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CAERDYDD

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Cardiff School of Engineering, 5th – 6th April 2004

ACME 2004

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Tuesday 6th April 2004

9:00 am – 10:40am Session 5a, Main Lecture Theatre (South 1.32)

9:00 – 9:20 A Thermodynamic Approach to Constitutive Modelling of Concrete
G.D. Nguyen and G.T. Houlsby.

9:20 – 9:40 Two-component contact for embedded planes in a 3D plastic-damage-contact constitutive model.
S. C. Hee and A.D. *Jefferson*

9:40 – 10:00 Size and Parabolic-Shape Optimization of Dome Structures with Buckling and Load Variation on the Joints Using Genetic Algorithm
M.R. Ghasemi and F. Azhdari

10:00 – 10:20 Constitutive Behaviour of a Pressure and Lode-Sensitive Material: Multiaxial Stiffness Change and Instabilities.
Roger Crouch and Mihail Petkovski

10:20 – 10:40 Recent developments in computational fracture mechanics at Cardiff
Q.Z. Xiao and B.L. Karihaloo.

10:40 – 11:10 Coffee Break, Room South 4.10(A)

9:00am – 10:40pm Session 5b, Lecture Theatre T4 (South 1.25)

9:00 – 9:20 Design Sensitivity of a Sequentially Coupled Problem: Casting.
R. Ahmad, D.T. Gethin, R.W. Lewis and E.W. Postek

9:20 – 9:40 Formulation of Lower Bound Limit Analysis as a Second Order Cone Programming (SOCP) Problem
A. Makrodimopoulos and C.M. Martin.

9:40 – 10:00 Finite element model of mould filling during squeeze forming processes.
E.W. Postek, R.W. Lewis, D.T. Gethin, R.S. Ransing.

10:00 – 10:20 Turbulence Modelling for Thermal Management of Electronic Systems
K. Dhinsa, C. Bailey, and K. Pericleous

10:20 – 10:40 Simulation of deformation of ductile pharmaceutical particles with finite element method.
L.L. Dong, R.W. Lewis, D.T. Gethin, and E.W. Postek.

10:40 – 11:10 Coffee Break, Room South 4.10(A)

Design Sensitivity of a Sequentially Coupled Problem: Casting.

R. Ahmad, D.T. Gethin, R.W. Lewis and E.W. Postek

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Abstract: Automatic Optimisation of manufacturing processes by numerical simulation is a key research area. The presented paper deals with the design sensitivity of a sequentially coupled thermo-mechanical problem. The key finding is to provide a tool for optimisation software (analytical design sensitivity gradients) for a sequentially coupled thermal-mechanical problem.

1. Introduction: Sensitivity analysis is a crucial step in process optimisation, reliability analysis and identification problems. Although established in linear systems, it presents challenges in the case of non-linear applications that are characteristic of manufacturing processes such as casting. In optimisation software, the design sensitivity gradients are often calculated using the Finite Difference Method (FDM) which require running the FE program twice at each optimisation iteration. The analytical design sensitivity gradients are more desirable as the calculated gradients are the accurate values and the FE program needs to be run only one time at each optimisation iteration.

In this paper, we begin by describing the nonlinear transient heat conduction system and design sensitivity analysis (DSA) that uses an analytical method. The problem that is being considered is the coupled thermo-mechanical analysis of the squeeze forming process. The final goal of this work is to determine process control, to achieve optimal part mechanical performance and toolset design through minimised die thermal stresses. The current stage focuses on the thermal stresses that are induced in the tool set. The scheme that is proposed adopts the design element approach.

2. Nonlinear Transient Heat Conduction and Structural linear static Problems:

It is well established that the transient heat conduction equation that describes the cooling and solidification cycle in the casting process can be descretised using the finite element method to give an equation of the form,

$$C(T)\{\dot{T}\} + K(T)\{T\} = F\ .$$

Within the die, subject to a linear mechanical response, following application of virtual work, the structural response is given by,

$$Kq=F.$$

3. Analytical Method for Sensitivity Gradients: The descriptor "Design elements" represents the designer's choice of the design parameters of the system. In sensitivity analysis using the design element concept, the discretised finite elements that are used to perform simulation are grouped together into zones that form fewer large design elements. This grouping is based on the designer's preference.

There are two approaches to evaluate gradients, a Direct Differentiation Method (DDM) and an Adjoint Variable Method (AVM). The DDM is used if the number of Design Constraints (DC) is greater than the Number of Design Variables (NDV). In DDM, the derivatives of the response with respect to design variables are solved as many times as there are design variables. Thus, the DDM is used if NDV < DC. In AVM, the adjoint equation is solved as many times as there are design constraints. Therefore, it is efficient to find the design sensitivity gradients using the AVM if DC ≤ NDV. In structural optimisation, AVM is the most efficient method as generally DC ≤ NDV.

3.1. Direct Differentiation Method (DDM): The current study focuses on establishing design sensitivity through the application of DDM and AVM methods. This has been applied to explore die thermal stress and the following explains the details.

Consider the equilibrium equation,

$$K(b)q=F$$

Where $\boldsymbol{b}$ is the design variable vector, $\boldsymbol{q}$ is the displacement vector, $\boldsymbol{F}$ is the global force vector and $\boldsymbol{K}$ is the global stiffness matrix. The goal is to find the sensitivity of a general scalar function $\psi\ (\boldsymbol{q(b)}, q_a, \boldsymbol{b})$ with respect to the design variables $\boldsymbol{b}$,

$$\nabla_b \psi \text{ subject to } \boldsymbol{K(b)q=F}$$

where $\nabla_b \psi$ is defined as

$$\nabla_b \psi = [\frac{\partial \psi}{\partial \boldsymbol{b}_1} \frac{\partial \psi}{\partial \boldsymbol{b}_2} \cdots \frac{\partial \psi}{\partial \boldsymbol{b}_n}]$$

and q_a is the displacement constraint. Assuming that the ***K*** matrix is not singular, both sides of the equilibrium equation are differentiated with respect to ***b***. The following expression for $\nabla_b \boldsymbol{q}$ can be derived:

$$\nabla_b \boldsymbol{q} = \boldsymbol{K}^{-1}[\nabla_b \boldsymbol{F} - \nabla_b(\boldsymbol{Kq})]$$

The exact sensitivities of ψ $(\boldsymbol{q(b)}, q_a, \boldsymbol{b})$ can be calculated by substituting $\nabla_b \boldsymbol{q}$

$$\nabla_b \psi = \nabla_b^e \psi + \nabla_q \psi . \nabla_b \boldsymbol{q}$$

Where $\nabla_b^e \psi$ is the gradient term for the explicit dependence of ψ $(\boldsymbol{q(b)}, q_a, \boldsymbol{b})$ on ***b***.

3.2. Adjoint Variable Method (AVM): We define an augmented functional $L(\boldsymbol{q},\boldsymbol{b},\lambda) = \psi - \lambda^T(\boldsymbol{Kq} - \boldsymbol{F})$, where λ is a Lagrange multiplier vector and the additional condition is the equilibrium equation. From the stationary condition, we have

$$\frac{\partial L}{\partial \boldsymbol{q}} = 0$$

Differentiating the augmented functional with respect to the design variable, we obtain

$$\frac{dL}{d\boldsymbol{b}} = \frac{d\psi}{d\boldsymbol{b}} - \lambda^T \frac{d}{d\boldsymbol{b}}(\boldsymbol{Kq} - \boldsymbol{F})$$

Since the state equation holds, we have

$$\frac{dL}{d\boldsymbol{b}} = \frac{d\psi}{d\boldsymbol{b}}$$

On the other hand, we define the sensitivity of augmented functional with respect to design variable as

$$\frac{dL}{d\boldsymbol{b}} = \frac{\partial L}{\partial \boldsymbol{b}} + \frac{\partial L}{\partial \boldsymbol{q}} \frac{d\boldsymbol{q}}{d\boldsymbol{b}}$$

By exploiting the stationary condition, we can find the adjoint vector as follows,

$$\boldsymbol{K}\lambda = \frac{\partial \psi}{\partial \boldsymbol{q}}$$

So, to obtain the sensitivities it is enough to find the partial design derivatives of the augmented functional. In consequence,

$$\frac{d\psi}{d\boldsymbol{b}} = \frac{\partial \psi}{\partial \boldsymbol{b}} + \lambda^T (\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{b}} - \frac{\partial \boldsymbol{K}}{\partial \boldsymbol{b}} \boldsymbol{q})$$

4. Design sensitivity example: The design sensitivity example of an axi-symmetric casting process is presented. The die material is Steel whereas the cast material is Aluminium LM 25. The die initial temperature was 200°C and heat was removed via convection in coolant channels. The initial temperature of the cast metal was just above the solidus temperature at 550°C.

4.1. Transient thermo-mechanical problem: Figure 1 shows the solidification in the cast part only at t=15s. Figure 2 shows the temperature field in the cast part and die at t=50s after the cast part has completely solidified. At t=50s, the temperature field in the die was directly used for the calculation of thermal stresses in the die for the structural evaluation as an illustrative step in the sequentially coupled thermo-mechanical problem. Figure 3 and 4 show the displacement and Mises stress in the die at t=50s. Figure 5 shows the design sensitivity of Mises stress with respect to the Young's Modulus. This has been calculated with respect to the Mises stress value close to the surface of the upper coolant channel. This has been chosen due to the high Mises stress gradients in the area. From figure 5, elements near the upper coolant channel and right bottom of the die have higher values of sensitivities as compared to the other elements. Figure 6 shows the design sensitivity of Mises stress with respect to the Young's moduli after applying the design element concept whereby the die has been zoned into four design elements. From figure 6, the element near the lower coolant channel has the greatest sensitivity value as compared to the other three elements.

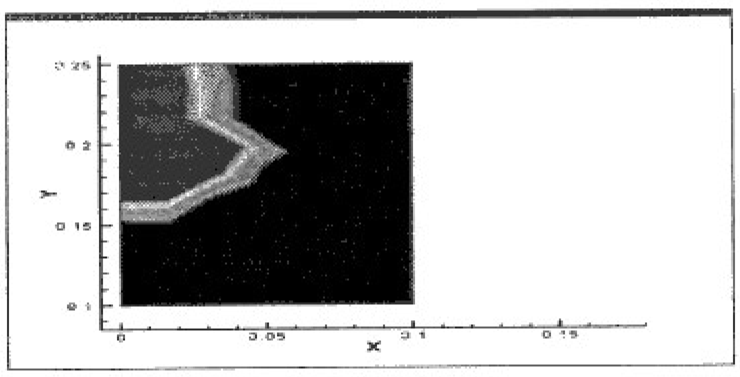

Fig. 1. Solidification in the cast part only at t=15s

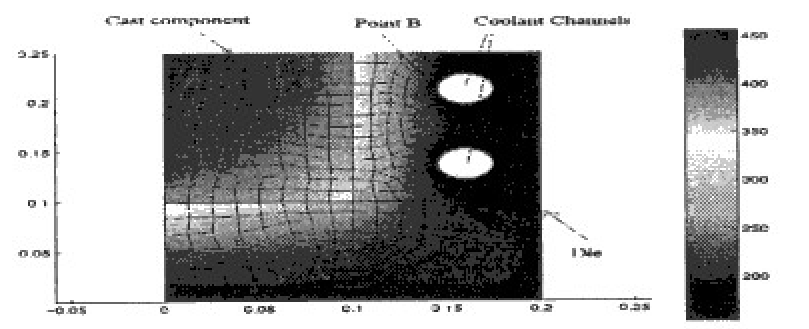

Fig. 2. Temperature field in the die and cast part at t=50s

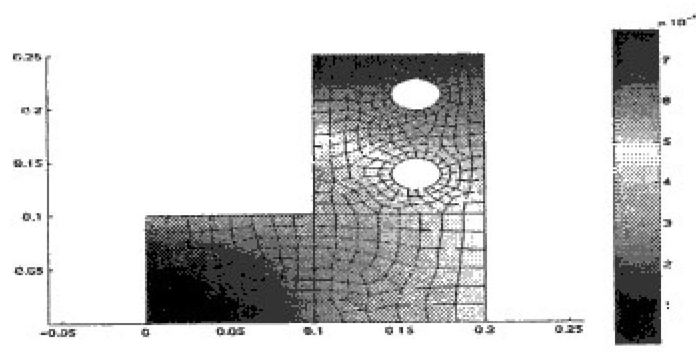

Fig. 3. Displacement in the die

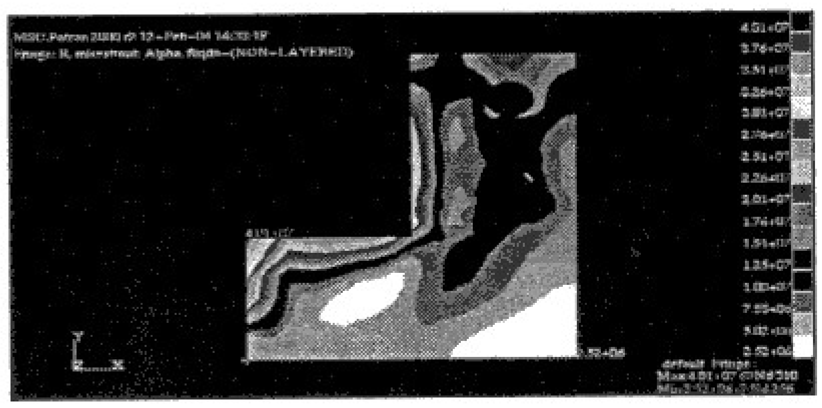

Fig. 4. Mises stress in the die

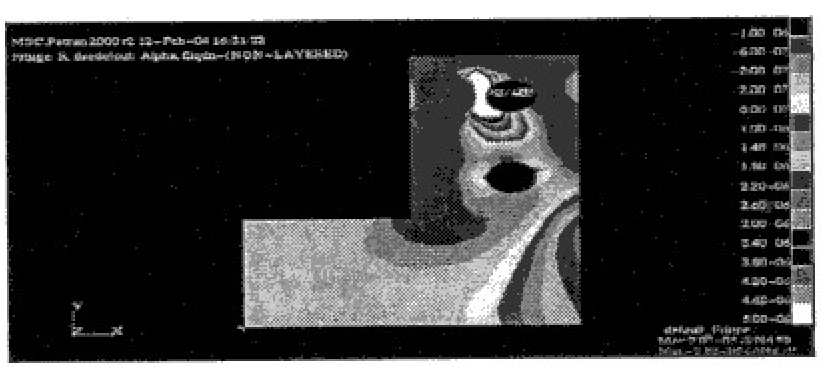

Fig. 5. Design sensitivity of Mises stress with respect to the Young's moduli in the die

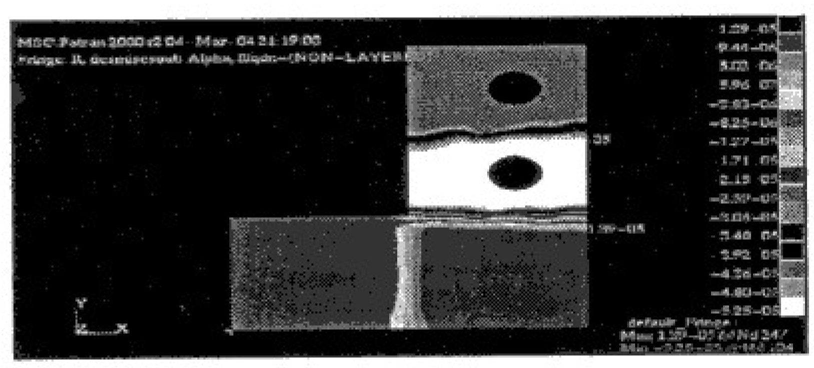

Fig. 6. Design sensitivity of Mises stress with respect to the Young's moduli in the die after applying design element concept

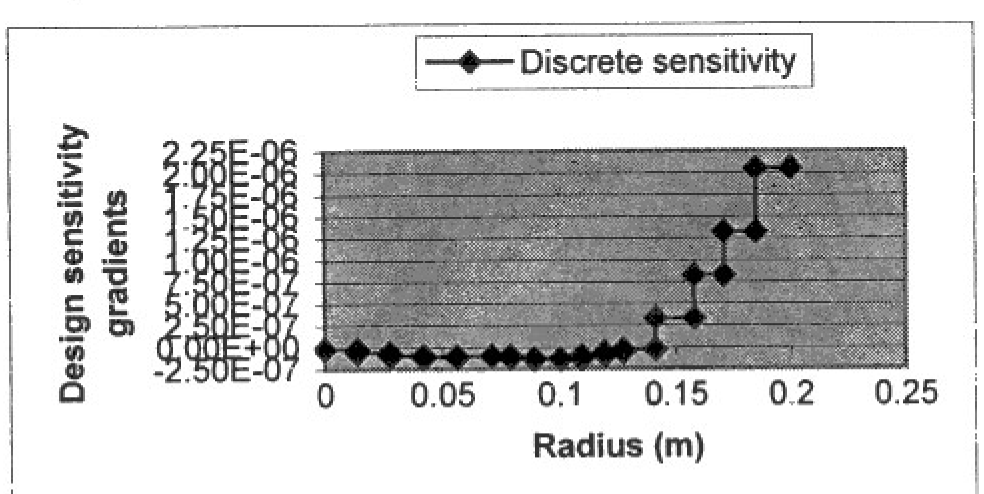

Fig. 7. Design sensitivity gradients vs radius in the die at the section line y=0.05m for the discretised solution

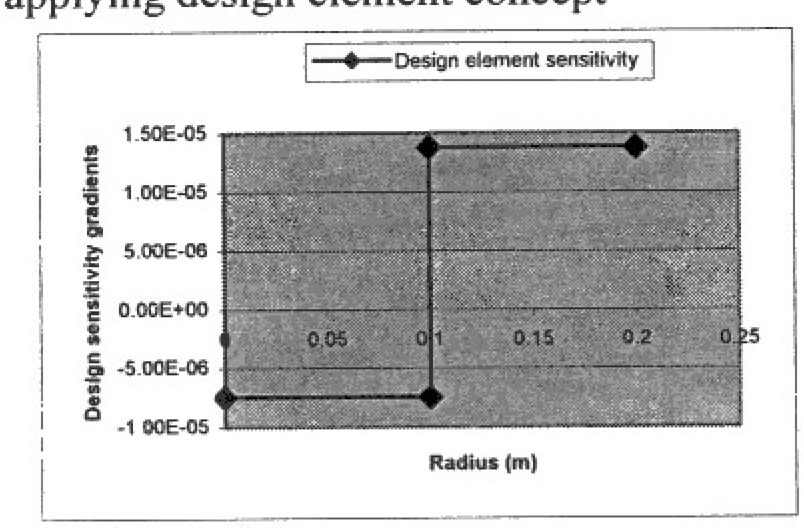

Fig. 8. Design sensitivity gradients vs radius in the die at the section line y=0.05m for the design element subdivision

Figure 7 and 8 show the graphs of the Design sensitivity gradients vs the radius of the die at the section along the line y=0.05m for the discretised solution and the design element subdivision. From figure 7 and 8, we can see that the trends of the design sensitivity gradients along the section line y=0.05m show some similarities in form, supporting the applicability of this concept.

5. Sensitivity Gradient Analytical Method vs Finite Difference Method: In this section, the calculated analytical design sensitivity gradients for a few elements are tabulated and then compared with the results for a FD based calculation. The calculated design sensitivity gradients can be described as accurate sensitivity gradients if the percentage errors are very small. Table 1 and 2 show the design sensitivities of Mises stress with respect to Young modulus at the middle part of the die just below the cast part. These small differences in the percentage errors apply to other elements as well.

	dSe/de (1e-8)
DDM	-9.36831
AVM	-9.36831

	% perturb	dSe/de (1e-8)	% error
FDM	0.2	-9.36838	0.0006
FDM	2	-9.24846	1.29

Table 1
Comparison of the analytical method and FDM design derivatives at the middle part of the die just below the cast part.

	dSe/de(1e-7)
DDM	-1.26683
AVM	-1.26683

	% perturb	dSe/de (1e-7)	% error
FDM	0.2	-1.28188	1.17
FDM	2	-1.26270	0.33

Table 2
Comparison of the analytical method and FDM design derivatives at the middle part of the die just below the cast part

6. Conclusions: This paper presents results on structural sensitivity that explores the impact of using dies having different material properties. The numerical results show good comparison between analytical and finite difference derived gradient values. The analytical design sensitivity gradients are ready to be supplied to the optimisation software for efficient optimisation process. The design element approach shows a promising method to assist with process design optimisation, but its robustness in application requires further exploration.

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