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ARCHING ACTION IN SLACKENED STRUCTURES

Elastic-plastic composite slabs with no-tension matrix are considered, with unilateral in-plane restraints at supports. In such conditions important compressive membrane forces (arching action) are generated by transversal loads. Because of unstable character of such flexural response it is very sensitive to slackening due to clearances at supports. An approximate method based on the post-yield approach (PYA) was adopted and permits to determine easily the ultimate supportable load for slackened structures. The method was calibrated and verified by the FEM incremental analysis.

Key words: slackened structures, arching action, elastic-plastic composite slabs

1. SLACKENED STRUCTURES AND GEOMETRICAL NON-LINEARITY

Analysis of imperfect structures, either slackened ones or undergoing structural damage due to precedent overloading, seems to be a promising domain of structural mechanics. In spite of a long-lasting interest in stability of imperfect shells and recently increasing interest in design and optimization accounting for tolerances (e.g., [2]), the domain is challenging and not sufficiently explored. Contributions by Andrzej Gawęcki (e.g., [4, 5, 6]) opened a new horizon in the domain of elastic-plastic structures slackened with intentionally created gaps at joints. It is hoped that this path will be continued by his followers.

The impact of clearances and imperfections on the structural response is, in general, enhanced by the geometrical non-linearity of the system. It arrives that a slackened structure becomes kinematically unstable as a result of some initial damage and is maintained only due to secondary (quasi-structural) agents. But after important geometry changes it becomes stout enough to withstand centuries of extreme environmental loading. This observation concerns first of all historical stone structures, e.g., the Roman wall at Tarsus shown in Fig. 1. It

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may be remarked that, as demonstrated in Andrzej Gawecki contributions, the slackening may permit geometry changes without excessive deformation. Historical structures may survive, when the system of clearances permit configuration changes and, e. g., apparition of the arching effect without deformation inducing local damage and fracture. Such restabilized segment of the stone wall in Fig.1 is marked with white contour.

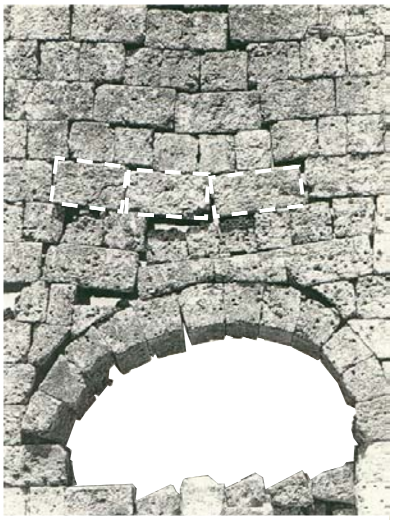


Fig. 1. Roman stone wall at Tarsus

On the other hand, slackening due to inevitable structural clearances may strongly reduce the structure performances, especially under geometrically non-linear conditions. In this case studying of the structure sensitivity to slackening is of a real technological interest. A case of an extreme sensitivity to slackening may be observed in the ultimate-load response of structures undergoing restrained flexure in the presence of arching action.

2. POST-YIELD ANALYSIS OF RESTRAINED BENDING

The term "restrained bending" concerns transversal flexure of flat structures with in-plane displacements at supports prohibited or restricted. For structures composed of metal-like ("symmetric") materials the in-plane restraints generate membrane effects and, therefore, influence the structure response only at very advanced deformation. However, if the material characteristics are different in tension and in compression ("non-symmetric materials") these restraints may change qualitatively the structure response from the very beginning of the deformation process. For example, in the case of brittle-matrix composite structures the end fixity generates important compressive membrane forces.

The difference between simple and restrained bending may be illustrated with an elementary case of plastic yielding in one-way bending of a uniform cross-section made of a non-symmetric material. The yield locus in the plane of stress resultants (bending moment – axial force, Fig. 2) is non-symmetric with respect to the *M*-axis. If the in-plane deformation rate is prohibited by the support restraints (λ=0) the plastic flow vector $\boldsymbol{\nu}(\lambda=0, \kappa)$ is normal to the locus at its apex, which corresponds to the ultimate moment M_u (well superior to the plastic flexural modulus M_f) and to a compression axial force $N = ½(N_c - N_t)$. Therefore, in this case purely bending response ($N = 0, \lambda=0$) is possible only for symmetric materials.

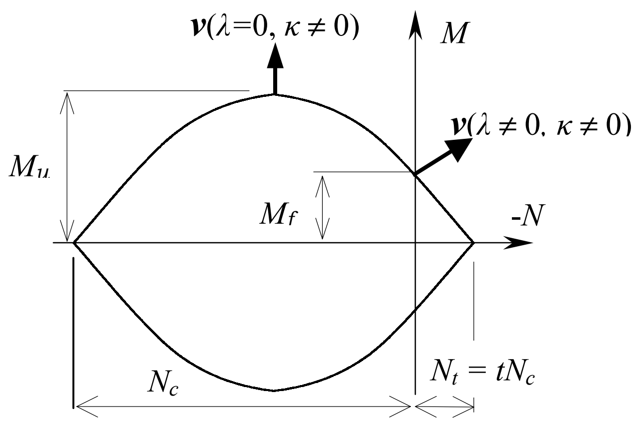


Fig. 2. Yield locus for a uniform cross-section of a one-way slab strip; non-symmetric material with the ratio of yield points in tension and compression σ_t/σ_c= t

The effect of in-plane support restraints, known as the "arching action" in RC beams and the "dome effect" in slabs strengthens considerably the structure but makes its response strongly unstable. It was discovered and discussed already long ago [7] but has been rarely taken into account in engineering practice because of its extremely unstable character. The geometrical non-linearity inherent to the behavior of eccentrically compressed slender bars is enhanced here by the deformation-dependence of the membrane forces.

Analysis of such behavior is possible nowadays even with commercial FEM codes and using sophisticated constitutive laws. However, in the case of brittle materials the corresponding procedures become slow, capricious and frustrating potential users. Moreover, they are very sensitive to input data and to modeling of support conditions. Important uncertainty level concerning the agents that are sometimes considered irrelevant by engineers makes the rigorous FEM approach of limited utility for practical purposes now. Therefore, a qualitative analysis and simplified methods, which were the only way possible in the pre-computer era, are still useful and need attention.

When geometrically non-linear response of elastic-plastic structures is concerned the post-yield approach (PYA) appears to be friendly and effective tool. It consists of determination of a sequence of instantaneous collapse loads for the structure configurations consecutively modified following the rigid-plastic flow mechanism. In this way the load-deflection relation corresponding to the quasi-static deformation path is established. This approach has been applied to restrained bending of RC slabs starting from early propositions [6] up to recent applications (e.g., [1]). The PYA gives reliable results for advanced deformation and, therefore, may be successful only in the absence of early geometrical softening. In the latter case the early response may be determinant for the ultimate-peak load and, hence, for the structure safety. Unfortunately, the restrained bending response falls into this category. However, the PYA may provide a realistic description of the early yielding if a spring model (Fig. 3) is used to account for elastic in-plane deformations.

This method was proposed long ago by the first author [8] and was revisited [9], when its pertinence has been proved by the incremental FEM analysis. As details of the method revisited here are given in [8] and repeated also in [9], we recall here only its principal assumptions leading to final results. It consists of applying the elementary limit analysis (plastic hinges, yield lines) in the framework of the PYA methodology, with the elastic in-plane compliance modifying the kinematical compatibility relations inherent to the plastic flow.

The load-deflection relation (see the formulae in Sec. 3) contains a term representing the plastic collapse load in simple bending and a term corresponding to the decreasing arching action as follows from the rigid-perfectly plastic PYA. The third term due to the elastic in-plane compliance of the slab is controlled by the rigidity parameter ε depending upon the span-to-thickness ratio $\Lambda = L/H$, resulting elastic compliance of the system C_{rs} and yield point in compression σ_c, as well as the geometry of the collapse mode (Fig. 3).

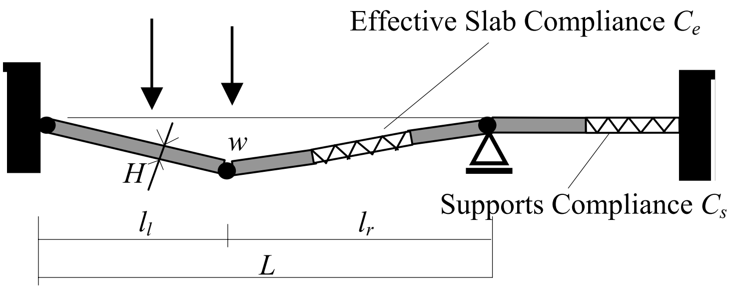


Fig. 3. Three-hinge collapse mode for one-way slabs completed with the elastic spring model; C_e – effective slab compliance, C_s – resultant support compliance

The resulting compliance $C_{rs} = C_e + C_s$ is composed of the effective in-plane compliance of the slab span C_e and of the compliance of the supports C_s (if any). An essential drawback of the method consists of an arbitrary choice of the effective value of the slab compliance. As a matter of fact, in the case of elastic-plastic deformation of a non-symmetric material the elastic axial compliance of the slab is different from its value in elastic behavior $C=L/EH,$ it depends upon the distribution of the stress resultants and evolves in the deformation process. To furnish a reliable effective (average) value of C_e calibration by the FEM analysis for benchmark cases is necessary.

Our attention is focused here on one-way bending but the approach is applicable also to two-way slabs [8]. However, in this case more attention has to be paid to the calibration of the rigidity parameter following the slab geometry.
It appears that for no-tension unreinforced structures a satisfactory fit in ultimate-peak loads furnished by the PYA and the FEM analysis (Fig. 4) is obtained if the effective slab compliance C_e is taken double of the compliance for the purely elastic slab strip $C = L/EH$. For reinforced structures with no-tension matrix a little better fit is obtained with this increase reduced depending upon the reinforcement intensity [9]. Neverthless, even without this correction errors are not excessive. As shown in Fig. 4a, the PYA and FEM results differ qualitatively at the beginning of the deformation process, because the PYA analysis requires deformation commencing at the simple bending collapse load. The ultimate-peak load occurs at deformation equal about a half of that corresponding to the maximum axial force. At this phase the fit of both curves is the most satisfactory and the discrepancy becomes again more important near the curve minimum corresponding to the transition from the compressive arching action to the tensile membrane response. However, the latter phase is of little practical interest. Of course, in the case of no-tension unreinforced slabs (Fig. 4b) all the discrepanciesare irrelevant, because the FEM and PYA curves (Fig.

4b) start at zero load (simple bending collapse) and, moreover, there is no ascending membrane branch of the curve.

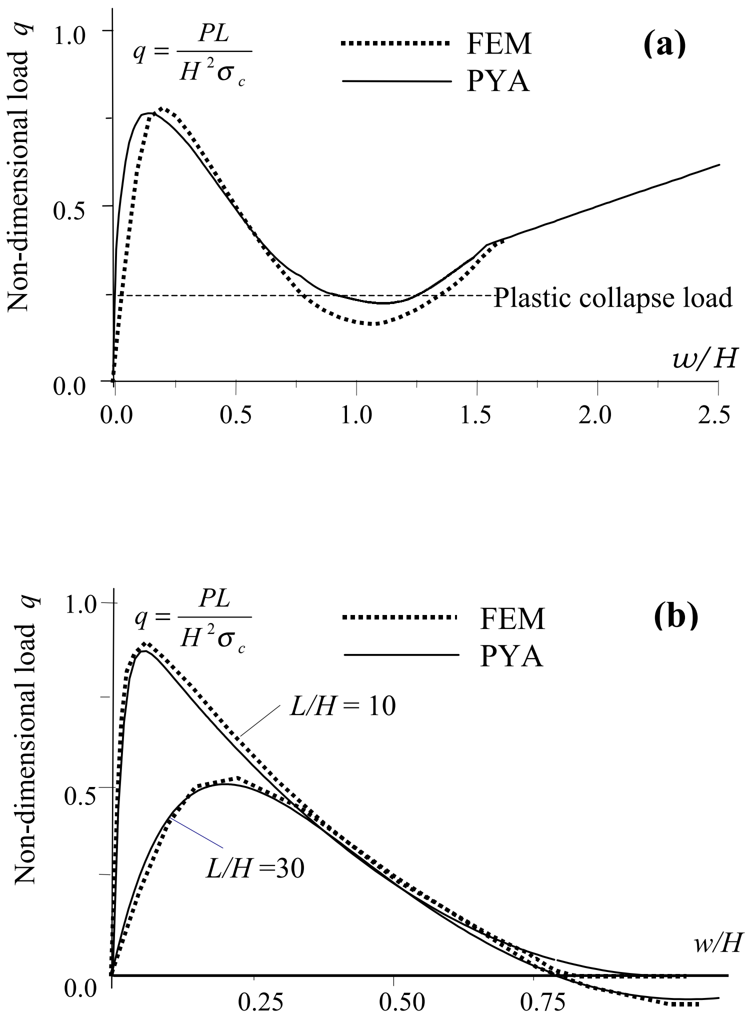


Fig. 4. Restrained bending response: non-dimensional load q vs. ratio of the mid-span deflection to the thickness w/H – curves for centrally loaded one-way clamped slabs. No-tension matrix with E/σ_c = 750; dashed lines – PYA, solid lines – FEM results; (**a**) Slender strip (Λ = L/H = 30), with bottom-face reinforcement; **(b)** Unreinforced strip

3. SLABS WITH SUPPORT CLEARANCES

Slackening of beams and slabs either introduced deliberately or due to imperfections may concern rotation allowances [3] or clearances at supports. The latter correspond to the presence of unilateral constraints or, more generally, to contact conditions. It is obvious that in-plane unilateral restraints generate compressive membrane forces accompanying transversal bending even in the case of "symmetric" elastic materials [11]. However, in this case the arching action produces the rise in the ultimate load of the order of few percents only. The effect becomes important for non-symmetric materials and, especially, for structures with a brittle or no-tension matrix, since in these cases the strengthening effect of arching action is especially important. Unfortunately, this effect appears to be very sensitive to clearances.

An illustrative example of such response may be given using the simplest case of a one-way slab with the length smaller by Δ than the restraining supports span $L \gg \Delta$. The structure responds in simple bending until the contact of the slab bottom faces appears at both supports and, then, restrained bending follows.

Elastic-perfectly plastic model is assumed for no-tension material of the matrix (with compression strength σ_c) and for face reinforcement (with the yield point σ_r and the surface area A_i per unit width of the strip). The reinforcement intensity is described by a reduced volume ratio η_i:

$$\eta_i = \frac{A_i \sigma_r}{H \sigma_c} \tag{1}$$

with $i=b$ and $i=t$ for the bottom and top reinforcement, respectively.

The FEM analysis of the contact problem may be performed using standard codes but its inconveniences are now even enhanced, when compared to the analysis of the restrained bending with perfect supports. Fortunately, the approximate PYA method described in Sec. 2 may be easily applied also to this case. The solution for flat slabs [9] has to be modified using a new initial condition for the differential equation describing evolution of the axial force N ($w = w_o$, $N = 0$); w_o stands for the deflection, at which the contact response commences. Following the rigid-plastic collapse mode used in the PYA analysis (Fig. 3) this deflection is determined depending upon the clearance Δ and the reinforcement intensities as:

$$\alpha_0 = \frac{w_0}{H} = k\left(1 - \sqrt{1 - \frac{\Delta}{\Delta_{\max}}}\right), \tag{2}$$

where Δ_{max} stands for the maximum clearance at which the contact may be effective:

$$\Delta_{\max} = \frac{LH^2k^2}{2l_l l_r} \ . \tag{3}$$

Dimensions are given in Fig. 3 and k depends upon the reinforcement intensity

$$k = 1 - \eta_b + \eta_t \, . \tag{4}$$

A non-dimensional parameterization of the load-deflection relation is introduced referring the current collapse load Q to the incipient rigid-plastic collapse load Q_{uo} in restrained bending of the structure, with its reinforcement neglected:

$$q = \frac{Q}{Q_{uo}} = \frac{M_{\max}}{2M_u} \ . \tag{5}$$

This non-dimensional load is equivalent to the ratio of maximum bending moment M_{max} in simple supported strip to the double ultimate moment (Fig. 2) for the no-tension matrix $2M_u = ¼\sigma_c H^2$. Such representation ensures the same load-deflection curve for any load configuration resulting in a three-hinge collapse mechanism of a slab that enters in contact with the restraining walls at the sag $\alpha_0 = w_0/H$, following (2). Equation for this curve is as follows:

$$q = q_Y + (k-\alpha)^2 - [k - \alpha_0 - (1 - e^{-\varepsilon(\alpha-\alpha_0)})(k - \alpha_0 + \varepsilon^{-1})]^2 . \tag{6}$$

The non-dimensional collapse load in simple bending q_Y is:

$$q_Y = 4\eta_b - 2(\eta_b - \eta_t)^2 \tag{7}$$

and the elastic rigidity parameter is

$$\varepsilon = \frac{2LH}{l_l l_r \sigma_c C_{rs}} , \tag{8}$$

with C_{rs} representing the slab and support compliance.

Depending upon the value of the resulting compliance C_{rs} equation (6) may describe restrained flexure from the rigid-plastic arching action (C_{rs}=0) up to simple bending response ($C_{rs} \rightarrow \propto$). The effective elastic compliance C_e has been calibrated in an extensive parametric study [10] to ensure the best fit in yhe ultimate-peak load q_U obtained from the PYA method and from the incremental FEM analysis. As remarked in Sec. 2. sufficient agreement is obtained for the reinforcement not eccesively strong if the effective compliance of the slab C_e is taken double of the purely elastic compliance of the matrix C. Hence, in the absence of support compliance (C_s=0) we can accept:

$$C_{rs} = 2C = 2L/EH. \tag{9}$$

The PYA and FEM results for centrally loaded slabs with different support clearances Δ are compared in Fig. 5. Slabs are bottom-reinforced, with its intensity (η_b = 0.062, η_t = 0) corresponding to about 1% of mild-steel reinforcement of concrete.

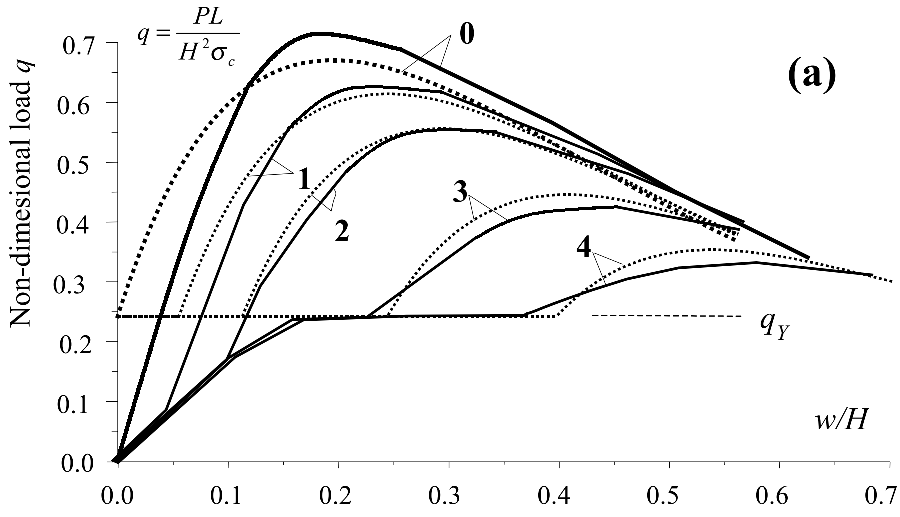
It was found [12] that in the case of perfect support restraints the maximum of the load-deflection curve falls nearly exactly at deformation equal to the half of that corresponding to the maximum of the axial force. It appears that this observation holds also when concerning the curve following eq. (6) for slackened slabs. The formulae from [10] are applicable when α and k are replaced by $\alpha' = \alpha - \alpha_o$, $k' = k - \alpha_o$, with α_o following eq. (2). Hence, the non-dimensional ultimate-peak load may be determined as follows:

$$q_U = q_Y + \varepsilon^{-2}[(k - \alpha_o - \ln\sqrt{(k-\alpha_o)\varepsilon + 1})^2 - (\sqrt{(k-\alpha_o)\varepsilon + 1} - 1)^2] \quad (10)$$

and is shown with dotted curves in Fig. 6. It may be remarked that the ultimate load is nearly linear function of the clearance size Δ varying from the simple bending response q_Y at $\Delta = \Delta_{max}$, following (3), up to the value for clamped slab [10] at perfect initial contact (Δ = 0). For moderately slender and/or weakly reinforced structures the PYA formula (10) gives rather satisfactory evaluation of the ultimate load from the FEM analysis. For very thick strongly reinforced slabs with large clearances the PYA solution overestimates the strength reserve due to arching action.

The PYA solution and the corresponding FEM analysis presented above concern the case when the bottom reinforcement does not attain the support cross-section. If the reinforcement is effective in compression zone at the contact interface the rise in the ultimate load will be of the order of 10%. This case is not covered by the solution (6), in which tension reinforcement in plastic hinges is assumed stronger than the reinforcement in compression zone. To avoid a heavy artmetics PYA solution corresponding the opposite case is not presented here. The use of the solution (6) with the parameter k = 1 (as for double reinforcement, see [8]), although incorrect, does not introduce substantial errors.

The best agreement of the PYA and FEM results is in the case of unreinforced structures and it decreases with increasing reinforcement strength. As already remarked, for strong reinforcement a little better agreement may be obtained if a certain reduction of the effective slab compliance C_e is introduced depending upon the reinforcement intensity η (see [10]).



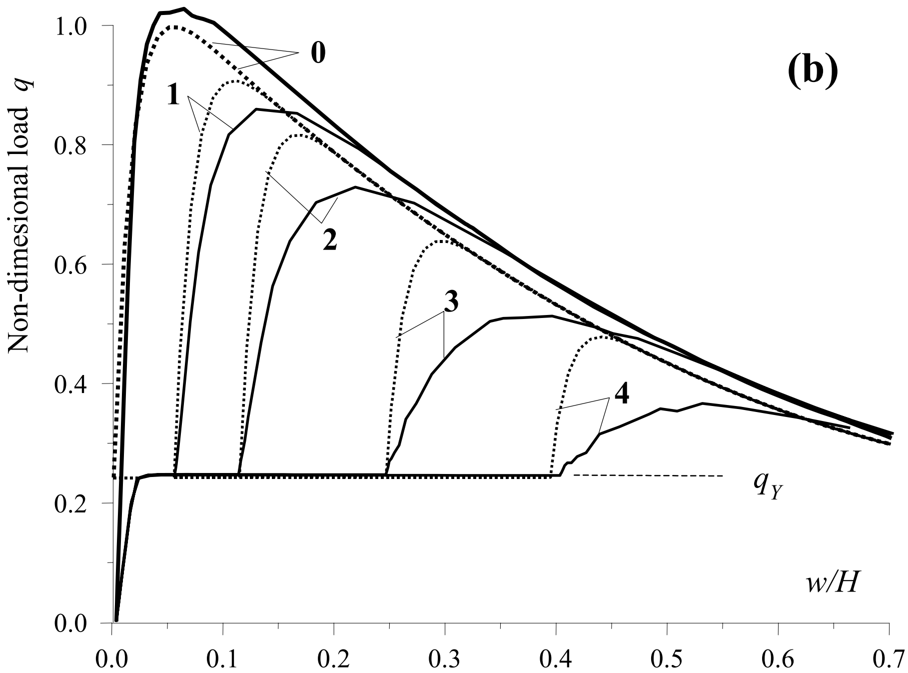


Fig. 5. Load-deflection curves for slackened slabs with different support clearances: $\delta = \Delta/\Delta_{max} = 0$ (i.e. no gap, curves **0**), $\delta = 0.11$ (**1**), $\delta = 0.23$ (**2**), $\delta = 0.46$ (**3**), $\delta = 0.76$(**4**); solid curves – FEM simulation, dotted curves – PYA results;
(a) slender slabs: $L/H = 30$, $\Delta_{max} = 0.6$ cm; **(b)** thick slabs: $L/H = 10$, $\Delta_{max} = 5.3$ cm

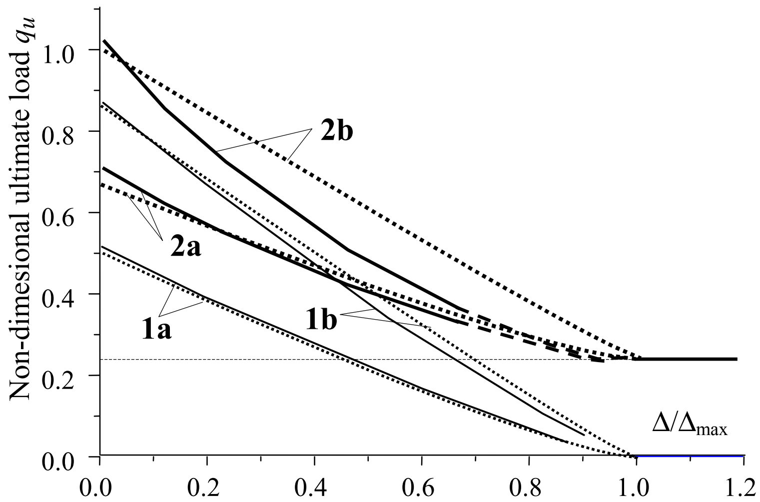


Fig. 6. Ultimate load q_Y vs. support clearance Δ: **1** – unreinforced slab (η = 0), **2 –** bottom reinforcement (η = 0,062); **a** – slender slab (*L/H* = 30), **b** – thick slab (*L/H* = 10);

solid lines – FEM results, dotted lines – PYA formula (10)

4. FINAL REMARKS

The ultimate strength of slabs undergoing restrained bending is very sensitive to clearances at unilateral in-plane supports. This sensitivity is particularly important for slender structures. In this case the clearances even of the order of only 5% of the slab thickness may practically annihilate the strengthening effect of the arching action. It is obvious that also a drop in the temperature may increase the clearance. Therefore, one should be very cautious when accounting for the arching action in presence of unilateral restraints, when any wedging/prestraining is absent. This fact was well known to ancient builders of stone-skeleton structures.

The PYA method gives simple and reasonably correct description of the load-deflection response of elastic-perfectly plastic composite structures with no-tension or brittle matrix when the reinforcement is weak or absent. These are the cases when a contribution of the arching action to the structure load-carrying capacity is most needed.

Applying this approach in the case of compliant supports needs only adding the support compliance C_s into the expression (9) for the resulting compliance of the system C_{rs} (see the analysis and test data in [10]).

The approach may be applied also to regular-shape two-way slabs (see [8]) but sufficient FEM parametric studies necessary for calibration of the rigidity parameter ε and, first of all, experimental verification are still lacking.

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EFEKT ROZPORU W PŁYTACH Z LUZAMI

S t r e s z c z e n i e

Rozpatrywane są pasma płytowe (kompozyty o matrycy z pomijalną wytrzymałością na rozciąganie) przy jednostronnych więzach w płaszczyźnie płyty. W takich warunkach poprzeczne obciążenia powodują powstanie znacznych ściskających sił osiowych (efekt rozporu). Ze względu na silnie niestateczny charakter procesu jest on bardzo czuły na występowanie na podporach ewentualnych luzów (nieuniknionych przy jedno-stronnych więzach). Przybliżona metoda oparta na podejściu pozagranicznym (ekstra-polacja metod nośności granicznej na zagadnienia nieliniowe) została zastosowana do pasm z luzami. Pozwala ona na łatwe określanie maksymalnego udźwigu. Kalibrację danych wyjściowych i weryfikację wyników metody wykonano za pomocą analizy przyrostowej MES.

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