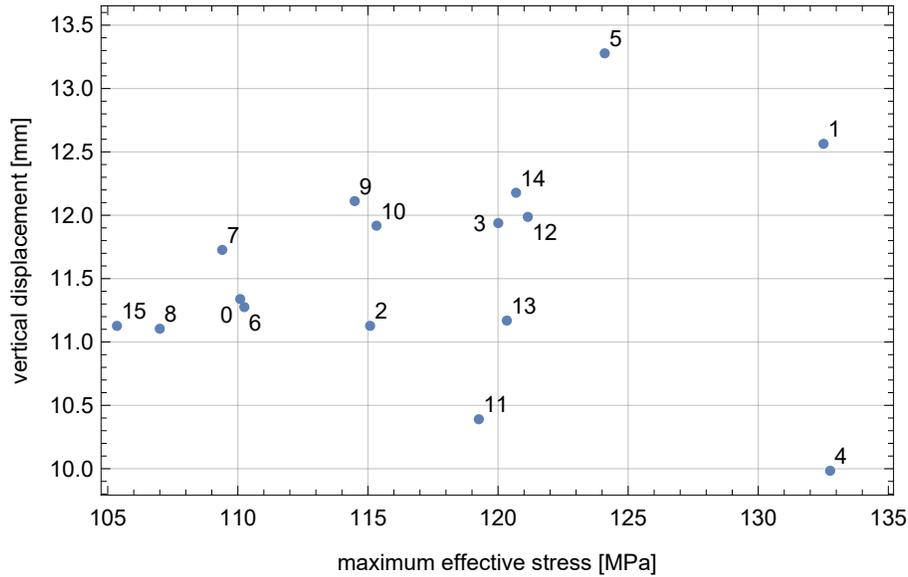
Figure 5: The 16 optimized modules that correspond to the 16 possible configurations of the diagonals, shown in the space of maximum effective stress vs. maximum vertical displacement. The specific configuration of the diagonals is shown in the label of each point.
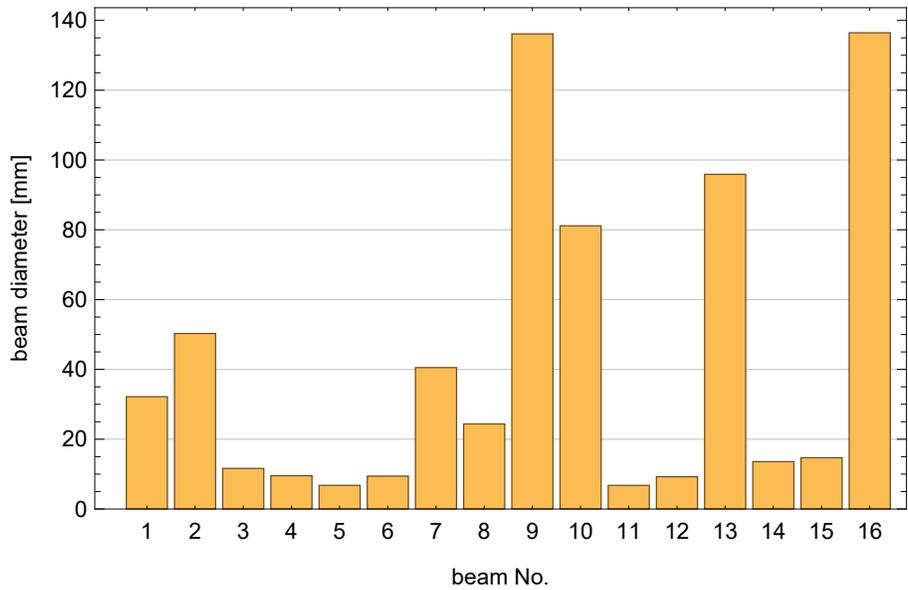
Figure 6: The optimized values $\bar{\mathbf{x}}_{15}$ of the beam diameters obtained in the optimum configuration of diagonals $d = 15$. The corresponding maximum effective stress has been reduced to 104 MPa from the initial critical level of 519 MPa (for the design with all diameters equal).
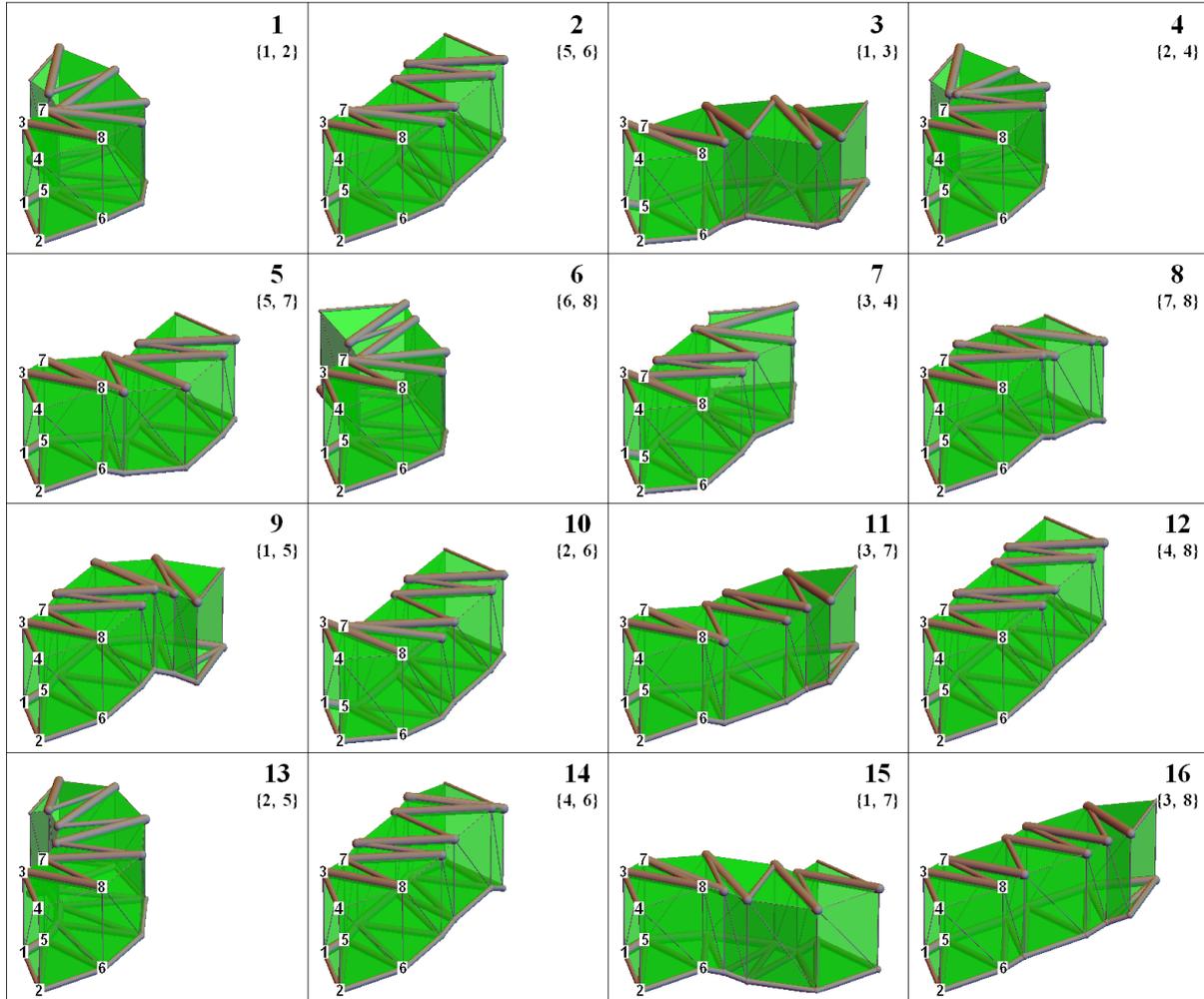
Figure 7: The 16 worst-case TZ configurations s that correspond to the optimum diagonal configuration $d = 0$. Each beam is drawn using its optimized diameter. The vertices defining beams are displayed at the first TZM in a sequence. The vertices identifying the most stressed beam are displayed for each case.

6   CONCLUSIONS

This contribution presents a preliminary study on structural optimization of the Truss-Z module. The outer geometry of the module is assumed fixed, and it is subjected to a static vertical load of 5 kN/m^2. The optimization variables are the diameters of the beams, as well as the internal topology of the module (configuration of its diagonals), as constrained by the assumption of the constant mass of the module. The optimization goal is to minimize the maximum effective von Mises stress that occurs in all 16 beams in all 5 modules in all 256 possible 5-module configurations. For the resulting optimized module, the maximum effective stress in each of the beams equals approximately 104 MPa, which is well below the yield strength of steel and which indicates that the mass of the module can be safely decreased. In future work, multicriterial optimization will be performed with respect to geometric parameters of the module, and the buckling effects will be taken into account.

Acknowledgments

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