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Parameter sensitivity of a nuclear containment shell

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ABSTRACT: This paper deals with the design sensitivity of structural systems giving an outline of a sensitivity algorithm for RC structures. As an example a RC nuclear containment vessel is choosen. The linear and nonlinear analyses are performed.

1. INTRODUCTION

The design sensitivity results are useful for optimization problems, systems identification and an estimation of the imperfections influence on the structural behaviour. Numerical aspects of the sensitivity analysis are presented in detail in Refs 1, 2. As a particularly important aspect of the sensitivity analysis the possibility of comparison of the significance of particular parameters for the structural behaviour is emphasized.

2. PROBLEM FORMULATION

Consider a general constraint function defined over the domain of a structure Ω and its boundary $\partial\Omega_\sigma$ with prescribed tractions. The function depends on the stresses $\mathbf{S}$ and nodal displacements $\mathbf{q}$, both implicitly depending on the design variable h and determined at time $t+\Delta t$ (at the end of the step). The function is of the form:

$$^{t+\Delta t}\Phi(\mathbf{S},\mathbf{q};h) \;=\; \int_{\Omega^t} G(^{t+\Delta t}\mathbf{S},{}^{t+\Delta t}\mathbf{q};h)\,d\Omega^t + \int_{\partial\Omega^t_\sigma} g(^{t+\Delta t}\mathbf{q};h)\,d(\partial\Omega^t_\sigma)\,. \qquad (1)$$

with $t+\Delta t$ referring to the end of the typical time step. The incremental equilibrium equation in the updated Lagrangian configuration is of the form:

$$\int_{\Omega^t} \mathbf{B}_L^T \Delta\mathbf{S}\, d\Omega^t = \Delta\mathbf{Q} \qquad (2)$$

where $\mathbf{B}_L$ is a linear operator such that $\mathbf{B}_L\Delta\mathbf{q}$ denotes the corresponding linear strain increment, $\Delta\mathbf{S}$ is the stresses increment and $\Delta\mathbf{Q}$ is the external forces increment.

The objective of the design sensitivity analysis is to calculate the derivative of the constraint function (1) w.r.t. the design variable, called the design derivative, as:

$$\frac{d^{t+\Delta t}\Phi}{dh} \;=\; \int_{\Omega^t}\left[\frac{\partial G}{\partial \mathbf{S}}\frac{d^{t+\Delta t}\mathbf{S}}{dh} + \frac{\partial G}{\partial \mathbf{q}}\frac{d^{t+\Delta t}\mathbf{q}}{dh} + \frac{\partial G}{\partial h}\right] d\Omega^t + \int_{\partial\Omega^t_\sigma}\left[\frac{\partial g}{\partial \mathbf{q}}\frac{d^{t+\Delta t}\mathbf{q}}{dh} + \frac{\partial g}{\partial h}\right] d(\partial\Omega^t_\sigma)\,. \qquad (3)$$

In the numerical example, a particular case of the function Eqn (1) will be taken in the form:

$$\Phi = |q| - q^a < 0 \qquad (4)$$

where q is a choosen displacement and q^a is its allowable value.

3. DIRECT DIFFERENTIATION METHOD.

To obtain the increment of the displacement design derivative the incremental equilibrium equation (5) is differentiated directly as follows:

$$\int_{\Omega^t} \mathbf{B}_L^T \frac{d\Delta \mathbf{S}}{dh}\, d\Omega^t = \frac{\partial \Delta \mathbf{Q}}{\partial h} . \tag{5}$$

To calculate the stress design derivative increment $\Delta\mathbf{S}$ we consider explicitly integrated constitutive equation where the linear strains increment $\Delta\mathbf{e}$ and elastic–plastic matrix $\mathbf{C}^{(e-p)}$ depend on the design variable. The elastic–plastic matrix is also a function of total stresses $\mathbf{S}$ and internal variables vector γ, both beeing determined at time t (at the beginning of the step):

$$\Delta \mathbf{S} = \mathbf{C}^{(e-p)}({}^t\mathbf{S}, {}^t\gamma, h)\Delta \mathbf{e}(h) . \tag{6}$$

By design differentiating of Eqn (6) the following relation for the stress design derivative is obtained:

$$\frac{d\Delta \mathbf{S}}{dh} = \left(\frac{\partial \mathbf{C}^{(e-p)}}{\partial \mathbf{S}} \frac{d^t\mathbf{S}}{dh} + \frac{\partial \mathbf{C}^{(e-p)}}{\partial \gamma} \frac{d^t\gamma}{dh} + \frac{\partial \mathbf{C}^{(e-p)}}{\partial h} \right) \Delta \mathbf{e} + \mathbf{C}^{(e-p)}({}^t\mathbf{S}, {}^t\gamma, h) \frac{d\Delta \mathbf{e}}{dh} . \tag{7}$$

The incremental stresses design derivative is substituted in the differentiated incremental equilibrium equation Eqn (5) resulting in the following sensitivity equation:

$$\left(\int_{\Omega^t} \mathbf{B}_L^T \mathbf{C}^{(e-p)} \mathbf{B}_L \, d\Omega^t \right) \frac{d\Delta \mathbf{q}}{dh} = \frac{\partial \Delta \mathbf{Q}}{\partial h} - \int_{{}^t\Omega} \mathbf{B}_L^T \frac{d\Delta \mathbf{S}}{dh} \bigg|_{\Delta \mathbf{q}(h)=const} d\Omega^t , \tag{8}$$

where the first term in the above equation represents the elastic–plastic tangent stiffness matrix and the last term is the design derivative of the internal incremental force taken at $\Delta\mathbf{q}$ treated formally as independent of h. Having solved Eqn (8) the design derivatives of the displacements, stresses and internal variables increments must be accumulated.

3. ADJOINT VARIABLE METHOD

The alternative way to calculate the sensitivity of the function Eqn (1) is the adjoint variable method (AVM). The objective of the method is to avoid the calculation of the displacement design derivative directly. The method may rationally be applied only to the problems of (linear or nonlinear) elasticity problems.

Additionally to the original structure an adjoint structure is defined. The adjoint structure has the same physical properties but is loaded by the adjoint load defined as the partial derivative of the function (1) w.r.t. the corresponding displacements.

The AVM may be understood as the Lagrange multiplier method. Let us define the augmented Lagrange function in the form:

$$L(\mathbf{q}, \boldsymbol{\lambda}, \mathbf{h}) = \Phi - \boldsymbol{\lambda}^T(\mathbf{K}\mathbf{q} - \mathbf{Q}) , \tag{9}$$

where $\boldsymbol{\lambda}$ is the N–dimensional Lagrange multipliers vector (adjoint variables).

The stationary condition w.r.t. the displacements imposed on the Eqn (9) is of the form:

$$\frac{\partial L}{\partial \mathbf{q}} = \frac{\partial \Phi}{\partial \mathbf{q}} - \boldsymbol{\lambda}^T \mathbf{K} = 0 . \tag{10}$$

The stationary condition makes it possible to calculate $\boldsymbol{\lambda}$. By design differentiating Eqn (9) w.r.t. the vector $\mathbf{h}$ we obtain:

$$\frac{dL}{d\mathbf{h}} = \frac{d\Phi}{d\mathbf{h}} - \boldsymbol{\lambda}^{\mathrm{T}} \frac{d}{d\mathbf{h}} (\mathbf{Kq} - \mathbf{Q}) - \left(\frac{d\boldsymbol{\lambda}}{d\mathbf{h}}\right)^{\mathrm{T}} (\mathbf{Kq} - \mathbf{Q}) \ . \tag{11}$$

Noting that the equilibrium equation is fulfilled for the nominal and the perturbed value of the parameter $\mathbf{h}$ the equation Eqn (11) takes the form:

$$\frac{dL}{d\mathbf{h}} = \frac{d\Phi}{d\mathbf{h}} \ . \tag{12}$$

The derivative of the augmented function Eqn (9) may be calculated and is expressed as follows:

$$\frac{dL}{d\mathbf{h}} = \frac{\partial L}{\partial \mathbf{h}} + \frac{\partial L}{\partial \mathbf{q}} \frac{d\mathbf{q}}{d\mathbf{h}} + \frac{\partial L}{\partial \boldsymbol{\lambda}} \frac{d\boldsymbol{\lambda}}{d\mathbf{h}} \ . \tag{13}$$

Considering the Eqns (13, 12) and (9) the total derivative of the function Eqn (1) takes the form:

$$\frac{d\Phi}{d\mathbf{h}} = \frac{\partial \Phi}{\partial \mathbf{h}} - \boldsymbol{\lambda}^{\mathrm{T}} \frac{d}{d\mathbf{h}} (\mathbf{Kq} - \mathbf{Q}) + \left(\frac{\partial \Phi}{\partial \mathbf{q}} - \boldsymbol{\lambda}^{\mathrm{T}} \mathbf{K}\right) \frac{d\mathbf{q}}{d\mathbf{h}} \ . \tag{14}$$

The last term in the equation given above Eqn (14) disappears because of the stationary condition Eqn (10). It is clearly seen, that it is possible to calculate the design derivative of the functional Eqn (1) without calculation the displacement design derivatives. However, it is necessary to solve the additional linear adjoint equation, Eqn (15):

$$\mathbf{K}\boldsymbol{\lambda} = \left(\frac{\partial \Phi}{\partial \mathbf{q}}\right)^{\mathrm{T}} \ . \tag{15}$$

The r.h.s of the equation Eqn (15) is the explicit partial derivative of the functional Eqn (1). Finally the expression for the design derivative of the function Eqn (1) takes the form:

$$\frac{d\Phi}{dh} = \frac{\partial \Phi}{\partial h} - \boldsymbol{\lambda}^{\mathrm{T}} \left(\frac{\partial \mathbf{K}}{\partial h} \mathbf{q} - \frac{\partial \mathbf{Q}}{\partial h}\right) \ . \tag{16}$$

For the particular case of the function Eqn (1) in the form of Eqn (4) the r.h.s. of the adjoint equation, Eqn (15) is expressed as follows:

$$\frac{\partial \Phi}{\partial \mathbf{q}} = sign(q) \left[0, \ldots, 0, \frac{1}{q^a}, 0, \ldots, 0\right] \ . \tag{17}$$

When applying the adjoint method it is necessary to solve as many adjoint equations as the design constraints. So, it is worthy to apply the AVM if the number of the constraints is lower than the number of the design variables. The direct differentiation method is useful in the opposite case.

4. SENSITIVITY OF REINFORCED CONCRETE STRUCTURES

Since the reinforced concrete structures behave nonlinearly and are path dependent, the direct differentiation method will be employed. An outline of an algorithm to obtain the design sensitivity gradients for reinforced concrete structures in the plane stress state is presented. The Chan–Scordelis constitutive model for concrete and an elastic–plastic with isotropic hardening model for steel are used, Ref 3.

The main assumption of the model is the concept of the uniaxial equivalent strains which allows to describe the behaviour of the material in the principal directions by relations depending on the state of the material. The possible states of the material are compression–compression, compression–tension and tension–tension. We describe here only the compression–compression state. The concrete in compression is described by the Saenz's equation, Ref 4:

$$\sigma_i = \frac{E_o \varepsilon_{iu}}{1 + \left(\frac{E_o}{E_s} - 2\right)\left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}}\right) + \left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}}\right)^2}, \quad E_i = \frac{E_o\left[1 - \left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}}\right)^2\right]}{\left[1 + \left(\frac{E_o}{E_s} - 2\right)\frac{\varepsilon_{iu}}{\varepsilon_{ic}} + \left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}}\right)^2\right]^2} \tag{18}$$

where E_o is the initial Young modulus, E_s is the secant modulus, ε_{iu} are the equivalent uniaxial strains, ε_{ic} are the maximum uniaxial strains, and σ_{ic} are the maximum uniaxial stresses. The relations for σ_{ic} and ε_{ic} for the compression–compression state are defined according to the Kupfer–Gerstle curve, Ref 5:

$$\begin{aligned} \sigma_{2c} &= \frac{1 + 3.65\beta}{(1+\beta)^2} f_c, \quad \varepsilon_{1c} = \varepsilon_c\left[3\frac{\sigma_{2c}}{f_c} - 2\right], \\ \sigma_{1c} &= \beta\sigma_{2c}, \qquad \varepsilon_{2c} = \varepsilon_c\left[-1.6\left(\frac{\sigma_{1c}}{f_c}\right)^3 + 2.25\left(\frac{\sigma_{1c}}{f_c}\right)^2 + 0.35\left(\frac{\sigma_{1c}}{f_c}\right)\right]. \end{aligned} \tag{19}$$

where β is the actual ratio of the principal stresses σ_1/σ_2. The specific form of the r.h.s. of the Eqn (8) for reinforced concrete structure in the plane stress state takes the form

$$\left.\frac{d\Delta \mathbf{F}}{dh}\right|_{\Delta \mathbf{q}(h)=const} = \left(\int_{\Omega^t} \mathbf{B}_L^T \frac{d\mathbf{C}(E_1, E_2, \nu)}{dh} d\Omega^t\right)\Delta \mathbf{q} + \left(\int_{\Omega^t} \mathbf{B}_L^T \frac{d\mathbf{C}^{(e-p)}}{dh} d\Omega^t\right)\Delta \mathbf{q}. \tag{20}$$

where $\mathbf{C}$ is the orthotropic plane stress constitutive matrix depending on the Young moduli in the principal directions and $\mathbf{C}^{(e-p)}$ is the elastic–plastic matrix for the reinforcement in the uniaxial stress state. To calculate the r.h.s. of the above equations it is necessary to know the derivatives of the tangent moduli depending on the state of the material i.e. the total stresses and strains, and on the design variable h. The stress increment may be expressed as follows:

$$\Delta\sigma_i = E_i(\varepsilon_{iu}, \varepsilon_{ic}(\beta, h), \sigma_{ic}(\beta, h))\Delta\varepsilon_{iu}. \tag{21}$$

The design derivative of the stress increment takes the form:

$$\frac{d\Delta\sigma_i}{dh} = E_i \frac{d\Delta\varepsilon_{iu}}{dh} + \frac{dE_{iu}}{dh}\Delta\varepsilon_{iu}, \tag{22}$$

The design derivative of Eqn (22) may be expressed as follows:

$$\begin{aligned} \frac{dE_i}{dh} = {} & \frac{\partial E_i}{\partial \varepsilon_{iu}}\frac{d\varepsilon_{iu}}{dh} + \frac{\partial E_i}{\partial \varepsilon_{ic}}\left(\frac{\partial \varepsilon_{ic}}{\partial \beta}\frac{d\beta}{dh} + \frac{\partial \varepsilon_{ic}}{\partial h}\right) + \frac{\partial E_i}{\partial \sigma_{ic}}\left(\frac{\partial \sigma_{ic}}{\partial \beta}\frac{d\beta}{dh} + \frac{\partial \sigma_{ic}}{\partial h}\right) \\ & + \frac{\partial E_i}{\partial h}. \end{aligned} \tag{23}$$

The partial derivatives $\partial E_i/\partial \varepsilon_{iu}$, $\partial E_i/\partial \varepsilon_{ic}$, $\partial E_i/\partial \sigma_{ic}$ and $\partial E_i/\partial h$ for the compression–compression state may be obtained differentiating the Kupfer–Gerstle curves, Eqns (18, 19) and the total derivatives $d\varepsilon_{iu}/dh$, $d\varepsilon\beta/dh$ have to be accumulated in time.

5. REACTOR CONTAINMENT SHELL

As an example, the sensitivity analysis of a reactor containment shell is choosen, Ref 6. Displacement sensitivity of a horizontal displacement of a node placed at the midspan of the wall of the cylinder is investigated.

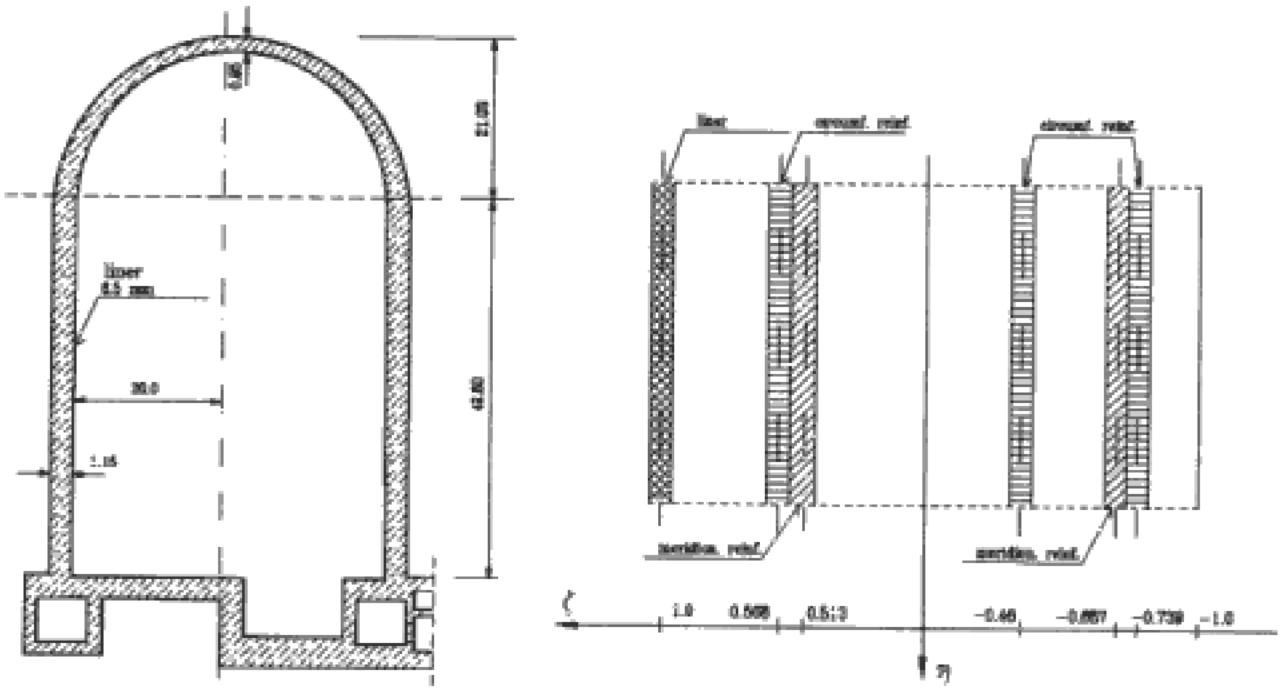

Figure 1. The vertical cross-section of the structure and the typical cross–section of the wall.

The containment shell consists of the 3 main parts: foundation with a place for the reactor, cylinder and the dome (Figure 1, left).

The diameter of the cylinder is 40 m, the height of the structure 64 m, the thickness of the wall 1.15 m. The thickness of the dome varies from 1.15 m at the connection of the dome and the cylinder to 0.95 m at the top of the dome. The structure is initially prestressed with cables. The reinforcement varies along the height of the cylinder and the dome. The material properties are as follows: Young modulus of concrete is 3.0E+7 kN/m^2, Young moduli of the liner and the passive reinforcement are 2.1E+8 kN/m^2. Young modulus of the prestressed reinforcement is 2.025E+8 kN/m2. The computational model of the structure is presented in Fig. 2 (left). It consists of 640 isoparametric layered shell elements (Refs 7,8,9); the number of the d.o.f is 12500. The reinforcement is modelled using the smeared model.

At the prestressing stage the external equivalent pressure is calculated according to the standards and the distribution of the pressure is given in Fig. 2.

During prestressing the structural materials behave elastically. The design derivatives of the investigated displacement w.r.t. the design parameters of the equivalent steel layers are considered. The design parameters are as follows: the equivalent steel layers thicknesses, the Young moduli of steel and the distances of the steel layers from the midsurface of the shell.

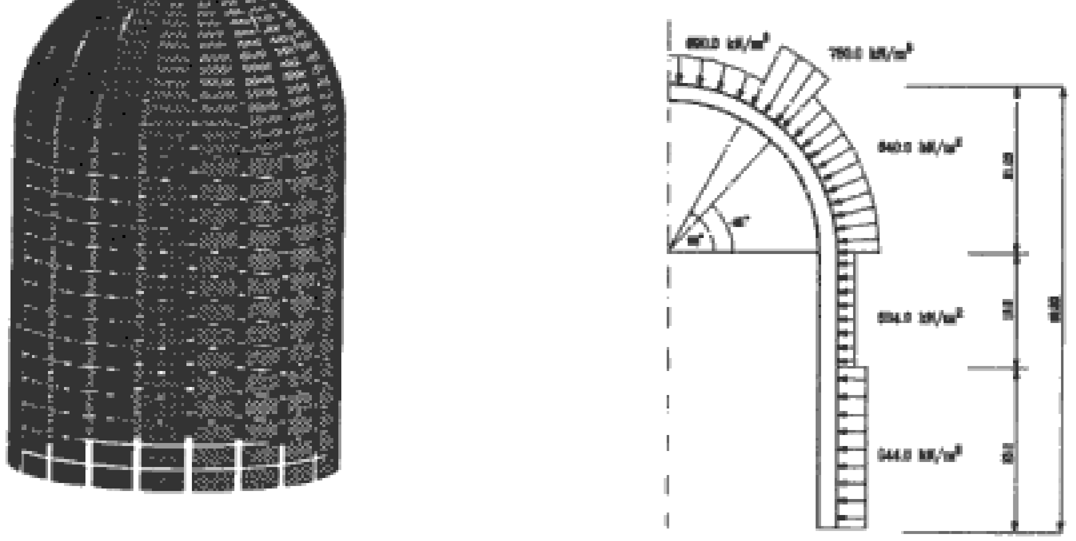

Figure 2. External equivalent pressure.

We compare the importance of the design parameters using the logarithmic design derivatives of the investigated displacement. Because of the fact that the stiffness matrix depends linearly on the Young modulus and the thickness, the respective logarithmic derivatives are equal. Thus, it is sufficiently to compare the logarithmic derivatives with respect to either Young modulus or thickness ("E/th") and the distance of the steel layer from the midsurface of the shell as is shown in Fig.3.

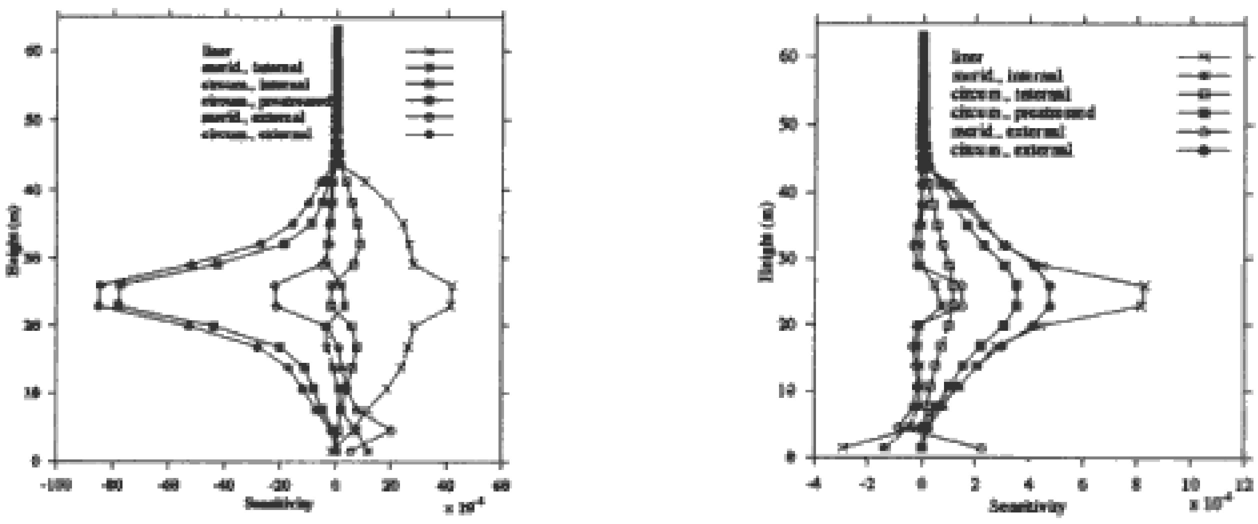

Figure 3. Logarithmic design derivatives distribution, design parameters: a) Young modulus or thickness of the layer b) the layer distance from the midurface of the shell.

Concerning the investigated displacement design derivatives w.r.t. the distance of a steel layer from the midsurface of the shell the highest sensitivity gradients are in the liner. The absolute values of the design derivatives w.r.t. the layer distance from the midsurface of the shell are lower than the design derivatives of the investigated displacement with respect to "E/th" (particularly in the prestressed external layer where the gradients are the highest). The highest values of the gradients are in the circumferential external layer close to the midspan of the cylinder.

The logarithmic design derivatives distribution plots allow to establish the most important place in the structure for the investigated design condition. In the case of a more complicated shape this may be a hint where a special care has to be taken on the agreement of the with the project.

Figure 4 illustrates the application of the direct differentiation method. Applying his method the displacement and stress sensitivity fields in the whole structure have een obtained.

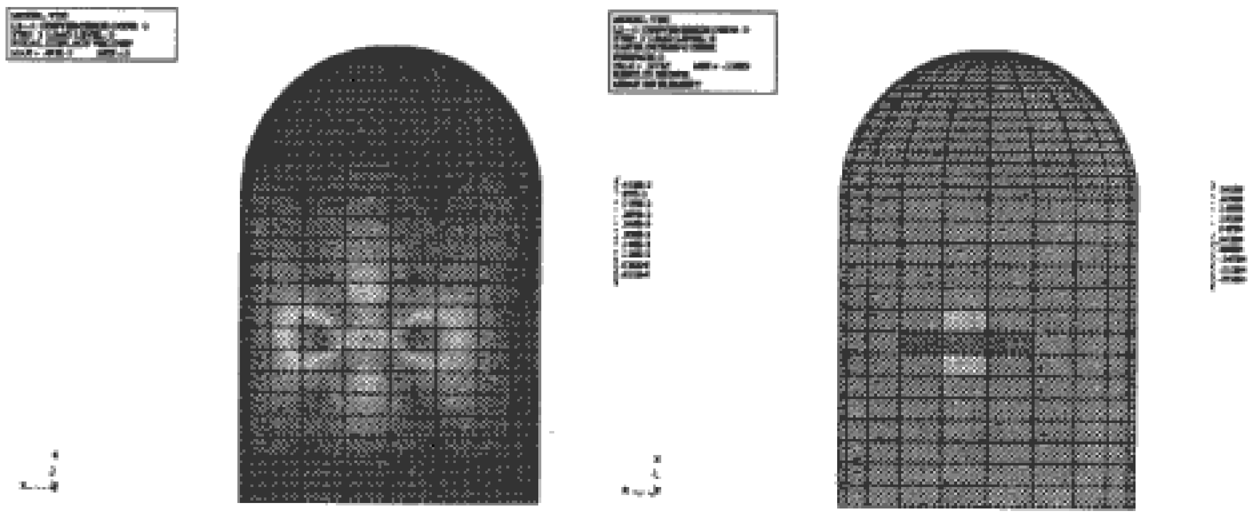

Figure 4. Displacement and hoop stress derivative (3rd concrete layer) fields due to variation of the distance of the third steel layer from the shell midsurface.

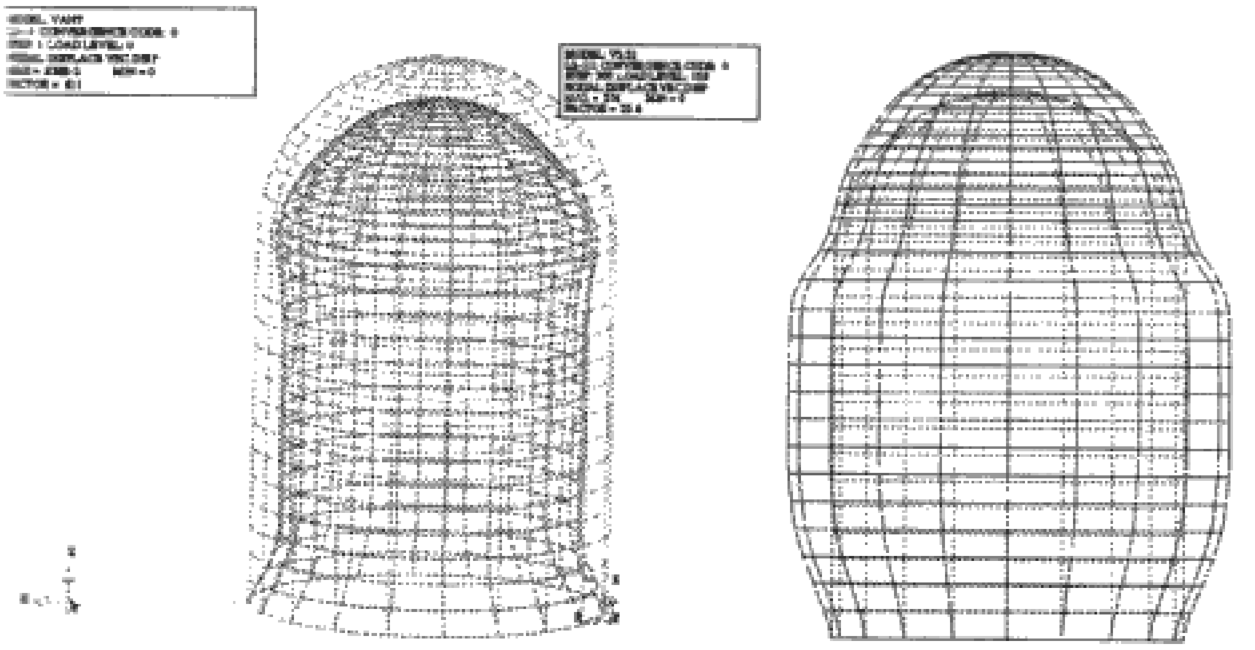

Figure 5. Shapes of the structure: prestressing phase, and due to failure pressure.

The nonlinear analysis was peformed following the monotonic increase of the in-ernal pressure up to the failure of the structure. The initial and final shapes of the ructure are shown in Fig. 5. The equilibrium path of the investigated displacements he vertical displacement of the top of the dome and the horizontal displacement ose to the midspan of the cylinder) is followed up to 20 cm of the horizontal displa-ments and is presented in Fig. 6 (left). The sensitivity variation of the investigated splacement is given in Figure 6 (right). The displacement design derivatives w.r.t. e thickness of the circumferential external equivalent steel layer in the element 141 re investigated.

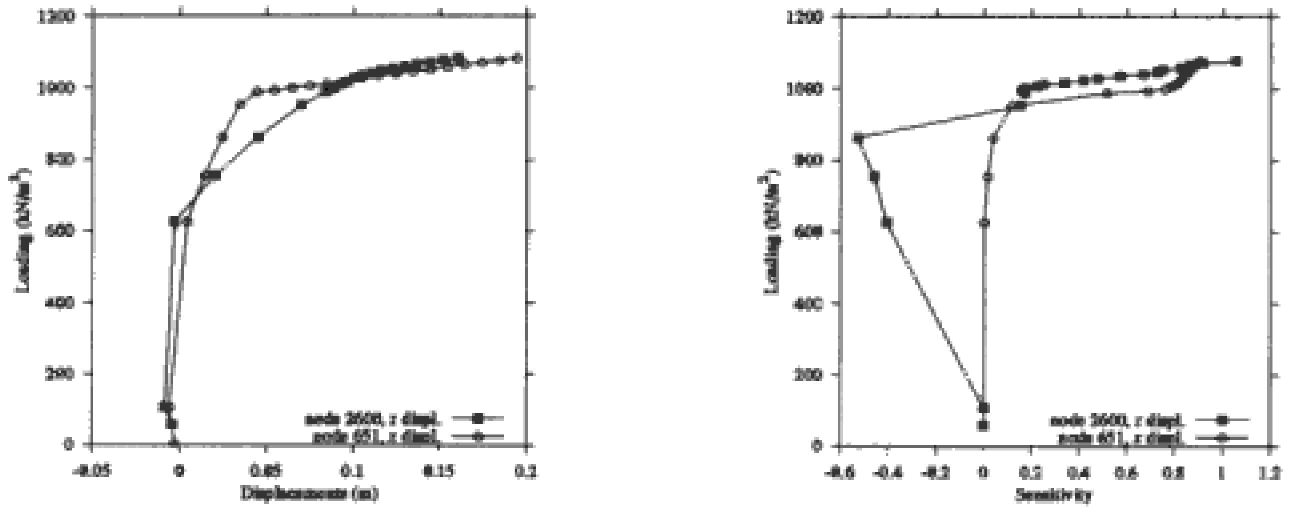

Figure 6. Equilibrium path (left) and the sensivity variation (right) of the investigated displacements.

5. FINAL REMARKS

The computer program developed is a significant extension of the program NASHL Ref (4) which allows to analyse large shell structural systems. The program was tested employing up to 30000 d.o.f.'s. Because of the memory dynamic allocation the only limitation in use the program is the massive memory capacity. The computations were performed on Sparc 2000 (IFTR PAS), CRAY YMP4E (Interdisciplinary Center of Mathematical Modelling) and CS6400 (Warsaw University of Technology).

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