Rapid Communications

Entanglement negativity as a universal non-Markovianity witness

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In order to engineer an open quantum system and its evolution, it is essential to identify and control the memory effects. These are formally attributed to the non-Markovianity of dynamics that manifests itself by the evolution being indivisible in time, a property which can be witnessed by a nonmonotonic behavior of contractive functions or correlation measures. We show that by monitoring directly the entanglement behavior of a system in a tripartite setting it is possible to witness all invertible non-Markovian dynamics, as well as all (also noninvertible) qubit evolutions. This is achieved by using negativity, a computable measure of entanglement, which in the usual bipartite setting is not a universal non-Markovianity witness. We emphasize further the importance of multipartite states by showing that non-Markovianity cannot be faithfully witnessed by any contractive function of single qubits. We support our statements by an explicit example of eternally non-Markovian qubit dynamics, for which negativity can witness non-Markovianity at arbitrary timescales.

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Introduction. Describing effective dynamics of any realistic quantum system that interacts with its environment inevitably requires the theory of open quantum systems [1,2]. In recent years, a growing interest has been devoted to the determination of dynamical properties that can be pinpointed when studying solely the system evolution; in particular, distinguishing memoryless—*Markovian*—dynamics from ones that exhibit memory effects. Various ways have been proposed on how to define the concept of memory or, more precisely, non-Markovianity at the level of quantum evolutions (see [3–6] for detailed reviews on the topic). Although recently questioned [7,8], the most commonly adopted definition [9–11] is the natural generalization of the Chapman-Kolgomorov equation, which assures the time divisibility of stochastic maps in the case of classical Markovian processes [12]. In particular, focusing on the family of quantum operations, i.e., completely positive (CP) trace-preserving (TP) maps Λ_t that represent the system evolution from the initial time $t = 0$ to each $t > 0$, one may verify their *CP divisibility* [13] by inspecting whether at any intermediate time $0 \leqslant s \leqslant t$ each of them could be decomposed (concatenated) as

$$\Lambda_t = V_{t,s} \circ \Lambda_s \tag{1}$$

with a valid dynamical (CPTP) map $V_{t,s}$.

Nevertheless, the above criterion is often weakened in order to construct witnesses of non-Markovianity that despite not always being able to certify the non-CP character of $V_{t,s}$ can have an operational motivation. The most commonly used notion is the temporal behavior of distinguishability, as measured by the trace distance $||\rho - \sigma||_1/2$ with the trace norm $||M||_1 = \mathrm{Tr}\sqrt{M^\dagger M}$, between a pair of evolving quantum states ρ and σ [14]. As the trace distance is nonincreasing for a larger class of *P-divisible* evolutions [3], i.e., ones decomposable as in Eq. (1) but with $V_{t,s}$ guaranteed to only be positive (P), it monotonically decreases for some dynamics that according to the criterion (1) are non-Markovian [10]. Still, its increase at a given time instance is interpreted as a manifestation of information backflow from the environment to the system [15,16].

However, when dealing with invertible [17] or image non-increasing [18] dynamical maps Λ_t, which describe almost all quantum evolutions, the CP-divisibility criterion can be restated in terms of the information backflow. By allowing for an ancilla of system dimension d, the condition (1) becomes equivalent to the statement [15]

$$\frac{d}{dt}\|\Lambda_t \otimes \mathbb{1}_d[p_1\,\rho_1 - p_2\,\rho_2]\|_1 \leqslant 0, \tag{2}$$

which must now be valid for all $t \geqslant 0$, all bipartite system-ancilla initial states ρ_1, ρ_2, and all probabilities $p_1 + p_2 = 1$ [19]. In this Rapid Communication, we will consider evolutions for which this equivalence holds, which in fact includes also all qubit dynamics [20]. That is why, from now on we will refer to non-Markovianity as defined by the violation of CP divisibility.

Still, it has remained unknown whether such notion of non-Markovianity can be faithfully verified by considering solely the evolution of correlations; in particular, dynamics of the entanglement between the system and some ancillae [9]. This would allow one to certify non-Markovianity by preparing the system and ancillae in an initial correlated state, in order to observe an increase of some entanglement measure [21,22] at a later time $t^* > 0$, without the need to consider ensembles of initial states and distinguishability tasks [16]. Previous results suggest that traditional correlation quantifiers, such as entanglement measures [23,24] and mutual information [25] fail to witness all non-Markovian evolutions, while a recently

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proposed correlation measure [23] can witness "almost all" of them.

In this work, we show that negativity, a well-known computable quantifier of bipartite entanglement [26,27], can witness all non-Markovian qubit dynamics Λ_t and all invertible evolutions of arbitrary dimension. After discussing the limitations in witnessing non-Markovianity in single-qubit systems, we present the general construction for negativity as a universal non-Markovianity witness. We provide an explicit example, witnessing violations of CP divisibility for eternally non-Markovian qubit evolutions [28] at arbitrary timescales.

Witnessing non-Markovianity with contractive functions. A general witness of non-Markovianity can be built from any contractive function $f(\rho,\sigma)$ of two quantum states ρ and σ, where contractivity means that

$$f(\Lambda[\rho],\Lambda[\sigma]) \leqslant f(\rho,\sigma) \tag{3}$$

for any quantum operation Λ. Important examples for contractive functions are the trace distance $||\rho-\sigma||_1/2$, infidelity $1-F(\rho,\sigma)$ with fidelity $F(\rho,\sigma)=||\sqrt{\rho}\sqrt{\sigma}||_1$, and the quantum relative entropy $S(\rho||\sigma)=\mathrm{Tr}[\rho\log_2\rho]-\mathrm{Tr}[\rho\log_2\sigma]$. Recently, a family of contractive functions, named quantum relative Rényi entropy, has been introduced as [29,30]

$$D^{\mathrm{q}}_\alpha(\rho||\sigma)=\frac{1}{\alpha-1}\log_2\mathrm{Tr}[(\sigma^{(1-\alpha)/2\alpha}\rho\sigma^{(1-\alpha)/2\alpha})^\alpha], \tag{4}$$

with $\alpha\geqslant 1/2$. In the limit $\alpha\to 1$ the function $D^{\mathrm{q}}_\alpha(\rho||\sigma)$ coincides with the relative entropy $S(\rho||\sigma)$, and for $\alpha=1/2$ we obtain $D^{\mathrm{q}}_{1/2}(\rho||\sigma)=-2\log_2 F(\rho,\sigma)$.

Noting that any contractive function is monotonically decreasing with t for any Markovian evolution, an increase of f for some $t>0$ serves as a witness of non-Markovianity. It is now reasonable to ask whether any non-Markovian evolution can be witnessed by some suitably chosen contractive function. As we show in Theorem 1 below, the answer to this question is negative for single-qubit systems. An important type of evolution in this context is given by Eq. (1), where $V_{t,s}$ admits the decomposition

$$V_{t,s}[\rho]=p\mathcal{E}_1[\rho]+(1-p)\mathcal{E}_2[\rho^T] \tag{5}$$

with probabilities p and CPTP maps $\mathcal{E}_1$ and $\mathcal{E}_2$ which can further depend on t and s with $s\leqslant t$. As maps $V_{t,s}$ that admit Eq. (5) are guaranteed to be P but not necessarily CP (see also the Supplemental Material [31]), they must lead to P-divisible evolutions. These, however, may still be non-Markovian according to the CP-divisibility criterion (1). An example of dynamics that is *not* CP divisible but admits the form (5) is presented below in Eq. (22). We are now ready to present the first main result of this work.

Theorem 1. For any non-Markovian evolution $\Lambda_t=V_{t,s}\circ\Lambda_s$ with $V_{t,s}$ fulfilling Eq. (5) it holds that

$$\frac{d}{dt}f(\Lambda_t[\rho],\Lambda_t[\sigma])\leqslant 0 \tag{6}$$

for any contractive function $f(\rho,\sigma)$ and any single-qubit states ρ and σ.

Proof. First, we will show that for any two single-qubit states ρ and σ there exists a CPTP map $\Phi_{t,s}$ (that may in general depend on both ρ and σ) such that

$$V_{t,s}[\rho]=\Phi_{t,s}[\rho],\qquad V_{t,s}[\sigma]=\Phi_{t,s}[\sigma]. \tag{7}$$

This statement can be proven by considering the Bloch vectors $\boldsymbol{r}$ and $\boldsymbol{s}$ of the states ρ and σ. The Bloch vector $\tilde{\boldsymbol{r}}$ of the transposed state ρ^T is related to $\boldsymbol{r}=(r_x,r_y,r_z)$ via a reflection on the x-z plane, i.e., $\tilde{\boldsymbol{r}}=(r_x,-r_y,r_z)$, and similar for σ. In particular, this means that transposition preserves the lengths of the two Bloch vectors and the angle between them. This implies that for any two states ρ and σ there exists a unitary rotation U such that

$$\rho^T=U\rho U^\dagger,\quad \sigma^T=U\sigma U^\dagger. \tag{8}$$

The CPTP map $\Phi_{t,s}$ fulfilling Eqs. (7) is thus given as

$$\Phi_{t,s}[\rho]=p\mathcal{E}_1[\rho]+(1-p)\mathcal{E}_2[U\rho U^\dagger], \tag{9}$$

where the unitary U is chosen such that Eqs. (8) hold. Note that, in general, the unitary U depends on the two states ρ and σ.

Combining the above arguments, we obtain the following for any contractive function f and any two single-qubit states ρ and σ:

$$\begin{aligned} f(\Lambda_t[\rho],\Lambda_t[\sigma]) &= f(V_{t,s}\circ\Lambda_s[\rho],V_{t,s}\circ\Lambda_s[\sigma]) \\ &= f(\Phi_{t,s}\circ\Lambda_s[\rho],\Phi_{t,s}\circ\Lambda_s[\sigma]) \\ &\leqslant f(\Lambda_s[\rho],\Lambda_s[\sigma]), \end{aligned} \tag{10}$$

which proves that any contractive function is monotonically decreasing with t. ■

While Theorem 1 applies only to single-qubit systems, this constraint can be lifted if one considers only specific functions, namely, the trace distance, the relative entropy, and the quantum relative Rényi entropy $D^{\mathrm{q}}_\alpha(\rho||\sigma)$ for $\alpha>1$. Noting that these functions are contractive under positive trace-preserving maps [32], it follows that they are monotonic under non-Markovian evolutions which are P divisible. We refer to the Supplemental Material for more details.

A question which is left open in Theorem 1 is whether it is still possible to detect non-Markovianity via the behavior of a contractive function f. Even if f is monotonically decreasing with t, its overall behavior might depend on whether the evolution is Markovian or not. We answer this question in the Supplemental Material, showing that the monotonic behavior of any contractive function can be reproduced by Markovian dynamics.

Witnessing non-Markovianity with entanglement. The results of the previous section tell us that to witness all non-Markovian evolutions, our input state must be of higher dimension, possibly a compound state of the system extended by ancillae, i.e., we need to consider the evolution $\Lambda_t^A\otimes\mathbb{1}^B$ acting on a bipartite state $\rho=\rho^{AB}$. The behavior of any entanglement measure $E^{A|B}$ of the final state

$$\sigma_t=\Lambda_t^A\otimes\mathbb{1}^B[\rho] \tag{11}$$

then serves as a witness of non-Markovianity, as for any Markovian evolution the entanglement must monotonically decrease [9]. However, this approach is not suitable to create a universal witness of non-Markovianity, as for any evolution Λ_t which consists of an entanglement breaking map at some finite time t' followed by an arbitrary non-Markovian evolution, the state σ_t will have zero entanglement for all $t \geqslant t'$ [23].

Even if the evolution is not entanglement breaking, we can show that certain entanglement quantifiers fail to detect non-Markovianity. In the following, we quantify the amount of entanglement via negativity [26,27]

$$E^{A|B}(\rho) = \frac{||\rho^{T_B}||_1 - 1}{2}, \tag{12}$$

where T_B denotes the partial transpose with respect to the subsystem B. As is shown in the Supplemental Material, negativity is monotonic under local positive maps of the form (5), i.e.,

$$P^A \otimes \mathbb{1}^B[\rho] = p\mathcal{E}_1^A \otimes \mathbb{1}^B[\rho] + (1-p)\mathcal{E}_2^A \otimes \mathbb{1}^B[\rho^{T_A}], \tag{13}$$

for any bipartite state $\rho = \rho^{AB}$ and probability p [33]. This implies that negativity is monotonically decreasing for any local evolution $\Lambda_t^A = V_{t,s}^A \circ \Lambda_s^A$ with $V_{t,s}$ being of the form (5). An example for a non-Markovian evolution admitting this form will be given in Eq. (22). As we further show in the Supplemental Material, negativity cannot be used to witness non-Markovianity if $E^{A|B}(\Lambda_t^A \otimes \mathbb{1}^B[\rho])$ is monotonically decreasing with t, as a decreasing behavior can always be reproduced by Markovian dynamics. From this, we conclude that negativity $E^{A|B}$ fails to witness some non-Markovian evolutions on subsystem A even if they are not entanglement breaking [34].

In light of these results, it is tempting to conclude that negativity is not suitable for construction of a universal non-Markovianity witness. Quite surprisingly, the situation changes completely by adding an extra particle C, and considering the negativity $E^{AB|C}$ of the state

$$\tau_t^{ABC} = \Lambda_t^A \otimes \mathbb{1}^{BC}[\rho^{ABC}], \tag{14}$$

where ρ^{ABC} is a suitably chosen initial state. In fact, taking additional ancilla systems into account has proven to be useful for relating different notions of non-Markovianity [see Eq. (2)]. The following theorem shows that in a tripartite setting negativity is a universal non-Markovianity witness for all invertible evolutions and for all dynamics of a single qubit.

Theorem 2. For any invertible non-Markovian evolution Λ_t there exists a quantum state ρ^{ABC} such that

$$\frac{d}{dt}E^{AB|C}\big(\Lambda_t^A \otimes \mathbb{1}^{BC}[\rho^{ABC}]\big) > 0 \tag{15}$$

for some $t > 0$. For single-qubit evolutions Λ_t the statement also holds for non-invertible dynamics.

Proof. We introduce the following state:

$$\rho^{ABC} = p_1\rho_1^{AB_1} \otimes |\Psi^+\rangle\langle\Psi^+|^{B_2C} + p_2\rho_2^{AB_1} \otimes |\Psi^-\rangle\langle\Psi^-|^{B_2C}, \tag{16}$$

where B_1 and B_2 are subsystems of $B = B_1B_2$, $|\Psi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$ are maximally entangled states, and the states ρ_i and probabilities p_i will be specified in more detail below. If now an evolution Λ_t^A acts on the state ρ^{ABC}, the time-evolved state takes the form

$$\tau_t^{ABC} = p_1\Lambda_t^A\big[\rho_1^{AB_1}\big] \otimes |\Psi^+\rangle\langle\Psi^+|^{B_2C} + p_2\Lambda_t^A\big[\rho_2^{AB_1}\big] \otimes |\Psi^-\rangle\langle\Psi^-|^{B_2C}. \tag{17}$$

To evaluate the negativity in the $AB|C$ cut we notice that the partial transposition with respect to C is given by

$$\begin{aligned}\tau_t^{T_C} &= \tfrac{1}{2}\Lambda_t^A\big[p_1\rho_1^{AB_1} + p_2\rho_2^{AB_1}\big] \otimes (|01\rangle\langle 01|^{B_2C} + |10\rangle\langle 10|^{B_2C}) \\ &\quad + \tfrac{1}{2}\Lambda_t^A\big[p_1\rho_1^{AB_1} - p_2\rho_2^{AB_1}\big] \\ &\quad \otimes(|\Phi^+\rangle\langle\Phi^+|^{B_2C} - |\Phi^-\rangle\langle\Phi^-|^{B_2C})\end{aligned} \tag{18}$$

with $|\Phi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$. Since the states $|\Phi^\pm\rangle$ are orthogonal to $|01\rangle$ and $|10\rangle$, the trace norm of $\tau_t^{T_C}$ can be evaluated as

$$\big\|\tau_t^{T_C}\big\|_1 = 1 + \big\|\Lambda_t^A\big[p_1\rho_1^{AB_1} - p_2\rho_2^{AB_1}\big]\big\|_1, \tag{19}$$

where we used the fact that $\mu := p_1\Lambda_t^A[\rho_1^{AB_1}] + p_2\Lambda_t^A[\rho_2^{AB_1}]$ is a valid quantum state, and thus $||\mu||_1 = 1$. The negativity of τ_t^{ABC} is thus given as

$$E^{AB|C}\big(\tau_t^{ABC}\big) = \frac{1}{2}\big\|\Lambda_t^A\big[p_1\rho_1^{AB_1} - p_2\rho_2^{AB_1}\big]\big\|_1. \tag{20}$$

To complete the proof of the theorem, recall that for any invertible evolution there exist states $\rho_i^{AB_1}$ and probabilities p_i such that Eq. (2) is violated if the evolution is non-Markovian [15,17]. The same is true for all (also noninvertible) single-qubit dynamics [20]. ■

Few remarks regarding Theorem 2 are in place. First, we note that invertible dynamics constitute the generic case of quantum evolutions, as noninvertible evolutions have zero measure in the space of all quantum evolutions [23,35]. Moreover, the statement of Theorem 2 can be lifted to include also dynamics which are image nonincreasing, by applying the same arguments [18]. We further notice that negativity is a faithful entanglement quantifier in the setting considered here, and the states in Eq. (16) are never bound entangled (see Supplemental Material for more details).

Applications. We apply the results presented above to qubit eternally non-Markovian (ENM) dynamics [28], an evolution exhibiting non-Markovianity at any $t > 0$, even at arbitrarily small and large timescales. Such a model falls into well-studied categories of random-unitary [36] and phase-covariant [37] qubit commutative evolutions. Yet, it constitutes an important example with its non-Markovian features being hard to witness [38,39]. In general, a random-unitary qubit dynamics is described by a time-dependent master equation:

$$\frac{d\rho(t)}{dt} = \sum_{i=1}^{3}\gamma_i(t)\{\sigma_i\rho(t)\sigma_i - \rho(t)\}, \tag{21}$$

which upon integration yields a dynamical map corresponding to a qubit Pauli channel, i.e.,

$$\Lambda_t[\rho] = \sum_{\mu=0}^{3} p_\mu(t)\sigma_\mu\rho\sigma_\mu, \tag{22}$$

where the mixing probabilities $p_\mu(t)$, and their time dependence, can be explicitly expressed as a function of $\gamma_i(t)$ [36]. For any such evolution the CP-divisibility condition (1) is equivalent to the statement that for all $t > 0$ all the decay rates are non-negative, $\gamma_i(t) \geqslant 0$, while the P-divisibility criterion corresponds to a weaker requirement that at all times $t > 0$ each pair $(i \neq j)$ of decay parameters satisfies $\gamma_i(t) + \gamma_j(t) \geqslant 0$ [36].

The ENM model introduced in Ref. [28] corresponds then to the choice

$$\gamma_1 = \gamma_2 = \alpha\frac{c}{2}, \quad \gamma_3(t) = -\alpha\frac{c}{2}\tanh(ct) \tag{23}$$

with $\alpha \geqslant 1$ and $c > 0$. Crucially, ENM dynamics exhibits non-Markovianity at all times, as $\gamma_3(t) < 0$ for all $t > 0$. In contrast, it is always P divisible due to $\gamma_\ell + \gamma_3(t) = \alpha\frac{c}{2}[1 - \tanh(ct)] \geqslant 0$ for $\ell \in \{1, 2\}$ and any $t \geqslant 0$ [40,41]. Still, the resulting CP map (22) is invertible, i.e., for every $t \geqslant 0$ one can find a linear map Λ_t^{-1} such that $\Lambda_t^{-1} \circ \Lambda_t = \mathbb{1}$. As a result, one can unambiguously define $V_{t,s} = \Lambda_t \circ \Lambda_s^{-1}$ in (1) and explicitly compute its Choi-Jamiołkowski (CJ) matrix, $\Omega_{V_{t,s}} := 2\, V_{t,s} \otimes \mathbb{1}[|\Phi^+\rangle\langle\Phi^+|]$, associated with it:

$$\Omega_{V_{t,s}} = \frac{1}{2}\begin{pmatrix} 1+\lambda_{t-s}^{2\alpha} & 0 & 0 & 2\Gamma_{t,s}^{\alpha} \\ 0 & 1-\lambda_{t-s}^{2\alpha} & 0 & 0 \\ 0 & 0 & 1-\lambda_{t-s}^{2\alpha} & 0 \\ 2\Gamma_{t,s}^{\alpha} & 0 & 0 & 1+\lambda_{t-s}^{2\alpha} \end{pmatrix}, \tag{24}$$

where $\lambda_\tau = e^{-c\tau}$ and $\Gamma_{t,s} = \lambda_{t-s}\cosh(ct)\mathrm{sech}(cs)$. It may be explicitly verified that $\Omega_{V_{t,s}}$ is nonpositive for any $0 < s < t$, confirming the "eternal non-Markovianity" of dynamics, unless $s = 0$ for which $\Omega_{V_{t,0}} = \Omega_{\Lambda_t} \geqslant 0$ assures the physicality of the overall evolution.

In the Supplemental Material, we explicitly show that the CJ matrix (24) admits a convex decomposition:

$$\Omega_{V_{t,s}} = p_1 P_{\Phi_+} + p_2 P_{\Phi_-} + (1 - p_1 - p_2) P_{\Psi_+}^{T_B} \tag{25}$$

with probabilities $p_1 = \frac{1}{2}(\lambda_{t-s}^{2\alpha} + \Gamma_{t,s}^{\alpha})$ and $p_2 = \frac{1}{2}(1 - \Gamma_{t,s}^{\alpha})$, and $P_\psi = 2\,|\psi\rangle\langle\psi|$. Hence, it follows (see Supplemental Material for a general discussion) that the decomposition (25) of the CJ matrix assures the map $V_{t,s}$ for the ENM dynamics to admit a decomposition (5). As a direct consequence, Theorem 1 applies to the ENM dynamics, implying that no contractive function $f(\rho, \sigma)$ evaluated on single-qubit states ρ and σ will be able to witness non-Markovianity of the ENM model. Moreover, as Eq. (5) naturally generalizes to Eq. (13), it becomes evident that negativity cannot be used in the usual bipartite setting $E^{A|B}(\Lambda_t^A \otimes \mathbb{1}^B[\rho])$ to witness the non-Markovianity of the ENM evolution.

However, we explicitly demonstrate that, in accordance with the Theorem 2, negativity in the tripartite setting, $E^{AB|C}$, can be used to faithfully witness the non-Markovianity of the ENM evolution for any $t^* > 0$. In order to choose the initial state ρ^{ABC} in Eq. (16)—in particular, its constituents $p_\ell \rho_\ell^{AB_1}$ ($\ell = 1, 2$) such that $E^{AB|C}$ increases at a given $t^* > 0$—we follow the constructive method of Bylicka *et al.* [17]. We choose $\rho_\ell^{AB_1} \in \mathcal{B}(\mathbb{C}_2 \otimes \mathbb{C}_3)$ and mixing probabilities p_ℓ such that the trace norm in Eq. (20) is assured to increase at time t^* [17]. The construction with the analytic proof can be found in the Supplemental Material. Yet, in Fig. 1, we plot the dynamical behavior of $E^{AB|C}$ for the ENM model (23) with $\alpha = 2$ and $c = \frac{1}{2}$ after setting ρ^{ABC}, so that the non-Markovianity of dynamics can be clearly witnessed at time $t^* = 1$ (and $t^* = 0.01$ within the inset).

Conclusions. In this Rapid Communication we discuss possibilities and limitations to detect non-Markovianity in qubit systems and beyond. It is shown that a very general class of quantities based on contractive functions fails to detect non-Markovianity of all qubit evolutions. This includes widely studied quantifiers such as trace distance, fidelity, and quantum relative entropy. It is shown that all of them fail to witness non-Markovianity in a certain class of evolutions, which includes eternal non-Markovian dynamics exhibiting non-Markovianity at all times $t > 0$.

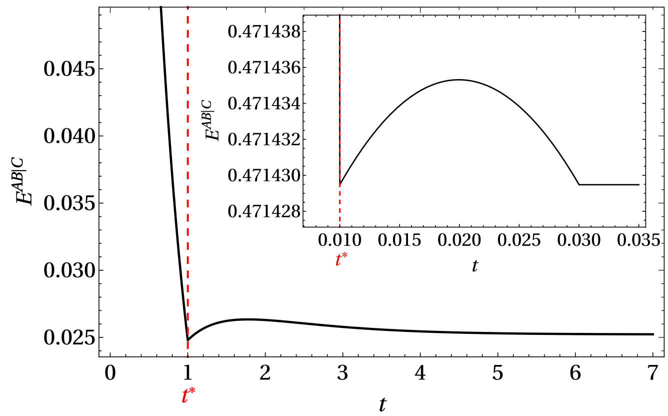

FIG. 1. Negativity $E^{AB|C}$ as a function of time t (in arbitrary units) for the eternally non-Markovian qubit dynamics (23) with $\alpha = 2$ and $c = 1/2$. The initial state ρ^{ABC} has been set as in Eq. (16) with probabilities p_i and states $\rho_i^{AB_1}$ chosen according to the constructive method of Bylicka *et al.* [17], leading to violation of Eq. (2) for a specific time $t^* > 0$. The plot shows detection of non-Markovianity at $t^* = 1$ ($t^* = 0.01$ in the inset), which is marked on the axis and with a dashed red vertical line.

If entangled systems are employed to witness non-Markovianity, we show that the situation strongly depends on the number of particles used. Surprisingly, for three particles A, B, and C it is possible to witness non-Markovianity of all invertible dynamics of system A by considering entanglement in the cut $AB|C$. We show this explicitly for entanglement negativity, a computable measure of entanglement, which is nonmonotonic for any non-Markovian invertible dynamics and a suitably chosen initial state. For single-qubit evolutions our results apply also when the dynamics is not invertible. As an example, we show results for the eternal non-Markovianity model, where the nonmonotonic behavior of negativity can be observed at arbitrary small times.

Our results demonstrate that well-established entanglement quantifiers can be useful as faithful non-Markovianity witnesses for very general classes of evolutions. An important question left open in this work is whether entanglement measures can universally witness non-Markovianity of all evolutions, incuding noninvertible dynamics beyond qubits. Recalling that entanglement theory is a prominent example of more general quantum resource theories, the fundamental connection between entanglement and non-Makovianity presented in our work can also be useful for the development of a resource theory of non-Markovianity [42,43].

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[1] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002).
[2] D. A. Lidar, arXiv:1902.00967.
[3] Á. Rivas, S. F. Huelga, and M. B. Plenio, Rep. Prog. Phys. **77**, 094001 (2014).
[4] H.-P. Breuer, E.-M. Laine, J. Piilo, and B. Vacchini, Rev. Mod. Phys. **88**, 021002 (2016).
[5] I. de Vega and D. Alonso, Rev. Mod. Phys. **89**, 015001 (2017).
[6] L. Li, M. J. Hall, and H. M. Wiseman, Phys. Rep. **759**, 1 (2018).
[7] F. A. Pollock, C. Rodríguez-Rosario, T. Frauenheim, M. Paternostro, and K. Modi, Phys. Rev. Lett. **120**, 040405 (2018).
[8] S. Milz, M. S. Kim, F. A. Pollock, and K. Modi, Phys. Rev. Lett. **123**, 040401 (2019).
[9] Á. Rivas, S. F. Huelga, and M. B. Plenio, Phys. Rev. Lett. **105**, 050403 (2010).
[10] D. Chruściński and S. Maniscalco, Phys. Rev. Lett. **112**, 120404 (2014).
[11] J. Bae and D. Chruściński, Phys. Rev. Lett. **117**, 050403 (2016).
[12] B. Vacchini, A. Smirne, E.-M. Laine, J. Piilo, and H.-P. Breuer, New J. Phys. **13**, 093004 (2011).
[13] M. M. Wolf and J. I. Cirac, Commun. Math. Phys. **279**, 147 (2008).
[14] H.-P. Breuer, E.-M. Laine, and J. Piilo, Phys. Rev. Lett. **103**, 210401 (2009).
[15] D. Chruściński, A. Kossakowski, and Á. Rivas, Phys. Rev. A **83**, 052128 (2011).
[16] F. Buscemi and N. Datta, Phys. Rev. A **93**, 012101 (2016).
[17] B. Bylicka, M. Johansson, and A. Acín, Phys. Rev. Lett. **118**, 120501 (2017).
[18] D. Chruściński, Á. Rivas, and E. Størmer, Phys. Rev. Lett. **121**, 080407 (2018).
[19] In general, it is enough to consider only the right derivative in Eq. (2).
[20] S. Chakraborty and D. Chruściński, Phys. Rev. A **99**, 042105 (2019).
[21] V. Vedral, M. B. Plenio, M. A. Rippin, and P. L. Knight, Phys. Rev. Lett. **78**, 2275 (1997).
[22] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).
[23] D. De Santis, M. Johansson, B. Bylicka, N. K. Bernardes, and A. Acín, Phys. Rev. A **99**, 012303 (2019).
[24] A. C. Neto, G. Karpat, and F. F. Fanchini, Phys. Rev. A **94**, 032105 (2016).
[25] S. Luo, S. Fu, and H. Song, Phys. Rev. A **86**, 044101 (2012).
[26] K. Życzkowski, P. Horodecki, A. Sanpera, and M. Lewenstein, Phys. Rev. A **58**, 883 (1998).
[27] G. Vidal and R. F. Werner, Phys. Rev. A **65**, 032314 (2002).
[28] M. J. W. Hall, J. D. Cresser, L. Li, and E. Andersson, Phys. Rev. A **89**, 042120 (2014).
[29] M. Müller-Lennert, F. Dupuis, O. Szehr, S. Fehr, and M. Tomamichel, J. Math. Phys. **54**, 122203 (2013).
[30] M. M. Wilde, A. Winter, and D. Yang, Commun. Math. Phys. **331**, 593 (2014).
[31] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevA.101.020303, containing Refs. [44,45,46], for more details on maps which are P but not CP. The Supplemental Material also contains Ref. [47] for a criterion of entanglement distillability.
[32] A. Müller-Hermes and D. Reeb, Ann. Henri Poincare **18**, 1777 (2017).
[33] Note that $P^A \otimes \mathbb{1}^B[\rho]$ might not be positive; we thus extend the definition of negativity in Eq. (12) to nonpositive Hermitian operators with unit trace.
[34] Note that negativity in general fails to detect non-Markovianity for evolutions Λ_t which consists of a non-positive partial transpose (NPT) breaking map for some $t' > 0$, followed by an arbitrary evolution for $t > t'$.
[35] W. Ott and J. A. Yorke, Bull. Am. Math. Soc. **42**, 263 (2005).
[36] D. Chruściński and F. A. Wudarski, Phys. Lett. A **377**, 1425 (2013).
[37] A. Smirne, J. Kołodyński, S. F. Huelga, and R. Demkowicz-Dobrzański, Phys. Rev. Lett. **116**, 120801 (2016).
[38] N. Megier, D. Chruściński, J. Piilo, and W. T. Strunz, Sci. Rep. **7**, 6379 (2017).
[39] H.-B. Chen, J.-Y. Lien, G.-Y. Chen, and Y.-N. Chen, Phys. Rev. A **92**, 042105 (2015).
[40] F. Benatti, D. Chruściński, and S. Filippov, Phys. Rev. A **95**, 012112 (2017).
[41] J. Kołodyński, J. B. Brask, M. Perarnau-Llobet, and B. Bylicka, Phys. Rev. A **97**, 062124 (2018).
[42] S. Bhattacharya, B. Bhattacharya, and A. S. Majumdar, arXiv:1803.06881.
[43] N. Anand and T. A. Brun, arXiv:1903.03880.
[44] M.-D. Choi, Linear Algebra Appl. **10**, 285 (1975).
[45] E. Størmer, *Positive Linear Maps of Operator Algebras* (Springer, Berlin, Heidelberg, 2013).
[46] S. L. Woronowicz, Rep. Math. Phys. **10**, 165 (1976).
[47] I. Devetak and A. Winter, Proc. R. Soc. London, Ser. A **461**, 207 (2005).