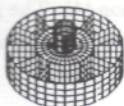# COMPUTER METHODS IN MECHANICS vol. II

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NONLINEAR STATIC ANALYSIS OF A GUYED MAST WITH FINITE ELEMENT METHOD

Eligiusz POSTEK, Jerzy ROJEK, Horacio ANTUNEZ

Summary

This paper deals with numerical aspects of nonlinear static analysis of a guyed mast of 649 m height. A concept of a new structure constructed of solid bars instead of tubes is analyzed. Due to decrease of member diameters the wind load declines, thus the horizontal displacements are smaller. A few structural variants of the proposed mast are calculated.

Introduction

The collapse of the guyed mast at Gąbin in 1991 drew public attention to the technical problems of design, construction and maintenance of guyed masts. There was begun a discussion on the possibility of rebuilding the mast at the same place but of different construction.

Basing on our experience from work integrating the numerical computations group for the government commission investigating the cause of the collapse [1] we could pass to the numerical analysis of a proposed new structure. This paper presents numerical aspects of nonlinear static analysis of the new proposed mast [2].

The fallen radio mast at Gąbin was constructed of steel tubes. A single tube is more efficient than a single bar of the same cross–section, since, having larger inertia moment, is stiffer to bending and torsion. But this may be not true for structures constructed with these elements, especially when wind loads are to be considered.

In fact, a well known Danish firm has preferred solid bars in its 300 m height television masts constructed in the last years [3]. When comparing both alternatives, attention should be focussed on the total structure weight, difficulties in construction, corrosion prevention, costs, *etc*. We are interested in the loading conditions and the mast resistance. Loads are given by the guys prestressing, the self–weight of the mast and of the guys, and the wind loads. Since the latter are linearly dependent on the exposed surfaces, it is desirable that the diameters of the elements in the mast be as small as possible.

In this paper we present a comparison of the behavior of masts built of tubes and solid bars, respectively. Each design alternative is analyzed with two models, namely a simplified one, which considers the mast to be a beam with the effect of guys accounted for by imposing appropriate forces where these are connected to the mast shaft, and a full discretization of the guys and the whole structure, as a frame of bars. Nonlinear geometry is considered for both models. The solution

is obtained with step–by–step analysis using U.L. (Updated Lagrangian) formulation [4]. The guys are prestressed and the effective forces they apply to the mast depend on the mast displacements and the curvature due to self–weight.
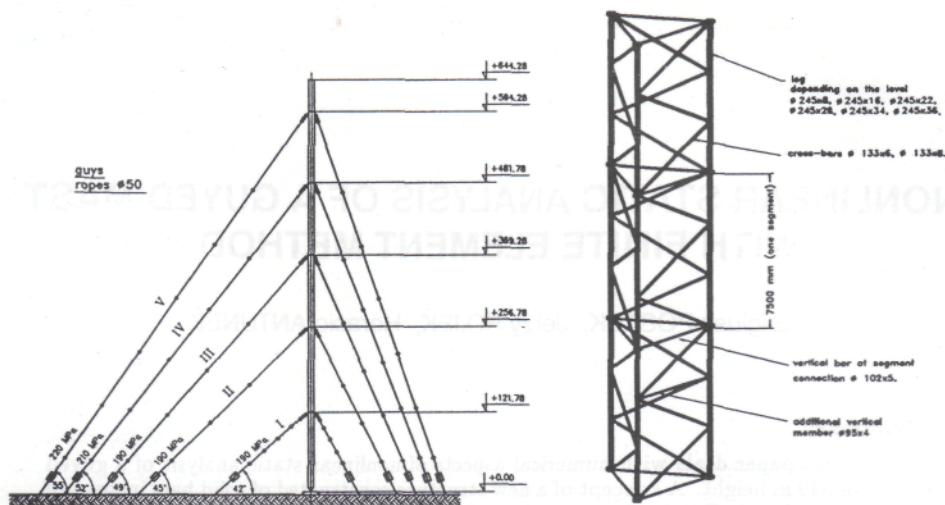

Fig. 1. Overall scheme of the structure.

The set of results show the comparative behavior of each design option, and the full model gives an idea about the accuracy of the beam–guys model, which is convenient for obvious computational time saving.

Numerical Model

The traditional method arising from engineering practice treats the mast as a beam on nonlinear spring supports whose stiffnesses are calculated earlier basing on separate calculation of each guy [5]. The method employed herein allows to analyze the whole system which consists of the shaft and the guys under complex load.

The outlay of the problem considered causes us to solve the problem in two steps. Firstly, to estimate the influence of changing the legs span on the behavior of the structure the simplified beam–guys model is considered, secondly to arrive closer to the possible design point and to verify the results obtained previously the full model is examined.

In the beam model of the mast the equivalent cross–sectional characteristics of the shaft should be used, but some of them are difficult to obtain: to estimate the stiffness properly it would be necessary to include the influence of shear stiffness which depends on the stiffness of the cross–bracings. This leads to an awkward procedure, in which it is not clear the determination of the different geometric properties of the beam. To avoid these problems in our analysis we use the Euler–Bernoulli theory of beam, neglecting the influence of shear stiffness. In our analysis we decide to cover a wide range of structural and loading patterns exploiting the beam–guys model and then after browsing the analyzed cases to refine our considerations we investigate the full model.

Since our analysis concerns a structure similar to the original mast at Gąbin [6] we start the study from the configuration of this mast. The calculations keep the main geometrical features — i.e. height, number of guy levels, frame structure of the shaft (Fig. 1), construction materials, and the environmental characteristics, as they are regarded by standards — wind speed, type of

terrain, *etc.* Meanwhile, variations in the design parameters are investigated: frame constructed of either tubes or solid bars, on the one hand, and in parallel, different diameters of the bars and different legs span.

The mast is a structure of triangular cross–section, kept in vertical position by five levels of guys. It is connected at the basis to the ground by a hinge which allows rotations. However, rotations with respect to the vertical axis are constrained in this point, in order to have the problem kinematically determined.

Loading. The loading consists of three following components: prestressing by the guys, self–weight and wind load.

The guys impose a prestressing to the mast shaft, which requires an initial stage of loading. From the point of view of the finite element implementation, this requires the possibility of introducing initial stresses at the element matrix generation stage.

To compute the body forces given by the self–weight, a specific weight of $\gamma = 78.50$ kN/m^3 is assumed for the steel. Only the elements constituting the frame and the guys are taken into account. This value is multiplied by a partial coefficient on loading $\gamma_f = 1.15$.

The IASS standards [7] are taken as guidelines for the calculation of wind loads, with the assumption that the mast is placed on a type 2 terrain, *i.e.* in a moderately windy place. An hourly speed of $v = 20$ m/s^2 is considered for the wind, together with a gust factor of 1.6. Again, no ancillary elements are considered, such as ladders, *etc.*

Since the wind load is proportional to the normal area facing the wind, it is assumed that the wind direction is coincident with a symmetry plane, thus maximizing the frontal area and, therefore, the wind load for a given wind speed.

Calculation of Stresses. To estimate the load carrying capacity of the structure the stresses in the legs are calculated and compared with the yield stress, then to check the structure with respect to buckling of members the ratio *normal force/critical load* assuming the buckling length as double length of a member is determined. Additionally, when dealing with the beam–guys model the stresses are calculated according to Polish Standard [8]. The yield stress is assumed 260 MPa.

Discretized Models of the Structure. Due to large displacements expected at the mast top and to the guys prestressing, the numerical model considers nonlinear geometry, in terms of the Updated Lagrangian formulation. We consider all the structure as working in the elastic range.

An introductory analysis is performed in order to calculate the necessary guys prestressing to have, after equilibrium is reached, the required forces in the guys, as given by measurements in the original mast on site. The magnitude of the force in a guy depends on its self–weight, prestressing and shaft displacements as well. The guys are treated as chains of truss elements with prestressing force where only tension is allowed. The analysis of such a chain is possible because of prestressing and the inclusion of geometric stiffness matrix. Depending on the length of the guys their models consist of 20 to 200 elements. Assuming different magnitudes of initial stresses the force at the lower end of each guy is calculated and compared with the measured values. The guy model is combined with both models of the mast: I, the beam–guys model, and II, the full discretized one.

The first model has obvious simplicity advantages, and with it a wide range of cases are calculated (Fig. 2). The shaft is discretized with 92 beam elements.

The model is three–dimensional, although, due to symmetry considerations, all displacements are symmetric with respect to the wind direction, and no torsion is present. The guys apply the load at the frame edges, not in the axis. To have the same situation concerning bending moments for the beam model the guys are attached to the extremes of rigid bars which are rigidly connected to the beam at each guy level. This is done by beam elements with a significantly higher stiffness
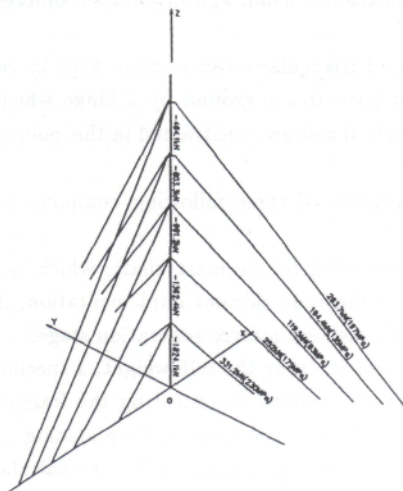
than the elements which model the mast shaft.

Fig. 2. Numerical model I.

The beam–guy model gives a good general approximation of the mast behavior, but in what concerns the stresses, it only gives an equivalent value along the axis. Finding out also the values of stresses in the frame elements requires an additional analysis to obtain the distribution of internal forces corresponding to the sectional resultant forces and moments in the mast. Here we can see the advantages of the full space–frame model in comparison with the model I. When we employ the model II in the analysis we can obtain directly internal forces and stresses in the legs, cross–bracings and vertical members.

So, some cases of the full model, namely, a configuration similar to the original mast at Gąbin, both with tubes and solid bars constituting the frame, and yet two more cases changing the legs span from 4.8 m to 3 m are analyzed. Loads are calculated and applied similarly to the beam–guys model, always concentrated at the nodes. This process was performed in 300 steps, and calculated on SPARC–2 stations, required a CPU time of 6 hours for about 2500 nodes and 3000 elements. The nonlinear systems of equations resulting for each load step are solved for both models by Newton–Raphson iterations.

In order to reach the final equilibrium configuration, the solution of each case involves the following steps: estimation of guys prestressing, this is done separately for each level of guys, estimation of initial equilibrium between systems of guys and the shaft, determination of displacements and stresses under self–weight and wind load.

Numerical Results

There are considered 9 structural patterns. Firstly, the mast erected 20 years ago is recalculated, however for homogeneity of all models, the wind load is assumed according to IASS code, secondly the next eight models are calculated assuming the solid bars. To simplify the comparisons and make them as clear as possible, there is assumed, that the cross–sections of the legs vary in the same way as in the original mast. There are two design variables: the diameters of the cross–bracings, which are of $\phi=80$ mm and $\phi=50$ mm, and the spans between the legs, which are assumed of 4.8 m, 3.0 m, 2.5 m and 1 m, respectively.

The original mast has a top displacement remarkably higher (4.71 m) than the masts constructed of solid bars which are for a cross–bracings diameter of 80 mm as follows, 3.11 m, 3.05 m, 3.08 m and 3.03 m. They are even lower for 50 mm, the results are: 2.62 m, 2.57 m, 2.56 m and 2.54 m (Fig. 3a).
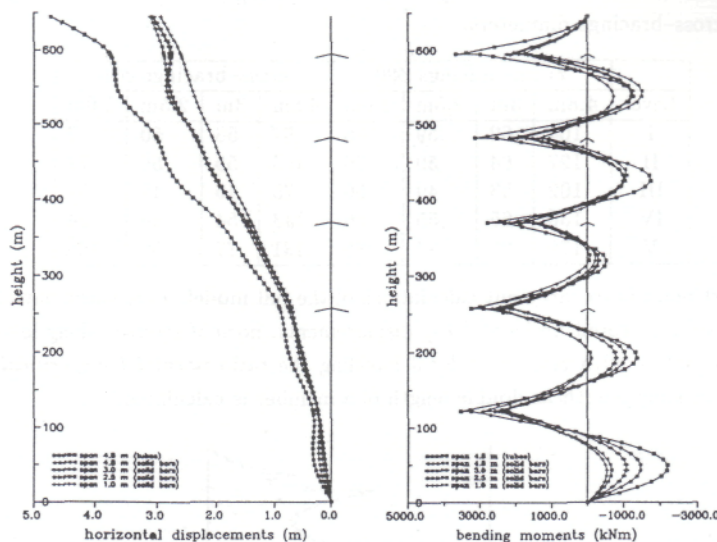

Fig. 3. Comparison of structures constructed of tubes and solid bars – model I, a) displacements, b) bending moments.

This is caused by the decreasing of the wind load. It is also seen a decrease of internal forces. The exemplary plot of bending moments varying along the shaft is given in Fig. 3b. The structures are checked for the load carrying capacity with respect to buckling of members by the two following methods; firstly, comparing the normal force in a member with the Euler force assuming the buckling length as double length of a member considered, secondly checking the ultimate limit state according to the Polish Standard [8].

Table 1. Ratio *normal force/critical force* depending on legs span and cross–bracings diameters.

Level	cross–bracings $\phi80$				cross–bracings $\phi50$			
	4.8m	3m	2.5m	1.0m	4.8m	3m	2.5m	1.0m
I	84	34	25	6	71	30	21	6
II	94	39	28	7	81	34	23	8
III	65	33	24	5	66	42	20	5
IV	101	48	37	8	84	40	29	8
V	133	77	61	22	97	57	44	29

Table 1 contains the ratios *normal force/critical (Euler) force*, Table 2 contains the ratios *normal stresses calculated according to the Standard* [8]/*yield stress* multiplied by 100. The stresses are monitored at the most crucial points of the structure, that is, close to the guy attachments.

The decreasing of dimensions of the structure implies improvement of the performance. The ratio decreases, however if the ultimate limit state is considered what involves inclusion of elastic–plastic buckling effect (according to the Standard [8]) it does not look so unique what arises

from Table 2. It appears that a certain minimum may exist, but the mast with the smallest legs span *i.e.* 1 m seems to be not acceptable from the very practical engineering point of view. So, it is beyond the scope of considerations.

Table 2. Ratio *normal stresses according to the Standard* [8]/*yield stress* depending on legs span and cross-bracings diameters.

Level	cross-bracings $\phi80$				cross-bracings $\phi50$			
	4.8m	3m	2.5m	1.0m	4.8m	3m	2.5m	1.0m
I	104	62	59	78	97	58	50	72
II	127	64	59	79	109	55	50	77
III	102	53	49	56	75	46	42	52
IV	136	65	55	56	113	54	44	54
V	179	82	84	86	131	77	59	105

The next point is to carry out calculations of the full model. Five static patterns are considered. There are monitored horizontal top displacements, normal stresses along leeward legs and to estimate the safety with respect to local buckling the ratio *normal force/critical load (Euler)* assuming the buckling length as double length of a member is calculated.
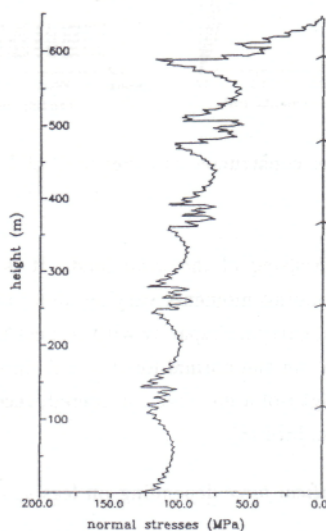

Fig. 4. Mast constructed of tubes, normal stresses along a leeward leg, model II.

Again we start from the original mast, which is constructed of tubes. The normal stresses in the compressed leeward leg along its length are given in Fig. 4. The highest level of stresses appears close to the base. The stresses do not exceed 130 MPa, what makes only half of the yield stress of the used steel material. There are also calculated the stresses in cross-bracings. The highest stresses are shown to be close to the guy attachments and never exceed 36 MPa.

Then, similarly to the case considered previously, the mast constructed of solid bars whose cross-sections are equal to the corresponding cross-sections of tubes, is analyzed. The cross-sections of bracings are assumed 80 mm. The decrease of wind load and significant increase of slenderness of the legs are the substantial features of the model. The path of equilibrium is followed until the loading reaches 30% of the wind load. After this point the tangent stiffness

matrix becomes singular, what means that the structure looses its overall stability. This is not demonstrated during the analysis of the beam–guys model.
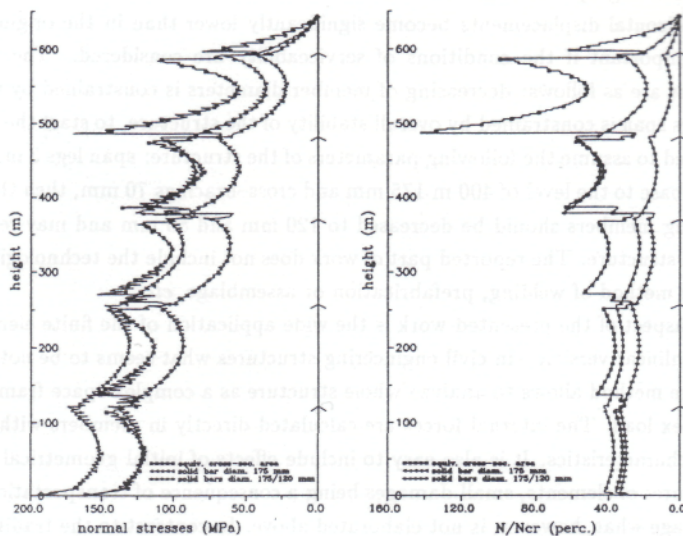

Fig. 5. Comparison of structures constructed of tubes and solid bars (model I), a) normal stresses b) ratio *normal force/critical load*.

The next point is to check if decreasing of the span legs improves the behavior of the structure. A mast of span legs 3.0 m and member diameters as taken into account previously is analyzed. Even though the slenderness of legs in the upper part of the mast is too high (about 250) the wind load decreases so the structure remains stable along the whole path of equilibrium. The maximum normal stresses in the legs do not exceed 160 MPa and the structure remains stable. However, this structure cannot be recommended to the design because the condition of member slenderness is not fulfilled.

Then, in consequence, we decide to assume the diameters of legs 175 mm constant along the whole shaft. The diameters of the cross–bracings are 90 mm, so their slenderness do not exceed 150 what agrees with the everyday design practice. In this case the highest normal stresses appear close to the base and reach 194 MPa. The stresses in cross–bracings do not exceed 16 MPa and the structure fulfills the conditions of the ultimate limit state and the state of serviceability according to the codes, however the self–weight of the structure cannot be accepted.

To arrive closer to the possible design point it is assumed a variation of the member diameters along the shaft. The diameters are changed two times in the lower part, that is, from the base to the level of 400 m the diameters of legs are assumed 175 mm and the cross–bracings 70 mm. The corresponding diameters in the upper part of the shaft are assumed as 120 mm and 50 mm. In this case the maximum normal stresses in the leeward leg close to the base are of value 159 MPa and above the third level of guy attachments 129 MPa. The top horizontal displacements are 2.8 m. The decrease of the wind load caused by the decreasing of all cross–sections and in addition the decrease of the self–weight allow to obtain a structure whose self–weight and the level of stresses are similar to the one in the original mast at Gąbin but of substantially lower horizontal displacement of the top.

Conclusions.

The analysis allows us to show the usefulness of the mast constructed of solid bars and to establish the first design point. The decrease of member diameters allows to decrease the wind load and the horizontal displacements become significantly lower than in the original mast. This is particularly important if the conditions of serviceability are considered. The further design recommendations are as follows: decreasing of member diameters is constrained by their buckling, decreasing of legs span is constrained by overall stability of the structure, to start the design process it is recommended to assume the following parameters of the structure: span legs 3 m, the diameters of legs from the base to the level of 400 m 175 mm and cross–bracings 70 mm, then the diameters of the corresponding members should be decreased to 120 mm and 50 mm and may remain constant to the top of the structure. The reported part of work does not include the technological constraints arising from the method of welding, prefabrication or assemblage, *etc.*

Another aspect of the presented work is the wide application of the finite element method – especially its nonlinear version – in civil engineering structures what seems to be not so common in our country. The method allows to analyze whole structure as a complex space frame–guys system with also complex load. The internal forces are calculated directly in members without employing any equivalent characteristics. It is also easy to include effects of initial geometrical imperfections, as initial curvatures of elements, small damages being a consequence of transportation and possible fault of assemblage what, however, is not elaborated above. In contrast to the traditional model of a beam with spring supports the problem of quality of results is first of all connected with proper load identification but not with equivalent characteristics.

In this case the finite element method when applied simultaneously with fast networked computers appeared to be very effective.

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