Limits for entanglement distribution with separable states

Alexander Streltsov,[1,2,*] Hermann Kampermann,[2] and Dagmar Bruß[2]
[1]*Institut de Ciències Fotòniques, 08860 Castelldefels, Barcelona, Spain*
[2]*Heinrich-Heine-Universität Düsseldorf, Institut für Theoretische Physik III, D-40225 Düsseldorf, Germany*
(Received 4 September 2013; published 22 September 2014)

Entanglement distribution with separable states has recently attracted considerable attention. Recent results suggest that quantum discord, a measure for quantum correlations beyond entanglement, is responsible for this counterintuitive phenomenon. In this work we study this question from a different perspective and find minimal requirements for a separable state to be useful for entanglement distribution. Surprisingly, we find that the presence of quantum discord is not sufficient to ensure entanglement distribution: There exist states with nonzero quantum discord that nevertheless cannot be used for entanglement distribution. As a result, we show that entanglement distribution is not possible with rank-2 separable states. Our work sheds light on the task of entanglement distribution with separable states and reveals a classification of quantum states with respect to their usefulness for this task.

DOI: 10.1103/PhysRevA.90.032323 PACS number(s): 03.67.Mn, 03.65.Ta

A fundamental task in quantum information processing is the distribution of entanglement between two distant parties. It has been shown in [1] that, counterintuitively, this task can be achieved by sending a particle that exhibits no entanglement with the rest of the system. Such entanglement distribution with separable states was also studied for Gaussian states [2,3] and experiments with continuous [4,5] and discrete variables [6] have been presented very recently (see also [7]). Quantum discord, a different type of quantum correlations going beyond entanglement [8–10], has been identified as the figure of merit for this puzzling phenomenon [11,12]. This finding is in accordance with earlier results, supporting the crucial role of quantum discord and related quantifiers of quantum correlations [13–18] in quantum information theory. These results include thermodynamic approaches [19,20] and the relations to entanglement creation in the quantum measurement process [21–23] and to entanglement consumption in quantum state merging [24,25]. Recently, the role of quantum discord for quantum metrology [26–28], encoding [29], and sharing [30–32] of information has also been subjected to scrutiny. Quantum discord was further proposed to be the figure of merit for the quantum computing protocol known as DQC1 (quantum computation with one quantum bit) [33] and for the task of remote state preparation [34]. Some of the arguments are still controversial [17,35,36] and the ongoing debate about the physical interpretation of quantum discord [37,38] is further amplified by the finding that quantum discord can be created by local operations [17,39–41]. On the one hand, these results suggest that quantum discord can also be regarded as a measure for the local quantumness of a state [42]. On the other hand, these findings underpin the role of general quantum correlations for tasks that are not covered by the traditional concept of entanglement.

In this paper we aim to find minimal requirements for entanglement distribution with separable states. To this end we consider a general distribution protocol and identify properties for a separable state to be a resource for entanglement distribution. In the following we call a state ρ^{AB} *useful* for entanglement distribution if it is possible to divide party B in two parties B_1 and B_2 in such a way that sending particle A from side 1 to side 2 leads to an increase of entanglement:

$$E^{B_1|B_2A} > E^{AB_1|B_2} \tag{1}$$

(see also Fig. 1). Certainly, any entangled state is useful for entanglement distribution, as can be seen by giving the full system B to side 1, i.e., $B_1 = B$. This implies that the presence of entanglement between A and B is sufficient for entanglement distribution. On the other hand, the finding that entanglement can be distributed with separable states [1] demonstrates that the presence of entanglement is not necessary and that, in general, some other kind of quantum correlation beyond entanglement is responsible for this process.

Recently, quantum discord was identified as the key resource for entanglement distribution: The distribution of any finite amount of entanglement needs the transmission of at least the same amount of quantum discord [11,12]. These results show that, in contrast to entanglement, quantum discord is implicitly required if one wishes to increase the amount of entanglement between two parties. As a consequence, all classical-quantum states, i.e., states of the form $\rho_{\mathrm{cq}} = \sum_i p_i \left|i\right\rangle \left\langle i\right|^A \otimes \rho_i^B$, cannot be used for entanglement distribution since all those states have zero quantum discord [9]. On the other hand, the results presented in [11,12] support the intuition that the presence of quantum discord in a state ρ^{AB} already ensures its usefulness for entanglement distribution. This idea leads us to the main question of this paper: Are all states with nonzero quantum discord useful for entanglement distribution?

To approach the answer to this question, we first consider the most simple class of potentially useful separable states

$$\rho^{AB} = p \left|\psi_1\right\rangle \left\langle\psi_1\right|^A \otimes \left|\phi_1\right\rangle \left\langle\phi_1\right|^B + (1-p) \left|\psi_2\right\rangle \left\langle\psi_2\right|^A \otimes \left|\phi_2\right\rangle \left\langle\phi_2\right|^B. \tag{2}$$

Noting that this state has nonzero discord for a generic choice of the states $|\psi_i^A\rangle$ and $|\phi_i^B\rangle$ and the probability p, it is reasonable to conjecture that this state is generically useful for entanglement distribution. Surprisingly, as we will see in

*alexander.streltsov@icfo.es

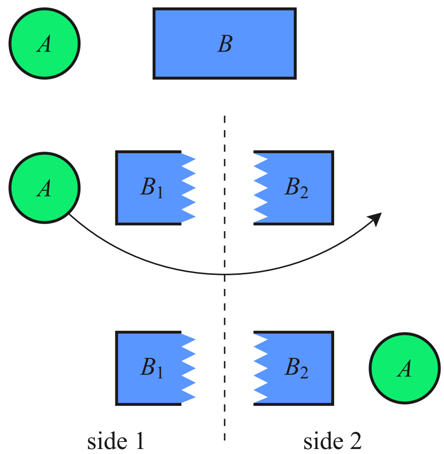

FIG. 1. (Color online) A quantum system consisting of two parties A and B (upper figure) is useful for entanglement distribution if party B can be divided in two parties B_1 and B_2 (middle figure) in such a way that sending particle A from side 1 to side 2 leads to an increase of entanglement (lower figure).

the following, this intuition is not correct. This implies that the answer to the question stated above is negative, leading to strong limitations on entanglement distribution with separable states.

In the following we will show that the state given in Eq. (2) cannot be used for entanglement distribution, regardless of the choice of states $|\psi_i^A\rangle$ and $|\phi_i^B\rangle$ and probability p. In particular, we will show that for any division of party B into two parties B_1 and B_2, sending particle A from one side to the other will never change the amount of entanglement [43]:

$$E_n^{B_1|B_2A} = E_n^{AB_1|B_2}. \tag{3}$$

Here E_n is the logarithmic negativity, defined for a state $\rho = \rho^{XY}$ as $E_n^{X|Y} = \log_2 ||\rho^{T_X}||_1$, where T_X denotes partial transposition and $\|M\|_1 = \mathrm{Tr}\sqrt{M^\dagger M}$ is the trace norm of an operator M [44]. Together with the closely related negativity $\mathcal{N}^{X|Y} = (||\rho^{T_X}||_1 - 1)/2$, which was first introduced in [45], the logarithmic negativity is a well-established quantifier of entanglement [46]. Its significance for quantum information theory comes from several desirable properties, such as being an entanglement monotone [47], which is computable and additive [44]. The logarithmic negativity is an upper bound on the distillable entanglement [44] and a lower bound on the positive partial transpose (PPT) entanglement cost, which is the entanglement cost under quantum operations preserving the positivity of the partial transpose [48]. Moreover, it was shown that E_n coincides with the PPT entanglement cost in a large number of scenarios, including all two-qubit states [49], Gaussian states [48], and general Werner states [48]. Since E_n is zero only on separable and bound entangled states [50] and bound entanglement is known to be absent in bipartite states with rank smaller than 4 [51–53], E_n is a faithful quantifier of entanglement for the states presented in Eq. (2).

To prove Eq. (3) we consider the partially transposed density matrices $\rho^{T_{B_1}}$ and $\rho^{T_{AB_1}}$ of the total state $\rho = \rho^{AB} = \rho^{AB_1B_2}$ given in Eq. (2), where B_1 and B_2 are two subsystems of B. In particular, we will show that the matrices $\rho^{T_{B_1}}$ and $\rho^{T_{AB_1}}$ are equal up to a unitary operation and thus share the same set of eigenvalues. To show this we start with the partially transposed density matrix $\rho^{T_{B_1}} = p\,|\psi_1\rangle\langle\psi_1|^A \otimes M_1^B + (1-p)\,|\psi_2\rangle\langle\psi_2|^A \otimes M_2^B$ with $M_i^B = (|\phi_i\rangle\langle\phi_i|)^{T_{B_1}}$. In the next step we will show that a partial transposition of this matrix $\rho^{T_{B_1}}$ with respect to the subsystem A is equivalent to a unitary operation, i.e., $\rho^{T_{AB_1}} = U\rho^{T_{B_1}}U^\dagger$. This can be seen by considering the Bloch vectors $\mathbf{r}$ and $\mathbf{s}$ corresponding to the states $|\psi_1^A\rangle$ and $|\psi_2^A\rangle$, i.e., $|\psi_1\rangle\langle\psi_1|^A = \frac{1}{2}(\mathbb{1} + \sum_i r_i\sigma_i)$ and $|\psi_2\rangle\langle\psi_2|^A = \frac{1}{2}(\mathbb{1} + \sum_i s_i\sigma_i)$ with Pauli matrices σ_i. The transposition of the states $|\psi_1^A\rangle$ and $|\psi_2^A\rangle$ takes them to new states $|\tilde{\psi}_1^A\rangle$ and $|\tilde{\psi}_2^A\rangle$ with Bloch vectors $\tilde{\mathbf{r}}$ and $\tilde{\mathbf{s}}$. At this point, it is crucial to note that the product of the Bloch vectors does not change under transposition: $\tilde{\mathbf{r}}\cdot\tilde{\mathbf{s}} = \mathbf{r}\cdot\mathbf{s}$. This implies that the transposition of subsystem A is equivalent to a joint rotation of the Bloch vectors $\mathbf{r}\to\tilde{\mathbf{r}}$ and $\mathbf{s}\to\tilde{\mathbf{s}}$, which, on the other hand, corresponds to a unitary operation acting on subsystem A. This proves that the matrices $\rho^{T_{B_1}}$ and $\rho^{T_{AB_1}}$ are equal up to a unitary operation, implying that the eigenvalues of both matrices must be the same. Starting from this result, Eq. (3) is seen to be correct by recalling that the logarithmic negativity $E_n^{X|Y}$ depends only on the eigenvalues of the partially transposed density matrix ρ^{T_X} [44].

The results presented so far imply crucial constraints on the possibility to distribute entanglement with separable states. In particular, we have seen that the distribution of entanglement is not possible if the corresponding separable state is a mixture of two pure product states [see Eq. (2)]. In the next step we will see that this limitation can be surpassed if the pure states $|\psi_i^A\rangle$ in Eq. (2) are replaced by mixed states ρ_i^A. In this case the total state takes the form

$$\rho^{AB} = p\rho_1^A \otimes |\phi_1\rangle\langle\phi_1|^B + (1-p)\rho_2^A \otimes |\phi_2\rangle\langle\phi_2|^B. \tag{4}$$

The use of this state for entanglement distribution can be demonstrated for the probability $p = 1/2$ by a proper choice of the states ρ_i^A and $|\phi_i^B\rangle$. This can be achieved by defining the states ρ_i^A of subsystem A as follows: $\rho_1^A = \frac{1}{6}|0\rangle\langle 0| + \frac{5}{6}|1\rangle\langle 1|$ and $\rho_2^A = \frac{2}{3}|a\rangle\langle a| + \frac{1}{3}|b\rangle\langle b|$. Here $|a\rangle$ and $|b\rangle$ are nonorthogonal qutrit states, defined as $|a\rangle = (|0\rangle + |1\rangle + |2\rangle)/\sqrt{3}$ and $|b\rangle = (|0\rangle + i\,|1\rangle)/\sqrt{2}$. Finally, party B consists of two subsystems B_1 and B_2 and the corresponding states $|\phi_i^B\rangle$ can be chosen as $|\phi_1^B\rangle = (|00\rangle + |01\rangle + i\,|11\rangle)/\sqrt{3}$ and $|\phi_2^B\rangle = \cos\alpha\,|00\rangle + \sin\alpha\,|11\rangle$. As can be seen from the difference $\Delta E_n = E_n^{B_1|B_2A} - E_n^{AB_1|B_2}$, shown in Fig. 2 as a function of α, this particular setting allows one to distribute a finite amount of entanglement $\Delta E_n > 0$ in the range $0 < \alpha < \pi/4$, where α is the parameter of the state $|\phi_2^B\rangle$.

The example presented above should be regarded as a proof of principle: It explicitly demonstrates that some separable states that are mixtures of only two product states can, in principle, be used for entanglement distribution. In particular, we have seen that a successful distribution of entanglement can be achieved by a specific choice of mixed qutrit states ρ_1^A and ρ_2^A. As we will see in the following, it is indeed crucial that the transmitted particle A is not a qubit: For entanglement distribution with separable states as given in Eq. (4) the dimension of A needs to be at least 3. To prove this statement it is enough to show that for a two-dimensional

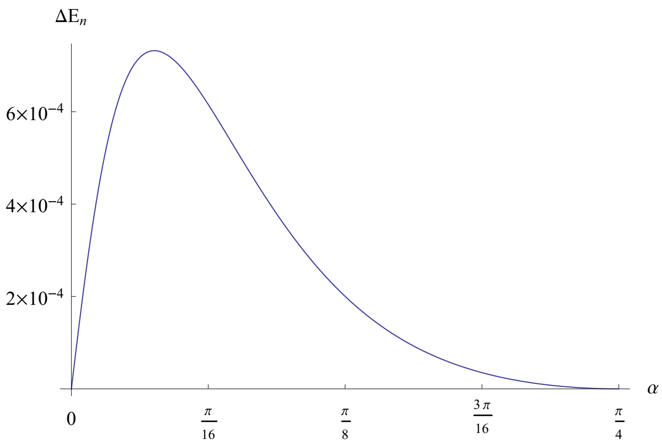

FIG. 2. (Color online) Entanglement distribution with separable states by sending a qutrit: The plot shows the amount of distributed entanglement $\Delta E_n = E_n^{B_1|B_2A} - E_n^{AB_1|B_2}$ for the state given in Eq. (4) as a function of the parameter α. For details see the main text.

subsystem A the state given in Eq. (4) cannot be used for entanglement distribution, i.e., Eq. (3) is satisfied. This can be seen by observing that the arguments given in the proof of Eq. (3) for mixtures of two *pure* product states remain valid if the pure states $|\psi_1^A\rangle$ and $|\psi_2^A\rangle$ are replaced by arbitrary qubit states ρ_1^A and ρ_2^A.

On the one hand, we have seen that entanglement distribution with separable states is impossible if the separable state is a mixture of two pure product states only. On the other hand, we have also demonstrated a possibility to avoid this problem by using two mixed states ρ_1^A and ρ_2^A for the exchanged particle A. In the next step we will show that mixedness of *both* states is essential: Entanglement distribution is not possible if ρ_1^A or ρ_2^A is pure, regardless of the dimension of the exchanged particle A. We will prove this statement by showing that Eq. (3) is satisfied for all states given in Eq. (4) as long as either ρ_1^A or ρ_2^A is pure. Without loss of generality we can assume that $\rho_1^A = |\psi\rangle\langle\psi|^A$ is pure and the state $\rho_2^A = \tau^A$ is diagonal in the computational basis $\tau^A = \sum_i \lambda_i |i\rangle\langle i|^A$. Using similar lines of reasoning as above we will prove the validity of Eq. (3) by showing that the partially transposed density matrices $\rho^{T_{B_1}}$ and $\rho^{T_{AB_1}}$ are equal up to a unitary operation. In particular, the matrix $\rho^{T_{B_1}}$ now has the form $\rho^{T_{B_1}} = p|\psi\rangle\langle\psi|^A \otimes M_1^B + (1-p)\tau^A \otimes M_2^B$ with $M_i^B = (|\phi_i\rangle\langle\phi_i|)^{T_{B_1}}$. From this expression we can obtain the matrix $\rho^{T_{AB_1}}$ by performing partial transposition on subsystem A: $\rho^{T_{AB_1}} = p|\tilde{\psi}\rangle\langle\tilde{\psi}|^A \otimes M_1^B + (1-p)\tau^A \otimes M_2^B$. Here we used the fact that $\tau^A = \sum_i \lambda_i |i\rangle\langle i|^A$ is diagonal in the computational basis and thus does not change under transposition. The relation between the state $|\psi^A\rangle$ and the transposed state $|\tilde{\psi}^A\rangle$ can be seen by expanding both states in the computational basis $|\tilde{\psi}^A\rangle = \sum_j c_j^* |j^A\rangle$, where c_j are the coefficients of the state $|\psi^A\rangle$, i.e., $|\psi^A\rangle = \sum_j c_j |j^A\rangle$. In the final step it is important to note that the states $|\psi^A\rangle$ and $|\tilde{\psi}^A\rangle = U|\psi^A\rangle$ are related by the unitary operation $U = \sum_j e^{-2i\phi_j}|j\rangle\langle j|^A$, where ϕ_j is the phase corresponding to the coefficient $c_j = |c_j|e^{i\phi_j}$. Since this unitary operation is diagonal in the computational basis, it does not change the state τ^A and thus we obtain the desired result $\rho^{T_{AB_1}} = U\rho^{T_{B_1}}U^\dagger$. This proves that for a successful distribution of entanglement with separable states as given in Eq. (4) both states ρ_1^A and ρ_2^A must be mixed.

The results presented above indicate that the structure of the separable state is crucial if the separable state is to be used as a resource for entanglement distribution. While a mixture of only two product states does not allow one to distribute any entanglement by sending a single qubit, this limitation disappears if the exchanged particle has dimension 3. We also note that this result remains valid if the pure states $|\phi_i^B\rangle$ of subsystem B are replaced by mixed states ρ_i^B. In particular, states of the form $\rho^{AB} = p\rho_1^A \otimes \rho_1^B + (1-p)\rho_2^A \otimes \rho_2^B$ can only be used for entanglement distribution if the transmitted particle A has at least dimension 3 and if both states ρ_1^A and ρ_2^A are not pure.

In the next step it is natural to ask about the situation where the separable state used for entanglement distribution is more general. As we will see in the following, qubits can still be used to distribute entanglement if the separable state is a mixture of at least *three* product states. This can be demonstrated on the following state:

$$\rho^{AB} = \frac{1}{3}\sum_{i=1}^{3} |\psi_i\rangle\langle\psi_i|^A \otimes |\phi_i\rangle\langle\phi_i|^B, \tag{5}$$

where the qubit states $|\psi_i^A\rangle$ are chosen as follows: $|\psi_1^A\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, $|\psi_2^A\rangle = (|0\rangle + i|1\rangle)/\sqrt{2}$, and $|\psi_3^A\rangle = |0\rangle$. Party B consists of two subsystems B_1 and B_2 and the corresponding states $|\phi_i^B\rangle$ are defined as $|\phi_1^B\rangle = |01\rangle$, $|\phi_2^B\rangle = (|00\rangle + i|11\rangle)/\sqrt{2}$, and $|\phi_3^B\rangle = \cos\alpha|00\rangle + \sin\alpha|11\rangle$. As can be seen from Fig. 3, this state allows one to distribute a finite amount of entanglement $\Delta E_n = E_n^{B_1|B_2A} - E_n^{AB_1|B_2}$ in the range $0 < \alpha < \pi/2$, where α is the parameter of the state $|\phi_3^B\rangle$.

As will become clear in a moment, the reason why the state in Eq. (5) is useful for entanglement distribution lies in the structure of the states $|\psi_i^A\rangle$. In particular, their Bloch vectors

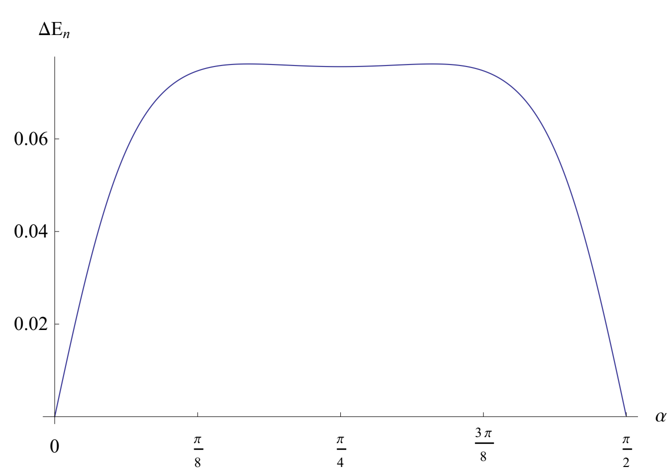

FIG. 3. (Color online) Entanglement distribution with separable states by sending a single qubit: The plot shows the amount of distributed entanglement $\Delta E_n = E_n^{B_1|B_2A} - E_n^{AB_1|B_2}$ for the state given in Eq. (5) as a function of the parameter α. For details see the main text.

are given by $\mathbf{r}_1 = (1,0,0)^T$, $\mathbf{r}_2 = (0,1,0)^T$, and $\mathbf{r}_3 = (0,0,1)^T$. Observe that these three vectors are linearly independent, i.e., they are not all in the same plane. We will see in the following that this feature is crucial for entanglement distribution, where at the same time we will generalize our results to arbitrary separable states, i.e., states of the form $\rho^{AB} = \sum_i p_i \rho_i^A \otimes \rho_i^B$, where the exchanged particle A is a qubit. Following the same arguments as in the preceding discussion, we consider the partially transposed matrices $\rho^{T_{B_1}} = \sum_i p_i \rho_i^A \otimes M_i^B$ and $\rho^{T_{AB_1}} = \sum_i p_i \tilde{\rho}_i^A \otimes M_i^B$ with $M_i^B = (\rho_i^B)^{T_{B_1}}$. Recall that the state ρ^{AB} cannot be used for entanglement distribution if the two matrices $\rho^{T_{B_1}}$ and $\rho^{T_{AB_1}}$ are equal up to a unitary operation. On the one hand, this is the case whenever the transposition on subsystem A of the matrix $\rho^{T_{B_1}}$ corresponds to a joint rotation of the Bloch vectors $\mathbf{r}_i \to \tilde{\mathbf{r}}_i$, i.e., whenever there exists a special orthogonal 3×3 matrix O such that $\tilde{\mathbf{r}}_i = O \cdot \mathbf{r}_i$ [54]. Here $\mathbf{r}_i$ and $\tilde{\mathbf{r}}_i$ are the Bloch vectors of ρ_i^A and the transposed state $\tilde{\rho}_i^A$, respectively. On the other hand, it is crucial to note that the Bloch vector $\tilde{\mathbf{r}}$ corresponding to a transposed state $\tilde{\rho} = \rho^T$ is related to the Bloch vector $\mathbf{r}$ of the initial state ρ via a reflection on the xz plane, i.e., $(\tilde{r}_1,\tilde{r}_2,\tilde{r}_3) = (r_1,-r_2,r_3)$. Combining these results, we can say that ρ^{AB} cannot be used for entanglement distribution if for all the Bloch vectors $\mathbf{r}_i$ a reflection on the xz plane is equivalent to a rotation. Note that this is always fulfilled if the number of Bloch vectors is 2, in accordance with the finding that mixtures of two product states cannot be used for entanglement distribution by sending a qubit. For more than two vectors a reflection does not necessarily correspond to a rotation, supporting the finding that qubits can be used for entanglement distribution if the number of product states is more than 2. Finally, we point out that reflection is equivalent to rotation for any number of Bloch vectors, whenever all the Bloch vectors are in the same plane. This immediately leads to a generalization of our previous results: Entanglement distribution with separable states by sending a single qubit is only possible if the corresponding Bloch vectors $\mathbf{r}_i$ are not all in the same plane.

In conclusion, we established minimal requirements for a separable state to be a resource for entanglement distribution. Here both the dimension of the exchanged particle and the number of product terms in the decomposition play a crucial role. Our results provide an answer to the main question of this paper: There are states with nonzero quantum discord that cannot be used as a resource for entanglement distribution. In particular, we have shown that a separable state cannot be used for this task if it is a mixture of only two pure product states. Since all rank-2 separable states are mixtures of two pure product states [55], we conclude that entanglement distribution with separable states requires states with rank of at least 3. Starting from this result, we also presented general criteria for entanglement distribution with separable states by sending a single qubit. In this case, entanglement distribution is impossible if the separable state is a mixture of two product states only, regardless of whether the corresponding product states are pure or mixed. Two possible solutions were presented to surpass this limitation: either by sending a qutrit or by using a separable state that is a mixture of at least three product states. Our results further imply that any tripartite state $\rho^{AB_1B_2}$ of rank 2 that is separable between A and B_1B_2 cannot be used for entanglement distribution since for all these states the logarithmic negativity does not depend on the location of the exchanged particle A: $E_n^{B_1|B_2A} = E_n^{AB_1|B_2}$. If the particle A is a qubit, the same arguments further apply to all separable states $\rho^{AB_1B_2} = p\mu^A \otimes \mu^{B_1B_2} + (1-p)\nu^A \otimes \nu^{B_1B_2}$ that are mixtures of two product states only. Noting that the corresponding tripartite state $\rho^{AB_1B_2}$ can, in principle, have an arbitrary amount of entanglement, we can say that the presence of entanglement in a multipartite quantum state alone does not ensure its usefulness for entanglement distribution. We also point out that the presented approach is significantly different from the recent approach by Kay [56]. There the author investigated the usefulness of Bell diagonal states for entanglement distribution with separable states. It is important to note that the definition of useful states in [56] is different from ours, making the two approaches independent and complementary contributions to the ongoing discussion of entanglement distribution with separable states. Finally, the results presented in this paper apply to two fundamental quantifiers of entanglement: the logarithmic negativity E_n and the closely related negativity $\mathcal{N}$, which has recently received a physical interpretation as an estimator of entanglement dimension [57]. Extension of our results to other entanglement measures is the subject of ongoing research and the investigation of this question may be instrumental for creation of new protocols for entanglement distribution and for the gain of new insights into the properties of quantum entanglement and general quantum correlations.

We acknowledge discussion with A. Kay and M. Piani, and financial support by the German Federal Ministry of Education and Research (BMBF, Project QuOReP). A.S. was also supported by a Feodor Lynen Research Fellowship from the Alexander von Humboldt-Foundation, the John Templeton Foundation, EU IP SIQS, ERC AdG OSYRIS and EU-Spanish Ministry CHISTERA DIQIP.

[1] T. S. Cubitt, F. Verstraete, W. Dür, and J. I. Cirac, Phys. Rev. Lett. **91**, 037902 (2003).
[2] L. Mišta, Jr. and N. Korolkova, Phys. Rev. A **77**, 050302 (2008).
[3] L. Mišta, Jr., Phys. Rev. A **87**, 062326 (2013).
[4] C. E. Vollmer, D. Schulze, T. Eberle, V. Händchen, J. Fiurášek, and R. Schnabel, Phys. Rev. Lett. **111**, 230505 (2013).
[5] C. Peuntinger, V. Chille, L. Mišta, Jr., N. Korolkova, M. Förtsch, J. Korger, C. Marquardt, and G. Leuchs, Phys. Rev. Lett. **111**, 230506 (2013).
[6] A. Fedrizzi, M. Zuppardo, G. G. Gillett, M. A. Broome, M. P. Almeida, M. Paternostro, A. G. White, and T. Paterek, Phys. Rev. Lett. **111**, 230504 (2013).
[7] C. Silberhorn, Physics **6**, 132 (2013).

[8] W. H. Zurek, Ann. Phys. (Berlin) **9**, 855 (2000).
[9] H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001).
[10] L. Henderson and V. Vedral, J. Phys. A **34**, 6899 (2001).
[11] A. Streltsov, H. Kampermann, and D. Bruß, Phys. Rev. Lett. **108**, 250501 (2012).
[12] T. K. Chuan, J. Maillard, K. Modi, T. Paterek, M. Paternostro, and M. Piani, Phys. Rev. Lett. **109**, 070501 (2012).
[13] S. Luo, Phys. Rev. A **77**, 022301 (2008).
[14] K. Modi, T. Paterek, W. Son, V. Vedral, and M. Williamson, Phys. Rev. Lett. **104**, 080501 (2010).
[15] P. Giorda and M. G. A. Paris, Phys. Rev. Lett. **105**, 020503 (2010).
[16] G. Adesso and A. Datta, Phys. Rev. Lett. **105**, 030501 (2010).
[17] B. Dakić, V. Vedral, and Č. Brukner, Phys. Rev. Lett. **105**, 190502 (2010).
[18] G. L. Giorgi, B. Bellomo, F. Galve, and R. Zambrini, Phys. Rev. Lett. **107**, 190501 (2011).
[19] J. Oppenheim, M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **89**, 180402 (2002).
[20] W. H. Zurek, Phys. Rev. A **67**, 012320 (2003).
[21] A. Streltsov, H. Kampermann, and D. Bruß, Phys. Rev. Lett. **106**, 160401 (2011).
[22] M. Piani, S. Gharibian, G. Adesso, J. Calsamiglia, P. Horodecki, and A. Winter, Phys. Rev. Lett. **106**, 220403 (2011).
[23] G. Adesso, V. D'Ambrosio, E. Nagali, M. Piani, and F. Sciarrino, Phys. Rev. Lett. **112**, 140501 (2014).
[24] V. Madhok and A. Datta, Phys. Rev. A **83**, 032323 (2011).
[25] D. Cavalcanti, L. Aolita, S. Boixo, K. Modi, M. Piani, and A. Winter, Phys. Rev. A **83**, 032324 (2011).
[26] K. Modi, H. Cable, M. Williamson, and V. Vedral, Phys. Rev. X **1**, 021022 (2011).
[27] D. Girolami, T. Tufarelli, and G. Adesso, Phys. Rev. Lett. **110**, 240402 (2013).
[28] D. Girolami, A. M. Souza, V. Giovannetti, T. Tufarelli, J. G. Filgueiras, R. S. Sarthour, D. O. Soares-Pinto, I. S. Oliveira, and G. Adesso, Phys. Rev. Lett. **112**, 210401 (2014).
[29] M. Gu, H. M. Chrzanowski, S. M. Assad, T. Symul, K. Modi, T. C. Ralph, V. Vedral, and P. K. Lam, Nat. Phys. **8**, 671 (2012).
[30] M. Zwolak and W. H. Zurek, Sci. Rep. **3**, 1729 (2013).
[31] A. Streltsov and W. H. Zurek, Phys. Rev. Lett. **111**, 040401 (2013).
[32] F. G. S. L. Brandao, M. Piani, and P. Horodecki, arXiv:1310.8640v1.
[33] A. Datta, A. Shaji, and C. M. Caves, Phys. Rev. Lett. **100**, 050502 (2008).
[34] B. Dakić, Y. O. Lipp, X. Ma, M. Ringbauer, S. Kropatschek, S. Barz, T. Paterek, V. Vedral, A. Zeilinger, Č. Brukner, and P. Walther, Nat. Phys. **8**, 666 (2012).
[35] G. L. Giorgi, Phys. Rev. A **88**, 022315 (2013).
[36] P. Horodecki, J. Tuziemski, P. Mazurek, and R. Horodecki, Phys. Rev. Lett. **112**, 140507 (2014).
[37] Z. Merali, Nature (London) **474**, 24 (2011).
[38] K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, Rev. Mod. Phys. **84**, 1655 (2012).
[39] A. Streltsov, H. Kampermann, and D. Bruß, Phys. Rev. Lett. **107**, 170502 (2011).
[40] S. Campbell, T. J. G. Apollaro, C. Di Franco, L. Banchi, A. Cuccoli, R. Vaia, F. Plastina, and M. Paternostro, Phys. Rev. A **84**, 052316 (2011).
[41] F. Ciccarello and V. Giovannetti, Phys. Rev. A **85**, 010102 (2012).
[42] M. Gessner, E.-M. Laine, H.-P. Breuer, and J. Piilo, Phys. Rev. A **85**, 052122 (2012).
[43] From the definition of useful states in Eq. (1) one might assume that those quantum states ρ^{AB} that cannot be used for entanglement distribution should obey the general condition $E^{B_1|B_2A} \leqslant E^{AB_1|B_2}$ for any division of party B in two subsystems B_1 and B_2. Remarkably, this inequality reduces to the equality $E^{B_1|B_2A} = E^{AB_1|B_2}$. This can be seen by contradiction, starting with a state ρ^{AB} that cannot be used for entanglement distribution and assuming the existence of two subsystems B_1' and B_2' such that $E^{B_1'|B_2'A} < E^{AB_1'|B_2'}$. Defining $B_1 = B_2'$ and $B_2 = B_1'$ we arrive at the inequality $E^{B_1|B_2A} > E^{AB_1|B_2}$. This implies that the corresponding state is useful for entanglement distribution, which is the desired contradiction.
[44] G. Vidal and R. F. Werner, Phys. Rev. A **65**, 032314 (2002).
[45] K. Życzkowski, P. Horodecki, A. Sanpera, and M. Lewenstein, Phys. Rev. A **58**, 883 (1998).
[46] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).
[47] M. B. Plenio, Phys. Rev. Lett. **95**, 090503 (2005).
[48] K. Audenaert, M. B. Plenio, and J. Eisert, Phys. Rev. Lett. **90**, 027901 (2003).
[49] S. Ishizaka, Phys. Rev. A **69**, 020301 (2004).
[50] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **80**, 5239 (1998).
[51] P. Horodecki, M. Lewenstein, G. Vidal, and I. Cirac, Phys. Rev. A **62**, 032310 (2000).
[52] P. Horodecki, J. A. Smolin, B. M. Terhal, and A. V. Thapliyal, Theor. Comput. Sci. **292**, 589 (2003).
[53] L. Chen and Y.-X. Chen, Phys. Rev. A **78**, 022318 (2008).
[54] Note that this argument applies only if the exchanged particle A is a qubit since in this case any rotation on the Bloch sphere corresponds to a unitary operation on the corresponding Hilbert space.
[55] A. Sanpera, R. Tarrach, and G. Vidal, Phys. Rev. A **58**, 826 (1998).
[56] A. Kay, Phys. Rev. Lett. **109**, 080503 (2012).
[57] C. Eltschka and J. Siewert, Phys. Rev. Lett. **111**, 100503 (2013).