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BRINKMAN'S REGULARIZATION OF DARCIAN SEEPAGE

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Henri Coenraad Brinkman proposed an additional term in the Darcy equation describing the velocity field v of seepage through a porous medium, [1]. This term contains Laplacian of the velocity v and is similar to the term accounting for the viscosity added to Euler's ideal fluid flow equation to obtain Navier-Stokes' equation. In this manner the boundary conditions imposed on the flow can be satisfied, what is impossible in Darcian theory.

Darcys law as patterned on others transport equations (Fourier's, Ohm's) does not render aptly the specificity of the water seepage. A basic objection is that any viscous shear tensor can be derived from it, as the viscous shearing has been neglected. Related to this objection are difficulties in posing the boundary conditions, for example for problems in which the fluid flows through porous medium and adjoining empty space.
To study the problem of Brinkman's regularization we consider an one-directional steady gravity flow in two layers. In the upper layer (A) it is a free flow, and in the bottom layer (B) we have a seepage through the porous medium. Thus, the flow A is above the flow B. The continuity of solutions of both regions is assured by boundary conditions on the velocity and shear stresses at the interface of A and B regions.
The parameters of the problem are: the permeability K of porous medium, the fluid viscosity coefficient η and typical for Brinkman's theory the coefficient η' (known as the effective viscosity). The η' is a modified fluid viscosity which may be different from the η. If η' approaches zero, Brinkman's equation is becoming Darcy's one. The fraction $\eta/\eta' K$ is crucial for the solution of Brinkman's equation.
For small values of the K the flow is similar to that described by Darcy's law. With the increasing permeability K, the growth of the velocity v in porous medium, and in consequence, the growth of the velocity of the whole flow are observed.
Finally, the limit passage to high permeabilities was performed, and it is shown that if only η' and η are identified, the asymptotic behaviour (when for $K \to \infty$) of our solution for the porous medium exhibits that of a suspension flow, found in [2].
From practical point of view, the considered system permits to study characteristic traits of the flows in beds of rivers (canals, pipes, lakes) with obstacles at the bottom (such as stones, plants or other structures). The Brinkman's equation gives satisfactory description in the case.

References

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