Optimization of A Steel Structure Taking Into Account The Randomness Of Design Parameters

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1. Introduction

The subject of the considerations are shallow lattice structures susceptible to global loss of stability by node snap through (Marcinowski [1], Waszczyszyn i in. [2], Thompson, Hunt [3]). In structures of this type, large displacement gradients may appear, therefore, when designing these covering, geometrically nonlinear relationships should be used. In the era of the more and more common trend of optimal design, an extremely important problem is to take into account the impact of the random nature of the parameters describing the structure. This work is an attempt to draw attention to this important aspect of the optimal dimensioning of bar structures using the formalism of the so-called robust optimization.
The problem of optimal design of ever larger and more complex structures forces engineers to minimize the cost of execution and the weight of the structure. Optimization methods are therefore becoming an indispensable tool in rational design. Thanks to them, it is possible to properly select material properties or dimensions of the structure, which is often a very labor-intensive task.

2. Optimization

It is worth noting that optimal structures are very sensitive to the random dispersion of model parameters and external interactions. Solutions that fulfill their function for nominal values may turn out to be unacceptable after taking into account the randomness of the parameters. Therefore, it seems natural to extend the formulation of deterministic optimization, which takes into account the random dispersion of parameter values. This formulation offers robust optimization. This term refers to the widely understood methodology of designing both structures, devices and production processes, in which, while maintaining the high functionality of the designed systems, the aim is to find a solution that is as resistant to changes in its parameters as possible. The solution obtained in the process of robust optimization is definitely less sensitive to the parameters of the model that are difficult to control or external influences. The advantages of this method include the fact that it leads to a solution that maintains quality and functionality in a wide range of working conditions. (Stocki [4], Gondzio et al. [5]).In rational design, it is necessary to strive to ensure the highest possible level of resistance of the structure to changes in the designed variables. Robust optimization can also be effectively used in the design of building structures. The methods known as 'robust' optimization are not, however, a frequent choice of designers in this field. Effective use of the random nature of parameters requires the improvement of the methods of stochastic analysis and work on engineering software enabling its application.

3. Single layer steel dome analysis

The example shows the analysis of a shallow single-layer steel structure, modeled with truss elements on pinned support. A force of P = 5kN was applied to the structure in each node. The bars were designed from S235 steel with the yield point fy = 235 MPa, Young's modulus E = 210 GPa and Poisson's ratio v = 0.3. The geometry and deformation of the considered structure is shown in Figure 1.

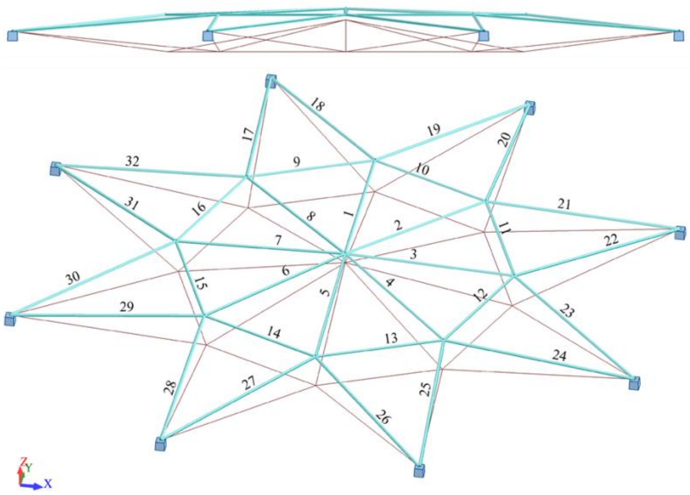

Figure 1. Geometry and deformation of the structure

On the basis of the static and strength analysis, the following cross-sections were assumed for individual groups of bars: RO54x3.6 for the Meridian_1 (bars 1 to 8), RO60.3x6.3 for Meridian_2 (bars 17 to 32) and RO 51x6.3 for the Ring (bars 9 to 16). The maximum effort of bars in the distinguished groups was verified, which amounted to 66%, 72% and 71%, respectively. The maximum vertical displacement was recorded at nodes 2 to 9 and was 13.62mm, while the limit value of vertical displacement according to Eurocode3 [6] is 46.67mm.

Then the structure was subjected to linear buckling analysis. The value of the critical load multiplier was α_{cr} = 1.714. Such a low value indicates the need to perform calculations that take into account geometric non-linearity.

The geometrically nonlinear analysis showed that the critical load multiplier was μcr = 1.184, while the limit displacement was 18.41mm. The effort of the bars was verified when the structure was loaded with the force P = 5.92kN, which was 71% for the Meridian_1, 90% for the Meridian_2 and 96% for the Ring. The maximum stresses in the bars did not exceed 62 MPa.

Reliability analysis.

The structure reliability was analyzed using the FORM method (Engelstad, Reddy [7]) when the structure was loaded with the force P = 0.98·5kN = 4.90 kN. The cross-sectional area of successive groups of bars Ai was assumed as random variables. The description of random variables is presented in Table 1. Variables are not correlated. The mass of the modeled structure is M = 813.51 kg. For the above case, the value of the coefficient of variation was set at 5%.

Table 1. Description of random variables

Random Variables	Mean Values	Standard deviations	Coefficient of variation

Xi	[cm^2]	[cm^2]	[%]
A1	5.7	0.285	5
A2	10.7	0.535	5
A3	8.85	0.443	5

On the basis of the analyzes, the maximum value of the node displacement was limited at w_{max} = 1.6 cm. For such a limit value, the limit function takes the form:,

$$f_s = 1 - \frac{w(\mathbf{x})}{w_{max}} = 1 - \frac{w(\mathbf{x})}{1.600} \quad (1)$$

where: w (x) - displacement in a given calculation step, w_{max} - assumed maximum displacement.

The determined value of the reliability index was β = 2.011, while the failure probability was pf = 0.022. The determined values of the elasticity index in relation to the mean value are as follows: for the random variable A1: 0.047, for random variable A2: 0.683 and for random variable A3: 0.729. There is a significant difference in the values of the elasticity index in relation to the mean value of variable A1 in relation to the elasticity indexes of the other variables. Therefore, it is worth verifying the influence of variable A1 on the reliability of the structure by re-performing the reliability analysis, taking it into account as a deterministic value. If only two random variables (A2 and A3) were taken into account, the value of the reliability index was β = 1.996, while the failure probability was p_f = 0.023. The difference in the value of the reliability index is 0.015. On this basis, for further calculations, we can accept two design random variables: A2 and A3.

Deterministic optimization

In the second part, we are looking for the optimal dimensions of the cross-sections of individual groups of bars: for the Meridian_2: A2 and for the Ring: A3 using the deterministic optimization algorithm (Błachowski et al. [8]).
The objective function will be the mass of the structure:

$$f_C = \text{minimum}\ (\rho \cdot (\sum_{j=17}^{32} L_j \cdot A2 + \sum_{9}^{16} L_k \cdot A3)) = \min(\text{Mass}) \quad (2)$$

where: L_j - length of the j-th rod from the Meridian_2 group, L_k - length of the k-th rod from the Ring group.

Simple constraints are described in Table 2. They represent the upper and lower limits of the searched design variables.

Table 2. Description of simple constraints

Design variable	Lower boundary [cm]	Upper boundary [cm]
A1	9.844	11.556
A2	8.142	9.558

For the case under consideration, 8% tolerance of the cross-sectional area was assumed.
The inequality constraint was formulated as the condition of not exceeding the permissible vertical displacement of the node, for w_{max} = 1.6 cm:

$$g(\mathbf{x}) = w(\mathbf{x}) - w_{max} = w(\mathbf{x}) - 1.600 < 0 \quad (3)$$

The deterministic optimization was carried out using the simplex method of Nelder Mead with the maximum number of iterations N = 1000 and the convergence parameter ε = 1.0 E-08.

The obtained dimensions of the cross-section are: A1=9.845cm^2, A2=8.331cm^2. The limit function value for this case was 761.706 kg. The probability of failure and the reliability index were also verified, which in this case was respectively: p_f = 0.5, while β = 0.0.

Robust optimization

In the case of robust optimization, random and design variables (μ_{A2}, μ_{A3}), the objective function and constraints were defined. The objective function is, as in the case of deterministic optimization, the mass of the structure, but assuming that it takes into account the weighting factor γ that determines the significance of each of the criteria (mean value and standard deviation). The value of the coefficient of variation was set at 5%.
For the case under consideration, the task of robust optimization takes the form:

1. Find the values of the variables: μ_{A2}, μ_{A3}
2. Minimizing: $f_C = \frac{1-\gamma}{\eta^*}[\text{Mass}] + \frac{\gamma}{\sigma^*}\sigma[\text{Mass}]$
3. With constrains:

$$E[w(\mathbf{x}) - 1.600] - \tilde{\beta}\cdot\sigma[w(\mathbf{x}) - 1.600] \geq 0$$
$$9.844 \leq \mu_{A2} \leq 11.556$$
$$8.142 \leq \mu_{A3} \leq 9.558$$

where: $\gamma \in [0, 1]$ - determines the meaning of each of the criteria, η^*, σ^* - normalizing constants, $w(\mathbf{x})$ - 1.600 - limitation of the permissible vertical displacement.

In order to confirm the correctness of the performed calculations, two methods of building the response surface were used: kriging (Simpson et al. [8]) and the second-order method (Box, Wilson [9]). The following parameters were assumed: γ=0.5, $\tilde{\beta}$=2.0.
Response surfaces are built using the kriging and second order method, while the experiments are generated according to the plan of optimal Latin hypercubes (Liefvendahl, Stocki [10]). After the robust optimization, the obtained values of the design variables and a mass of the structure are summarized in Table 3.

Table 3. The values of the random variables and a mass of the structure obtained in the robust optimization

	Kriging	**Second order method**
A2	9.926 cm^2	9.924 cm^2
A3	9.558 cm^2	9.558 cm^2
Mass	786.401 kg	786.309 kg

As a result of robust optimization, an increase in the values of the cross-sectional areas of individual bar groups and the weight of the structure was obtained. However, this results in a significant improvement in the safety of the structure, which is indicated by the values of the reliability index and the probability of failure, which in this case are respectively: for kriging: β = 2.038 and p_f = 0.021, for second order method: β = 2.036 oraz p_f = 0.021.

Summary

The results of both analyzes show a clear influence of the selection of the optimization method on the obtained values. The result of robust optimization is a structure resistant to random variable deviations. In analysis, it can be observed that the structure optimized by the robust method is lighter than the original structure by more than 27 kg, while the reliability index indicates its reinforcement.

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