Assisted Distillation of Quantum Coherence

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We introduce and study the task of assisted coherence distillation. This task arises naturally in bipartite systems where both parties work together to generate the maximal possible coherence on one of the subsystems. Only incoherent operations are allowed on the target system, while general local quantum operations are permitted on the other; this is an operational paradigm that we call local quantum-incoherent operations and classical communication. We show that the asymptotic rate of assisted coherence distillation for pure states is equal to the coherence of assistance, an analog of the entanglement of assistance, whose properties we characterize. Our findings imply a novel interpretation of the von Neumann entropy: it quantifies the maximum amount of extra quantum coherence a system can gain when receiving assistance from a collaborative party. Our results are generalized to coherence localization in a multipartite setting and possible applications are discussed.

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Introduction.—Quantum coherence represents a basic feature of quantum systems that is not present in the classical world. Recently, researchers have begun developing a resource-theoretic framework for understanding quantum coherence [1–9]. In this setting, coherence is regarded as a precious resource that cannot be generated or increased under a restricted class of operations known as incoherent operations [2,3]. A resource-theoretic treatment of coherence is physically motivated, in part, by certain processes in biology [10–12], transport theory [2,13,14], and thermodynamics [7,15,16], for which the presence of quantum coherence plays an important role.

In this Letter, we consider the task of assisted coherence distillation. It involves (at least) two parties, Alice (A) and Bob (B), who share one or many copies of some bipartite state ρ^{AB}. Their goal is to maximize the quantum coherence of Bob's system by Alice performing arbitrary quantum operations on her subsystem, while Bob is restricted to just incoherent operations on his. The duo is further allowed to communicate classically with one another. Overall, we refer to the allowed set of operations in this protocol as local quantum-incoherent operations and classical communication (LQICC). As we will show, the operational LQICC setting reveals fundamental properties about the quantum coherence accessible to Bob. In particular, the von Neumann entropy of his state, $S(\rho^B)$, quantifies precisely how much extra coherence can be generated in Bob's subsystem using LQICC than when no communication is allowed between him and any correlated party.

Alice and Bob's objective here is analogous to the task of assisted entanglement distillation. In the latter, entanglement is shared between three parties, A, B, C, and the goal is for B and C to obtain maximal bipartite entanglement when all parties use (unrestricted) local operations and classical communication (LOCC). The corresponding maximal entanglement that can be generated between B and C is known as the "entanglement of collaboration" [17]. Henceforth, here we define the "coherence of collaboration" as the maximum coherence that can be generated on subsystem B by LQICC operations. In general, both LOCC and LQICC protocols can be very complicated, involving many multiple rounds of measurement and communication [18]. A simplified scenario considers one-way protocols in which Alice holds a purifying system, and only she is allowed to broadcast measurement data. The maximum entanglement for B and C (maximum coherence for B) that can be generated in this manner is called the "entanglement of assistance" (EOA) [19] [the "coherence of assistance" (COA)]. In the asymptotic setting the entanglement of assistance is known to be equal to the entanglement of collaboration if the overall state is pure [20]. We show an analogous result for coherence: for pure states the coherence of assistance is equal to the coherence of collaboration in the asymptotic setting, and a closed expression for these quantities is also provided. Moreover, when Bob's system is a qubit and the overall state is pure, the coherence of assistance and the coherence of collaboration are equivalent even in the single-copy case. Finally, we also present a generalization to a multipartite setting where many assisting players collaborate to localize coherence onto a target system, and discuss possible applications to quantum technologies.

Resource theory of coherence.—The starting point of our work is the resource theory of coherence, introduced recently in [2–4,8]. In particular, a quantum state ρ is said to be incoherent in a given reference basis $\{|i\rangle\}$ if the state is diagonal in this basis, i.e., if $\rho = \sum_i p_i |i\rangle\langle i|$ with some probabilities p_i. For a bipartite system, the reference basis is assumed to be a tensor product of local bases [4,5,8].

A quantum operation is said to be incoherent if each of its Kraus operators K_α is incoherent, i.e., if $K_\alpha \mathcal{I} K_\alpha^\dagger \subseteq \mathcal{I}$, where $\mathcal{I}$ is the set of incoherent states. In this theory, a general completely positive trace-preserving map Λ is said to be incoherent if it can be represented by at least one set of incoherent Kraus operators. Completely dephasing any state ρ in the incoherent basis will generate the incoherent state $\Delta(\rho) \coloneqq \sum_i q_i |i\rangle\langle i|$ with $q_i = \langle i|\rho|i\rangle$. Note this is entire motivation for defining incoherent states as being diagonal in some particular basis: they are the density matrices obtained by erasing all off-diagonal terms through the decoherence map Δ. If d is the dimension of the Hilbert space of the system, the maximally coherent state is $|\Phi_d\rangle = \sqrt{1/d}\sum_i |i\rangle$, and we let $|\Phi\rangle \coloneqq |\Phi_2\rangle$ denote the "unit" coherence resource state [3].

Similar to the framework of entanglement distillation [21,22], general quantum states can be used for asymptotic distillation of maximally coherent states via incoherent operations. Formally, the distillable coherence C_d of a state ρ is defined as $C_d(\rho) = \sup\{R : \lim_{n\to\infty}(\inf_\Lambda \|\Lambda[\rho^{\otimes n}] - \Phi^{\otimes \lfloor Rn \rfloor}\|) = 0\}$, where $\|M\| = \mathrm{Tr}\sqrt{M^\dagger M}$ is the trace norm, and the infimum is taken over all incoherent operations Λ. Even more, a closed expression for the optimal distillation rate was found recently by Winter and Yang [8], and turns out to be equal to the relative entropy of coherence introduced in [1,3]. Recall the relative entropy of ρ to σ is defined as $S(\rho\|\sigma) = -\mathrm{Tr}(\rho\log\sigma) - S(\rho)$, with $S(\rho) = -\mathrm{Tr}(\rho\log\rho)$ being the von Neumann entropy of ρ.

Lemma 1.—The distillable coherence of ρ is [8]

$$C_d(\rho) = C_r(\rho) = S[\Delta(\rho)] - S(\rho), \tag{1}$$

where $C_r(\rho)$ is the relative entropy of coherence, defined as $C_r(\rho) = \min_{\sigma\in I} S(\rho\|\sigma)$.

Note that $C_d(\rho) > 0$ if and only if ρ is not incoherent.

Coherence of collaboration.—We now move to the main topic of this Letter, namely, the assisted distillation of coherence. As mentioned earlier, in this setting two parties Alice and Bob share many copies of a joint state $\rho = \rho^{AB}$ and aim to maximize coherence on Bob's system by LQICC operations.

In order to make a quantitative analysis, we define the distillable coherence of collaboration as the optimal rate, i.e., the optimal number of maximally coherent states on Bob's side per copy of the shared resource state ρ, in the assisted setting,

$$C_d^{A|B}(\rho) = \sup\{R : \lim_{n\to\infty}(\inf_\Lambda \|\Lambda[\rho^{\otimes n}] - \Phi^{\otimes\lfloor Rn\rfloor}\|) = 0\}, \tag{2}$$

where the infimum is taken over all LQICC operations Λ. When Alice is uncorrelated from Bob, i.e., $\rho^{AB} = \rho^A \otimes \rho^B$, then $C_d^{A|B}(\rho^{AB})$ reduces to the distillable coherence $C_d(\rho^B)$ which can be evaluated exactly using Lemma 1 [8]. In the following, we are interested in understanding how the assistance of Alice can improve Bob's distillation rate, i.e., how larger $C_d^{A|B}(\rho^{AB})$ can be in comparison to $C_d(\rho^B)$. To answer this question, we first note that the set of bipartite states which can be created via LQICC operations, that will be referred to as the set $\mathcal{QI}$ of quantum-incoherent (QI) states, admits a simple characterization. Namely, all such states have the following form:

$$\chi^{AB} = \sum_i p_i \sigma_i^A \otimes |i\rangle\langle i|^B. \tag{3}$$

Here, σ_i^A are arbitrary quantum states on A, and the states $|i\rangle^B$ belong to the local incoherent basis of B. Note that QI states have the same form as general quantum-classical states [23] (i.e., states with vanishing quantum discord [24]), except the "classical" part must be diagonal in the fixed incoherent basis.

It is obvious that any QI state has $C_d^{A|B}(\rho^{AB}) = 0$, and the following theorem shows that the converse is true as well.

Theorem 2.—A state ρ^{AB} has $C_d^{A|B}(\rho^{AB}) > 0$ if and only if the state ρ^{AB} is not quantum incoherent.

This theorem shows that any state which cannot be created for free via LQICC operations constitutes a resource for extracting coherence on Bob's side. For the proof of the theorem we refer to the Supplemental Material [25].

In the next step, we will provide an upper bound on the distillable coherence of collaboration. For this, we introduce the QI relative entropy,

$$C_r^{A|B}(\rho^{AB}) = \min_{\chi^{AB}\in\mathcal{QI}} S(\rho^{AB}\|\chi^{AB}), \tag{4}$$

with the minimization taken over the set of QI states. We note that $C_r^{A|B}$ is different from the relative entropy of discord introduced in [28,29], as the latter involves a minimization over all bases of B, while Eq. (4) is defined for a fixed incoherent basis $\{|i\rangle^B\}$. Using the same reasoning as in [29] (see Theorem 2 there), it is straightforward to see that $C_r^{A|B}$ can also be written as

$$C_r^{A|B}(\rho^{AB}) = S[\Delta^B(\rho^{AB})] - S(\rho^{AB}) \tag{5}$$

with $\Delta^B(\rho^{AB}) \coloneqq \sum_i (\mathbb{1}\otimes|i\rangle\langle i|)\rho^{AB}(\mathbb{1}\otimes|i\rangle\langle i|)$. Moreover, since the relative entropy does not increase under general quantum operations, $C_r^{A|B}$ is monotonically nonincreasing under LQICC operations. The following theorem shows that the QI relative entropy is an upper bound on $C_d^{A|B}$.

Theorem 3.—Given a state ρ^{AB} shared by Alice and Bob, the distillable coherence of collaboration is bounded above according to

$$C_d^{A|B}(\rho^{AB}) \le C_r^{A|B}(\rho^{AB}). \tag{6}$$

The proof can be found in [25]. This result shows that in the task considered here, the relative entropy plays similar role as in the task of entanglement distillation [30], bounding the distillation rate from above. Note that for standard coherence distillation the relative entropy of coherence is in fact equal to the optimal distillation rate [8], see also Lemma 1. It is an open question if this is also true for the task considered here, i.e., if the inequality (6) is an equality for all quantum states ρ^{AB}. As we will see in Theorem 4 below, the answer is affirmative at least for pure states.

Coherence of assistance.—We now introduce the COA for a state ρ as the maximal average coherence of the state,

$$C_a(\rho) = \max \sum_i q_i C_r(\psi_i) = \max \sum_i q_i S[\Delta(\psi_i)], \quad (7)$$

where the maximization is taken over all pure-state decompositions of $\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|$, and ψ_i is denoting $|\psi_i\rangle\langle\psi_i|$.

To provide the COA with an operational interpretation, it is instrumental to compare it with the EOA originally proposed by DiVincenzo *et al.* [19]. For a bipartite state ρ^{BC}, one identifies a decomposition of maximal average entanglement,

$$E_a(\rho^{BC}) = \max \sum_i q_i E(\psi_i^{BC}) = \max \sum_i q_i S(\mathrm{tr}_B \psi^{BC}), \quad (8)$$

for $\rho^{BC} = \sum_i q_i |\psi_i\rangle\langle\psi_i|^{BC}$. The interpretation of EOA is that by using local measurement and one-way classical communication, Alice can help Bob and Charlie obtain an average entanglement of at most $E_a(\rho^{BC})$ when they all share $|\Psi\rangle^{ABC}$, a purification of ρ^{BC}. In this case, any possible pure-state decomposition of ρ^{BC} can be realized when Alice performs a suitable measurement and announces the result [31]. If all the parties have access to arbitrary number of copies of the total state $|\Psi\rangle^{ABC}$, the figure of merit is the regularized EOA $E_a^\infty(\rho) = \lim_{n\to\infty}(1/n)E_a(\rho^{\otimes n})$. For an arbitrary density matrix ρ^{BC}, the regularized EOA is simply given by [20]

$$E_a^\infty(\rho^{BC}) = \min\{S(\rho^B), S(\rho^C)\}. \quad (9)$$

The COA defined in Eq. (7) has an analogous operational meaning if we assume that the state $\rho = \rho^B$ belongs to Bob, who is assisted by another party (Alice) holding a purification of ρ^B. Through local measurement, Alice can prepare any ensemble for Bob that is compatible with ρ^B, which is why we take the maximization in Eq. (7). Together with Lemma 1, then, $C_a(\rho^B)$ quantifies a one-way coherence distillation rate for Bob when Alice applies the same procedure for each copy of the state. In the many-copy setting, higher one-way distillation rates can typically be obtained when Alice performs a joint measurement across her many copies. Thus, we consider the regularized COA defined as $C_a^\infty(\rho) \coloneqq \lim_{n\to\infty}(1/n)C_a(\rho^{\otimes n})$.

As we prove in [25], the COA of a state $\rho = \sum_{i,j}\rho_{ij}|i\rangle\langle j|$ is equal to the EOA of the corresponding maximally correlated state [32] $\rho_{\mathrm{mc}} = \sum_{i,j}\rho_{ij}|ii\rangle\langle jj|$,

$$C_a(\rho) = E_a(\rho_{\mathrm{mc}}). \quad (10)$$

Clearly, Eq. (10) implies that this equality is also true for the regularized quantities, $C_a^\infty(\rho) = E_a^\infty(\rho_{\mathrm{mc}})$. Using Eq. (9), the regularized COA thus acquires the simple expression

$$C_a^\infty(\rho) = S[\Delta(\rho)]. \quad (11)$$

Equipped with these tools, we are now in position to provide a closed expression for $C_d^{A|B}$ for all pure states.

Theorem 4.—For a pure state $|\Psi\rangle^{AB}$ shared by Alice and Bob, the following equality holds:

$$C_d^{A|B}(|\Psi\rangle^{AB}) = C_a^\infty(\rho^B) = C_r^{A|B}(|\Psi\rangle^{AB}) = S[\Delta(\rho^B)]. \quad (12)$$

The proof of the theorem can be found in [25]. With Theorem 4 in hand, we give the von Neumann entropy an alternative operational interpretation. Namely, let $\delta C_d(\rho^B)$ denote the maximal increase in distillable coherence that Bob can obtain when exchanging classical communication with a correlated party: i.e., $\delta C_d(\rho^B) = \max_{\rho^{AB}}[C_d^{A|B}(\rho^{AB}) - C_d(\rho^B)]$, where the maximization is taken over all extensions ρ^{AB} of ρ^B. Noticing that the maximum is attained if ρ^{AB} is pure, Lemma 1 and Theorem 4 imply that

$$\delta C_d(\rho^B) = S(\rho^B). \quad (13)$$

Interestingly, this result does not depend on the particular choice of the reference incoherent basis.

Let us turn to the obvious inequality $C_a(\rho^B) \le C_a^\infty(\rho^B)$ and ask whether C_a is additive, in which case the inequality becomes tight. This question is especially interesting when one considers Ref. [8] where the coherence of formation, defined with a minimization rather than a maximization in Eq. (7), and thus a dual quantity to the COA, is shown to be additive. Below, we will show that in contrast, COA fails to exhibit additivity in general. Nevertheless, when restricting attention to n copies of an arbitrary single-qubit state ρ, additivity of COA can be proven. The latter finding is quite noteworthy since no analogous result is known for EOA in two-qubit systems.

Theorem 5.—COA is n-copy additive for qubit states ρ,

$$C_a(\rho) = C_a^\infty(\rho) = S[\Delta(\rho)]. \quad (14)$$

However, in general, the COA is not additive.

We refer to [25] for the proof. It is interesting to note that we prove non-additivity for systems with dimension 4 and above. Thus, it remains open if C_a is additive for qutrits. Note that by Theorem 4, this result implies that optimal coherence distillation for single-qubit systems involves just one-way communication and single-copy measurements from a purifying auxiliary system.

Multipartite scenario.—We now extend our results to the multipartite setting. When more than one party is providing assistance, the process of collaboratively generating coherence for Bob's system will be called coherence localization, in analogy to the task of entanglement localization [33].

We consider $(N+1)$-partite states $\rho^{A_1,\ldots,A_N B}$, where the parties $A_1,\ldots,A_N$ are allowed to perform arbitrary local quantum operations, and the party B is restricted to incoherent operations only. Additionally, classical communication is allowed between all the parties. The aim of all the parties is to localize as much coherence as possible on the subsystem of B. The corresponding asymptotic coherence localization rate can be defined just as in Eq. (2) and will be denoted by $C_d^{A_1,\ldots,A_N|B}(\rho^{A_1,\ldots,A_N B})$. For total pure states with B being a qubit we find that, quite remarkably, individual measurements on the auxiliary systems can generate the same maximal coherence for the target system B as when a global measurement is performed across all the auxiliary systems $A_1,\ldots,A_N$.

Theorem 6.—Let $|\Psi\rangle^{A_1,\ldots,A_N B}$ be an arbitrary multipartite state with system B being a qubit. Then

$$C_d^{A_1,\ldots,A_N|B}(|\Psi\rangle^{A_1,\ldots,A_N B}) = C_d^{A_{\mathrm{tot}}|B}(|\Psi\rangle^{A_{\mathrm{tot}}B}) = S(\Delta(\rho^B)), \tag{15}$$

where $A_{\mathrm{tot}} = A_1,\ldots,A_N$ is viewed as one party with the locality constraint removed among the A_i.

The proof is deferred to [25]. This theorem implies that for asymptotic coherence localization the assisting parties $A_1,\ldots,A_N$ do not need access to a quantum channel: local quantum operations on their subsystems together with classical communication are enough to ensure maximal coherence localization. This is true if the total state is pure, and if coherence is localized on a qubit.

LQICC versus SLOCC protocols.—The proof of Theorem 4 relied on relating the tasks of assisted coherence distillation and assisted entanglement distillation. This further supports a conjecture put forth in Ref. [8] that the resource theory of coherence is equivalent to the resource theory of entanglement for maximally correlated states [32]. We can prove a more general connection between LQICC operations in the coherence setting and LOCC operations in the entanglement setting.

For a given bipartite state ρ^{AB} we define the association

$$\rho^{AB} = \sum_{ij} M_{ij}^A \otimes |i\rangle\langle j|^B \Rightarrow \tilde{\rho}^{ABC} = \sum_{ij} M_{ij}^A \otimes |ii\rangle\langle jj|^{BC}, \tag{16}$$

where M_{ij} are operators acting on Alice's space and $\{|i\rangle\}$ is the fixed incoherent basis. As we show in [25], if two states ρ^{AB} and σ^{AB} are related via a bipartite LQICC map, i.e., $\sigma^{AB} = \Lambda_{\mathrm{LQICC}}[\rho^{AB}]$, then the corresponding states $\tilde{\rho}^{ABC}$ and $\tilde{\sigma}^{ABC}$ are related via a tripartite stochastic LOCC (SLOCC) map, i.e., $\tilde{\sigma}^{ABC} = \Lambda_{\mathrm{SLOCC}}[\tilde{\rho}^{ABC}]$. Thus any procedure implementable "for free" in the framework of assisted coherence has an equivalent probabilistic "free" implementation on the level of maximally correlated states. We find that, in fact, for many LQICC transformations $\rho^{AB} \to \sigma^{AB}$, the corresponding LOCC transformation $\tilde{\rho}^{ABC} \to \tilde{\sigma}^{ABC}$ can be implemented with probability one. It is an interesting open question whether the (tripartite) LOCC analog to every (bipartite) LQICC transformation has always a deterministic implementation.

In the case where the subsystem A is uncorrelated, Eq. (16) reduces to $\rho = \sum_{ij}\rho_{ij}|i\rangle\langle j| \Rightarrow \rho_{\mathrm{mc}} = \sum_{ij}\rho_{ij}|ii\rangle\langle jj|$. For this situation, the above results imply that for any two states ρ and $\sigma = \Lambda_{\mathrm{i}}[\rho]$ related via an incoherent operation Λ_{i}, the corresponding maximally correlated states ρ_{mc} and σ_{mc} are related via bipartite SLOCC: $\sigma_{\mathrm{mc}} = \Lambda_{\mathrm{SLOCC}}[\rho_{\mathrm{mc}}]$. Moreover, in the asymptotic setting where many copies of ρ are available, the SLOCC procedure becomes deterministic whenever the entanglement cost of σ_{mc} is not larger than the distillable entanglement of ρ_{mc}. This criterion can be easily checked, recalling that for these states the entanglement cost is equal to the entanglement of formation [34,35], and their distillable entanglement admits a simple expression [32].

Conclusions.—The results presented above are mainly based on the new set of LQICC operations which were introduced and studied in this Letter. This type of operations arises naturally if two parties have access to a classical channel, and one of the parties can perform arbitrary quantum operations, but the other is limited to incoherent operations only. The results presented here can be regarded as one application of this set of operations. Very recently, alternative applications for LQICC were presented in [36,37], including creation and distillation of entanglement [37] and implementation of quantum teleportation in a fully incoherent manner [36]. LQICC operations have also been extended to the class of local incoherent operations (for both parties) supplemented by classical communication [36,37]. Further applications closely adhering to realistic physical limitations are expected in the near future.

There are in fact many scenarios of practical relevance where the task of assisted coherence distillation can play a central role. For instance, think of a remote or unaccessible system on which coherence is needed as a resource (e.g., a biological system): our results give optimal prescriptions to inject such coherence on the remote target by acting on a controllable ancilla. In a multipartite setting, one can imagine distributing a correlated state among many parties and implementing an instance of open-destination quantum metrology, in which one party is selected to estimate an unknown parameter [38] and the other parties act locally on their subsystems in order to localize as much coherence as possible on the chosen target, so as to enhance the estimation precision. Similarly, the task can be a useful primitive within a secure quantum cryptographic network

[39], in which the distribution of nonorthogonal states (and thus coherence) is required [12].

The approach presented here can also be extended to other related scenarios. As an example, we mention the resource theory of frameness and asymmetry [40,41]. The relation of these concepts to the resource theory of coherence proposed by Baumgratz *et al.* [3] has been studied very recently [42]. In this context, an important set of quantum operations is known as thermal operations [15,16]. These operations are a subset of general incoherent operations [42]. It will be very interesting to see how the results provided here change when local incoherent operations for one party are further restricted to local thermal operations. This can be of direct relevance to the design of optimal ancilla-assisted work extraction protocols in thermodynamical settings [7].

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