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EFFECTIVE VISCOSITY OF A DILUTE SUSPENSION OF SPHERES BETWEEN PARALLEL SLIP WALLS

Néjiba Ghalya[1], Antoine Sellier*[2], Maria L. Ekiel-Jeżewska[3], and François Feuillebois[4]
[1]Laboratoire de Modélisation Mathématique et Numérique pour les Sciences de l'Ingénieur-LAMSIN, Université de Tunis El Manar, Ecole Nationale d'Ingénieurs de Tunis, BP37, 1002 Tunis Le Belvédère, Tunisia
[2]LadHyX, Ecol ePolytechnique, 91128 Palaise au cedex.France
[3]Institute of Fundamental Technological Research, Polish Academy of Sciences, Pawińskiego5b, 02-106 Warsaw, Poland
[4]LIMSI-CNRS, UPR 3251, BP 133, 91403 Orsay cedex, France

Summary The energy cost for transporting suspensions in micro-channels may be reduced by using slipping walls. A theoretical model is presented here for the effective viscosity, μ_{eff}, of a dilute suspension of freely suspended identical solid no-slip spherical particles carried by a Poiseuille flow between parallel slip walls. Compared with no-slip walls (Feuillebois et al., *J. Fluid Mech.*, 800:111, 2016), this viscosity strongly depends on the ratio of the wall slip length to particle radius, λ/a, for given ratio of the gap between walls to particle diameter $H/(2a)$: in the very confined case $H/(2a) = 2$ and for a very large slip $\lambda/a = 1$, the suspension intrinsic viscosity $[\mu] = (\mu_{\text{eff}} - \mu_0)/(\mu_0\phi)$, where μ_0 is the fluid viscosity and ϕ is the low suspension volume fraction, is divided by 3 compared with the no-slip case and divided by 5 compared with Einstein's result (5/2) for an unbounded suspension.

The effective viscosity of a confined flowing suspension is the relevant quantity to consider for particle transport (see e.g. [1] [2]). This viscosity depends on the suspension volume fraction and on the ambient flow field. Assuming a dilute suspension (i.e. at small volume fraction $\phi \ll 1$) of either spherical or elongated particles and a high-frequency ambient flow field, in the sense that all particles occupy all possible positions and orientations with equal probability, made it possible to calculate the suspension effective viscosity in Couette [3] and Poiseuille [4] flows between two parallel planar no-slip walls.

The same approach is used here for slip walls and no-slip spherical particles. On each wall the impermeability condition reads $\mathbf{u} \cdot \mathbf{n} = 0$ where $\mathbf{u}$ is the fluid velocity and $\mathbf{n}$ is the unit vector normal to the wall and pointing into the liquid. The usual Naviers's [5] slip boundary condition is applied on each wall:

$$\mathbf{u} = \lambda \frac{\partial \mathbf{u}}{\partial n}, \tag{1}$$

where $\lambda \geq 0$ is the wall slip length, assumed to be uniform and identical for both walls and n is the normal coordinate along $\mathbf{n}$. The Poiseuille flow velocity profile $u_0(z)$ with these slip conditions (1) on walls W_1, W_2 is sketched in Fig. 1.

The average pressure drop for driving particles in this flow field is derived by using Lorentz reciprocal theorem. The suspension effective viscosity is obtained from the relationship between this pressure drop and the volume flow rate. The result for the suspension intrinsic viscosity $[\mu]$ (defined in the summary) is:

$$[\mu] = \frac{9}{4\pi\left(1 + 6\dfrac{\widetilde{\lambda}}{\widetilde{H}}\right)\left(\widetilde{H} - 2\right)} \int_1^{\widetilde{H}-1} \left[\left(1 - 2\frac{\widetilde{z}_c}{\widetilde{H}}\right)\widetilde{S}_{xz} - \frac{1}{\widetilde{H}}\widetilde{Q}_{xzz}\right] d\widetilde{z}_c\,, \tag{2}$$

where $\tilde{H}, \tilde{\lambda}, \tilde{z}_c$ are made dimensionless with a sphere radius a. We denote $\widetilde{H}$ the normalized gap between walls and $\widetilde{z}_c$ the normalized position of the centre of a test particle (see Fig. 1). Eq. (2) involves (as is classical for the viscosity of suspensions) the stresslet component S_{xz} on a particle, that is a symmetric first moment of stresses on its surface. It also involves the quadrupole component Q_{xzz}, second moment of stresses. The normalized components of the stresslet $\widetilde{S}_{xz}$ and quadrupole $\widetilde{Q}_{xzz}$ appearing in (2) are defined as $\widetilde{S}_{xz} = S_{xz}/(\mu_0 a^3 k_1)$, $\widetilde{Q}_{xzz} = Q_{xzz}/(\mu_0 a^4 k_1)$, where k_1 is the linear shear rate at wall W_1.

Taking into account the full interactions of a sphere with both slip walls is not presently accessible. Two single-wall models are used here: the "nearest wall" (nw) and "wall superposition" (ws) approximations. The stresslet and quadrupole on a sphere near a single slip wall are calculated from analytical solutions of the Stokes equations in bispherical coordinates [6, 7]. Both nw and ws models are validated by comparing to earlier accurate results [3, 4] in the case of no-slip walls.

The suspension intrinsic viscosity $[\mu]$ is evaluated in the range $2 \leq H/(2a) \leq 100$ and $\widetilde{\lambda} \leq 1$. It depends strongly on $\widetilde{\lambda}$ for small $H/(2a)$, as detailed in the summary and shown in Fig. 2. The main contribution to the dependence on $\widetilde{\lambda}$ comes from the factor $1/(1 + 6\widetilde{\lambda}/\widetilde{H})$ in (2), which is independent of particle-wall interaction. This strong variation of $[\mu]$ on $\widetilde{\lambda}$ may have practical applications for particle transport in microfluidics. A handy fitting formula for $[\mu]$ is provided. Finally, outlooks for experiments are proposed: for instance, a non-optical microcapillary rheometer [8] may allow to measure first the slip length in pure fluid and then the suspension effective viscosity for comparison with the present theory. Details of this paper are available in [10].

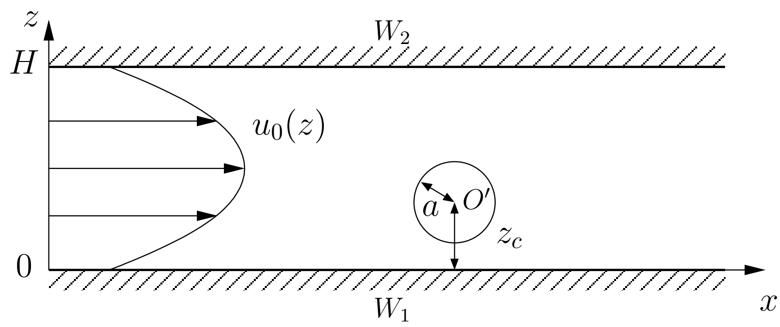

Figure 1: Sketch of a single sphere with centre O' and radius a freely suspended in a Poiseuille flow between two parallel solid slip walls W_1 and W_2.

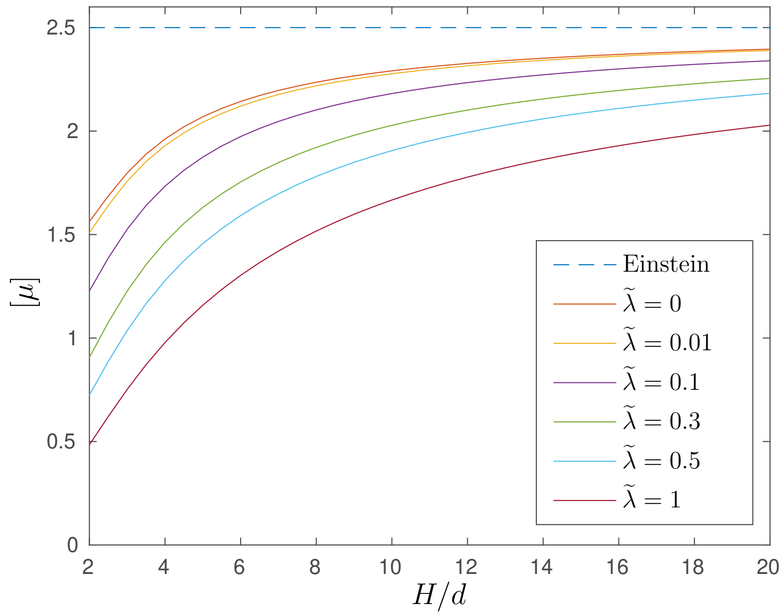

Figure 2: Variations of the suspension intrinsic viscosity $[\mu]$ calculated with the "wall superposition" model versus H/d (with $d = 2a$) for various values of $\widetilde{\lambda}$. From top to bottom: Einstein's [9] result 5/2 for an unbounded suspension and present results for a bounded suspension with $\widetilde{\lambda} = 0, 0.01, 0.1, 0.3, 0.5, 1$.

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*Corresponding author. E-mail: sellier@ladhyx.polytechnique.fr