Quantifying nonclassicality: Global impact of local unitary evolutions

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We show that only those composite quantum systems possessing nonvanishing quantum correlations have the property that *any* nontrivial local unitary evolution changes their global state. We derive the exact relation between the global state change induced by local unitary evolutions and the amount of quantum correlations. We prove that the minimal change coincides with the geometric measure of discord (defined via the Hilbert-Schmidt norm), thus providing the latter with an operational interpretation in terms of the capability of a local unitary dynamics to modify a global state. We establish that two-qubit Werner states are maximally quantum correlated, and are thus the ones that maximize this type of global quantum effect. Finally, we show that similar results hold when replacing the Hilbert-Schmidt norm with the trace norm.

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I. INTRODUCTION

Although the existence of quantum correlations more general than entanglement has been known for some time [1–3], they have begun to attract increasing interest only after the recent suggestion that they might constitute key resources for quantum information and computation tasks, such as the computational speedup in the model of deterministic quantum computation with one pure qubit (DQC1) [4]. In this model, the use of a mixed separable state appears to allow for the efficient, i.e., polynomial time, computation of the trace of any n-qubit unitary matrix [5], which is a problem believed to fall in the NP class on a classical computer [6,7]. Given the absence of entanglement, and assuming the essential nonclassicality of the protocol, this has led to the suggestion that a particular measure of bipartite quantum correlations, the quantum discord [1], is the figure of merit for quantum computation with mixed states [8]. Despite much progress, the issue is, however, not yet conclusively settled [9–12]. More recently, various operational interpretations of the quantum discord and other measures of quantum correlations have been established [10,13–20]. Quantum discord in its entropic definition, i.e., as the difference between two classically equivalent forms of mutual information [1], has been given its first information-theoretic operational meaning in terms of entanglement consumption in an extended quantum-state-merging protocol. Its asymmetry, i.e., the fact that, in general, the discord between parties A and B given that party A is measured is different from the discord given that party B is measured has been related to the performance imbalance in quantum state merging and dense coding [15]. The quantum discord has also been shown to be equal to the minimal partial distillable entanglement, that is, the part of entanglement that is lost when one ignores the subsystem, which is not measured in a local projective measurement [16]. Finally, a different measure of nonclassicality, i.e., the relative entropy of quantumness, has been shown to be equivalent to the minimum distillable entanglement generated between a system and local ancillae in a suitably devised activation protocol [17].

Notwithstanding this recent progress, several fundamental questions on the nature and properties of quantum correlations are yet to be addressed. Among them, a conceptually appealing one is determining a unified mathematical framework for the quantification of entanglement and quantumness. Such a framework would allow one to devise a basic physical interpretation of quantum correlations and formulate sharp quantitative questions on the ensuing measure of nonclassicality, such as the definition and properties of maximally quantum-correlated states. In the present work, we define a distance-based measure of quantumness that for pure states reduces to a particular distance-based measure of entanglement, the so-called stellar entanglement [21,22]. The latter associates pure-state bipartite entanglement to the minimal change of a state induced by local unitary operations. It is a *bona fide* entanglement monotone for $M \times N$-dimensional composite quantum systems and extends to mixed states via the convex roof construction. Indeed, the research program on the global effects of local unitary operations acting on composite quantum systems has turned out to be fruitful in the investigation of various other issues [23,24], including the quantification of measurement-induced nonlocality [25] and the theory and applications of ground-state factorization in the study of complex quantum systems [26–28]. Very recently, the possibility of quantifying quantum correlations via the effect of local unitary operations has been discussed in Ref. [29].

In the present work, we shall show that the minimal disturbance on mixed bipartite quantum states under the action of local unitary (Hamiltonian) time evolutions on only one of the parties defines a faithful measure of quantum correlations vanishing if and only if the state is classically correlated and reducing to the stellar entanglement for pure states. This measure enjoys a clear physical interpretation in terms of the impact power of local unitary time evolutions, i.e., the ability to induce a global state change. Moreover, at least for two-qubit systems, it coincides with the geometric measure of discord defined as the distance from the set of classically correlated states using the Hilbert-Schmidt norm [9]. In the case of two-qubit systems and for any value of the global state purity, we find that the measure is maximized by the class of two-qubit Werner states. Furthermore, for the general case of $m \times n$-dimensional systems, we show that the impact power

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is an upper bound to the geometric discord. Finally, we will briefly comment on the extension of the present investigation when the Hilbert-Schmidt norm is replaced by other norms.

II. QUANTIFYING QUANTUM CORRELATIONS BY LEAST PERTURBING LOCAL UNITARY EVOLUTIONS

Let us begin by considering a bipartite quantum system composed of two subsystems, A and B, so that the Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. Under the evolution driven by a local Hamiltonian H_A acting on only subsystem A, the global density matrix ρ^{AB} evolves according to the unitary Schrödinger dynamics,

$$\rho^{AB}(t) = e^{-iH_At}\rho^{AB}e^{iH_At}. \tag{1}$$

In order to quantify the effect of such a local unitary time evolution on any given initial global state, we define the *impact* of the Hamiltonian H_A as the Hilbert-Schmidt distance between the evolved state at time t and the initial state,

$$I(\rho^{AB},H_A,t) = \tfrac{1}{2}\|\rho^{AB}(t) - \rho^{AB}\|^2, \tag{2}$$

where $\|\rho - \sigma\|^2 = \mathrm{Tr}[(\rho - \sigma)^2]$ is the Hilbert-Schmidt distance. The impact vanishes if the time evolution does not affect the initial state, as in the trivial cases in which either $t = 0$ or $H_A \propto \mathbb{1}_A$. On the other hand, it can never exceed unity, as can be seen by noticing that for any two arbitrarily chosen quantum states ρ and γ, one has $\frac{1}{2}\|\rho - \gamma\|^2 = \frac{1}{2}(\mathrm{Tr}[\rho^2] + \mathrm{Tr}[\gamma^2] - 2\mathrm{Tr}[\rho\gamma]) \leqslant \frac{1}{2}(\mathrm{Tr}[\rho^2] + \mathrm{Tr}[\gamma^2]) \leqslant 1$. The above inequality also implies that the impact reaches unity if and only if the time evolution driven by H_A takes an initial pure state into another pure state orthogonal to it.

Given the Hamiltonian H_A and the initial state ρ^{AB}, we aim to determine the maximum possible value of the impact I with respect to time t. Hence, we introduce the *impact power* P of a Hamiltonian H_A with respect to the initial state ρ^{AB},

$$P(\rho^{AB},H_A) = \max_t I(\rho^{AB},H_A,t). \tag{3}$$

If H_A is trivial, i.e., $H_A \propto \mathbb{1}_A$, then $P(\rho^{AB},H_A) \equiv 0$. Let us consider the case in which A is a qubit while B can be any d-dimensional system. Any nontrivial local Hamiltonians H_A can then be written as $H_A = E_0\Pi_0^A + E_1\Pi_1^A$, where $E_0 \neq E_1$ are the two nondegenerate energy eigenvalues and Π_i^A are the orthogonal projectors onto the two energy eigenstates $|0\rangle$ and $|1\rangle$. With this expression of H_A, the impact power reads

$$P(\rho^{AB},H_A) = \max_t\{a - b\cos(\Delta E t)\}, \tag{4}$$

where the energy gap $\Delta E = E_1 - E_0$ and the time-independent quantities a and b are

$$a = \mathrm{Tr}[(\rho^{AB})^2] - \mathrm{Tr}\left[\rho^{AB}\sum_{i=0}^{1}\Pi_i^A\rho^{AB}\Pi_i^A\right], \tag{5}$$

$$b = 2\mathrm{Tr}\left[\rho^{AB}\Pi_1^A\rho^{AB}\Pi_0^A\right]. \tag{6}$$

Notice that b is non-negative, since it can be written as $2\mathrm{Tr}[XX^\dagger]$ with $X = \Pi_0^A\rho^{AB}\Pi_1^A$. The fact that a and b are constants and $b \geqslant 0$ implies that the impact reaches its maximum $a + b$ at times $t_{\max}^{(k)} = \frac{(2k+1)\pi}{\Delta E}$, with k integer. Exploiting completeness, $\sum_i \Pi_i^A = \mathbb{1}_A$, one has $\mathrm{Tr}[(\rho^{AB})^2] = \mathrm{Tr}[\rho^{AB}(\Pi_0^A + \Pi_1^A)\rho^{AB}(\Pi_0^A + \Pi_1^A)]$. As a consequence, $a = b$. Indeed, this result can be obtained straightforwardly from Eq. (4) by setting $t = 0$ and reminding that at $t = 0$, it must be $P = 0$. Exploiting the equality $a = b$, we then have

$$P(\rho^{AB},H_A) = 2\left\{\mathrm{Tr}[(\rho^{AB})^2] - \mathrm{Tr}\left[\rho^{AB}\sum_{i=0}^{1}\Pi_i^A\rho^{AB}\Pi_i^A\right]\right\}. \tag{7}$$

The impact power P cannot exceed unity and one has strictly $P < 1$ if the initial state is mixed. By maximizing over all H_A, we can define the maximal possible impact power for any given initial state ρ^{AB} as $P_{\max}(\rho^{AB}) = \max_{H_A} P(\rho^{AB},H_A)$. From this definition, it follows immediately that $P_{\max}(\rho^{AB}) < 1$ for all mixed states. On the other hand, it is known that an initial *pure* state is a product state if and only if there exists at least one local unitary traceless operation that leaves it invariant [21,22]. For any given initial state ρ^{AB}, we can then introduce the smallest possible impact power $P_{\min}(\rho^{AB})$, defined by minimizing P over all local Hamiltonians that are not proportional to the identity,

$$P_{\min}(\rho^{AB}) = \min_{H_A \neq \alpha\mathbb{1}_A} P(\rho^{AB},H_A). \tag{8}$$

It is evident from the definition that $P_{\min}(\rho^{AB})$ vanishes if and only if ρ^{AB} is a product pure state. For entangled pure states, $P_{\min}(\rho^{AB})$ cannot vanish because, due to the presence of the entanglement, any local perturbation acting on a subsystem will affect the entire system. Starting from this result, when we move from the case of pure entangles states to that of mixed nonclassical states, we find a similar behavior, but for the important difference that the role previously played by the entanglement is now played by the quantum correlations. Indeed, we will now show that $P_{\min}(\rho^{AB})$ is directly related to a well-defined measure of bipartite quantum correlations, that is, the geometric measure of discord $D_A^{(2)}(\rho^{AB})$ [9], defined as

$$D_A^{(2)}(\rho^{AB}) = \min_{\omega^{AB}\in CQ}\|\rho^{AB} - \omega^{AB}\|^2. \tag{9}$$

In the definition of the geometric discord, the minimization is taken over the set CQ of all classically correlated states, that is, states of the form $\omega^{AB} = \sum_i p_i\,|i\rangle\,\langle i|^A \otimes \omega_i^B$, where ω_i^B is a state on subsystem B. Using Eq. (7) together with the equality $\mathrm{Tr}[\rho^{AB}\sum_{i=0}^{1}\Pi_i^A\rho^{AB}\Pi_i^A] = \mathrm{Tr}[(\sum_{i=0}^{1}\Pi_i^A\rho^{AB}\Pi_i^A)^2]$, one can immediately verify by inspection that for any nondegenerate single-qubit Hamiltonian $H_A = E_0\Pi_0^A + E_1\Pi_1^A$, the impact power can be written as $P(\rho^{AB},H_A) = 2\|\rho^{AB} - \sum_{i=0}^{1}\Pi_i^A\rho^{AB}\Pi_i^A\|^2$. This implies the following order relation between the impact power and the geometric measure of discord:

$$P(\rho^{AB},H_A) \geqslant 2D_A^{(2)}(\rho^{AB}). \tag{10}$$

Equation (10) shows that the change in the global state due to a local unitary dynamics is bounded from below by the geometric measure of discord and hence cannot vanish in the presence of quantum correlations. Actually, one can prove a much stronger relation between the minimum impact power $P_{\min}$, which henceforth will be named the *impact power gap*, and the geometric measure of discord according to the following theorem:

Theorem 1. If ρ^{AB} is a state of a bipartite system, where subsystem A is a qubit, then the impact power gap $P_{\rm min}$ is given by

$$P_{\rm min}(\rho^{AB}) = 2D_A^{(2)}(\rho^{AB}). \tag{11}$$

Proof. We will prove this equality by identifying a Hamiltonian which explicitly minimizes the impact power $P(\rho^{AB},H_A)$. To this end, it is useful to recall that the geometric measure of discord is related to local von Neumann measurements, with local projectors Π_i^A, according to the following [32]:

$$D_A^{(2)}(\rho^{AB}) = \min_{\{\Pi_i^A\}} \left\| \rho^{AB} - \sum_i \Pi_i^A \rho^{AB} \Pi_i^A \right\|^2 . \tag{12}$$

Now let $\hat{\Pi}_0^A$ and $\hat{\Pi}_1^A$ be the projectors that achieve the minimum and consider the Hamiltonian $H_A = E_0\hat{\Pi}_0^A + E_1\hat{\Pi}_1^A$ with nondegenerate spectrum $E_1 \neq E_0$. Evaluating the impact power of H_A along the same lines discussed in the cases above yields $P(\rho^{AB},H_A) = 2D_A^{(2)}(\rho^{AB})$.

Theorem 1 exemplifies the relation between the impact power gap and quantum correlations (see also Fig. 1). If subsystem A is a qubit, then $P_{\rm min}$ can be computed explicitly by exploiting Theorem 1 and the explicit expression for $D_A^{(2)}$ provided in Refs. [9,30]. In fact, we can go one step further and provide independent closed expressions both for $P_{\rm min}$ and for the maximal impact power $P_{\rm max}$ in terms of the global state *purity*:

Theorem 2. If system A is a qubit, then the maximal impact power $P_{\rm max}$ reads

$$P_{\rm max}(\rho^{AB}) = {\rm Tr}[(\rho^{AB})^2] - m_{\rm min}, \tag{13}$$

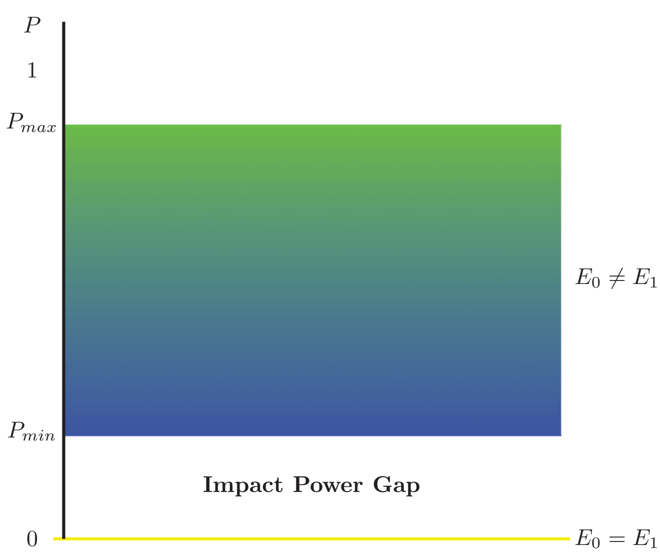

FIG. 1. (Color online) Possible values of the impact power P for an arbitrary initial state ρ^{AB}. The impact power is zero if the spectrum of the local Hamiltonian H_A is degenerate: $E_0 = E_1$ (yellow line). For $E_0 \neq E_1$, the impact power can only take values between $P_{\rm min}$ and $P_{\rm max}$ (green-blue area). The impact power gap is the region between 0 and $P_{\rm min}$. Its width is measured by the amount of quantum correlations present in the initial state ρ^{AB}, as measured by the geometric measure of discord: $P_{\rm min} = 2D_A^{(2)}$. See main text for details.

where $m_{\rm min}$ is the smallest eigenvalue of the matrix M with elements $M_{ij} = {\rm Tr}[\rho^{AB}\sigma_i^A\rho^{AB}\sigma_j^A]$, where σ_i^A (with $i = x,y,z$) are the Pauli operators of subsystem A. Moreover, given the largest eigenvalue $m_{\rm max}$ of the matrix M, the impact power gap $P_{\rm min}$ reads

$$P_{\rm min}(\rho^{AB}) = {\rm Tr}[(\rho^{AB})^2] - m_{\rm max}. \tag{14}$$

Proof. Since the impact power is identically vanishing if the single-qubit Hamiltonian H_A is degenerate, we need consider only the nondegenerate case. The unitary operator $U_A = e^{iH_A t_{\rm max}^{(0)}}$ is then traceless with the spectrum composed of the two complex roots of the unity. Let us recall Eq. (7) for the impact power $P(\rho^{AB},H_A)$ and the fact that we can always rewrite a local unitary operator in the form $U_A = \Pi_0^A - \Pi_1^A$. We can then express the impact power as follows:

$$P(\rho^{AB},H_A) = {\rm Tr}[(\rho^{AB})^2] - {\rm Tr}[\rho^{AB}U_A\rho^{AB}U_A^\dagger]. \tag{15}$$

Using the Bloch representation to write the projectors as $\Pi_0^A = \frac{1}{2}(\mathbb{1}_A + \sum_i r_i\sigma_i^A)$ and $\Pi_1^A = \frac{1}{2}(\mathbb{1}_A - \sum_i r_i\sigma_i^A)$, the unitary operator U_A in Eq. (15) takes the form $U_A = \Pi_0^A - \Pi_1^A = \sum_i r_i\sigma_i^A$. The final expression for the impact power becomes

$$P(\rho^{AB},H_A) = {\rm Tr}[(\rho^{AB})^2] - \sum_{i,j} r_i M_{ij} r_j, \tag{16}$$

where we defined the matrix M with the elements $M_{ij} = {\rm Tr}[\rho^{AB}\sigma_i^A\rho^{AB}\sigma_j^A]$. It is easy to see that M is symmetric, since $M_{ij} = M_{ji}$. Moreover, all entries of M are real. This implies that in order to compute $P_{\rm max}$, we have to minimize $\boldsymbol{r}^T M \boldsymbol{r}$ over all unit vectors $\boldsymbol{r}$ for a real symmetric matrix M. This problem is solved by finding the smallest eigenvalue of M [31]. The impact power gap $P_{\rm min}$ can be computed similarly by considering the largest eigenvalue of M.

By continuity in the Bloch vector $\boldsymbol{r}$, the impact power $P(\rho^{AB},H_A)$ may assume any real value in the range $[P_{\rm min},P_{\rm max}]$.

III. MAXIMALLY QUANTUM-CORRELATED TWO-QUBIT STATES

Equipped with these results, we can look for the class of states that, at fixed global purity, maximizes the impact power gap and thus the quantum correlations. When both subsystems are qubits ($d_A = d_B = 2$), the following theorem holds:

Theorem 3. For any state ρ^{AB} of two qubits,

$$P_{\rm min}(\rho^{AB}) \leqslant \tfrac{4}{3}{\rm Tr}[(\rho^{AB})^2] - \tfrac{1}{3}, \tag{17}$$

with equality achieved by the Werner states ρ_w.

Proof. In the Bloch sphere representation, any arbitrary two-qubit state can be written as

$$\rho^{AB} = \frac{1}{4}\left(\mathbb{1}\otimes\mathbb{1} + \sum_i x_i\sigma_i\otimes\mathbb{1} + \sum_i y_i\mathbb{1}\otimes\sigma_i + \sum_{ij} T_{ij}\sigma_i\otimes\sigma_j\right), \tag{18}$$

and the state purity ${\rm Tr}[(\rho^{AB})^2]$ can be expressed as ${\rm Tr}[(\rho^{AB})^2] = \frac{1}{4}(1 + \boldsymbol{x}^2 + \boldsymbol{y}^2 + \|T\|^2)$. By tracing out the first or second qubit, the purities of the reduced states are, respectively, $\mathrm{Tr}[(\rho^B)^2] = \frac{1}{2}(1+\boldsymbol{y}^2)$ and $\mathrm{Tr}[(\rho^A)^2] = \frac{1}{2}(1+\boldsymbol{x}^2)$. Using representation (18), it is possible to evaluate the geometric measure of discord for any two-qubit state [9], and hence the expression for $P_{\min}$:

$$P_{\min}(\rho^{AB}) = \tfrac{1}{2}(\boldsymbol{x}^2 + \|T\|^2 - k_{\max}), \tag{19}$$

where $k_{\max}$ is the largest eigenvalue of the matrix $K = \boldsymbol{x}\boldsymbol{x}^T + TT^T$, and $\|T\|^2 = \mathrm{Tr}[T^T T]$. Since $k_{\max}$ is the largest eigenvalue of the 3×3 matrix K, we have that $3k_{\max} \geqslant \boldsymbol{x}^2 + \|T\|^2$. Using this inequality in Eq. (19) and taking into account the expressions of the global and reduced purities, we have

$$\begin{aligned} P_{\min}(\rho^{AB}) &\leqslant \tfrac{1}{3}(\boldsymbol{x}^2 + \|T\|^2) \\ &= \tfrac{4}{3}\big(\mathrm{Tr}[(\rho^{AB})^2] - \tfrac{1}{2}\mathrm{Tr}[(\rho^B)^2]\big). \end{aligned} \tag{20}$$

Finally, noticing that for a single-qubit state the purity cannot be smaller than $\frac{1}{2}$, we arrive at inequality (17). On the other hand, a generic two-qubit Werner state can be written as $\rho_w = \frac{2-x}{6}\mathbb{1} + \frac{2x-1}{6}F$, where $x \in [-1,1]$ and $F = \sum_{k,l} |k\rangle\langle l| \otimes |l\rangle\langle k|$ is the permutation operator. For such a state, the purity is given by $\mathrm{Tr}[\rho_w^2] = \frac{1}{3}(x^2 - x + 1)$, while the geometric measure of discord reads [32] $D_A^{(2)}(\rho_w) = \frac{(2x-1)^2}{18}$. Recalling the relation between the impact power gap and the geometric discord, one has that inequality (17) is saturated by the Werner states. Werner states are thus maximally quantum-correlated two-qubit states at fixed global purity.

We could not yet clarify whether the Werner states are the only ones maximizing the two-qubit quantum correlations. Some preliminary analysis suggests that other classes of highly symmetric states, such as the isotropic states, might also saturate the bound given by Eq. (17).

IV. CONCLUSIONS AND OUTLOOK: DIFFERENT NORMS, HIGHER DIMENSIONS

In order to investigate systems with larger local dimension $d_A > 2$, we generalize our approach considering the fully nondegenerate local Hamiltonians of the form $H_A = \sum_{i=0}^{d_A-1} E_i \Pi_i^A$, with spectrum $E_i \neq E_j \,\forall\, i \neq j$. Following the same route of reasoning as in the qubit case, we find that the impact power of H_A over an arbitrary initial state ρ^{AB} can be expressed as

$$P(\rho^{AB}, H_A) = \max_t \left\{ a - \sum_{l>k} b_{lk} \cos(\Delta E_{lk} t) \right\}, \tag{21}$$

where $\Delta E_{lk} = E_l - E_k$, and the coefficients a and b_{lk} are

$$a = \mathrm{Tr}[(\rho^{AB})^2] - \mathrm{Tr}\left[\rho^{AB} \sum_{i=0}^{d_A-1} \Pi_i^A \rho^{AB} \Pi_i^A\right], \tag{22}$$

$$b_{lk} = 2\mathrm{Tr}\big[\rho^{AB}\Pi_l^A \rho^{AB} \Pi_k^A\big]. \tag{23}$$

Taking into account that $a = \sum_{l>k} b_{lk}$, we arrive at

$$P(\rho^{AB}, H_A) = \max_t \left\{ \sum_{l>k} b_{lk}\,[1 - \cos(\Delta E_{lk} t)] \right\}. \tag{24}$$

Since $P(\rho^{AB}, H_A) \geqslant \sum_{l>k} b_{lk}[1 - \cos(\Delta E_{lk} t)]$ for all times $t \neq t_{\max}$, it follows that $P(\rho^{AB}, H_A) \geqslant 2\max_{l>k} b_{lk}$. Using the fact that $a = \sum_{l>k} b_{lk} \leqslant N \max_{l>k} b_{lk}$, we obtain that $\max_{l>k} b_{lk} \geqslant \frac{1}{N}\sum_{l>k} b_{lk} = \frac{a}{N}$, where $N = (d_A - 1)d_A/2$ is the number of different b_{lk} terms. Collecting these results and recalling the definition of the geometric measure of discord $D_A^{(2)}(\rho^{AB})$, we find that the impact power of any nondegenerate, finite-dimensional local Hamiltonian H_A is bounded from below by a simple linear function of the geometric measure of discord,

$$P(\rho^{AB}, H_A) \geqslant \frac{4D_A^{(2)}(\rho^{AB})}{d_A(d_A - 1)}. \tag{25}$$

From Eq. (25), in complete analogy with the qubit case, it follows that if the initial state has vanishing quantum correlations, then there always exists at least one nontrivial local Hamiltonian H_A with vanishing impact power. Therefore, a nonvanishing impact power implies and quantifies a nonvanishing degree of quantumness, regardless of the local Hilbert-space dimension of party A.

It is worth noticing that while throughout this paper we have made use of the Hilbert-Schmidt norm, we are by no means limited to this choice. Similar conclusions hold as well for the trace distance, which is directly related to the distinguishability of quantum states [33]. Indeed, given two density matrices ρ and ω, their squared trace distance is $(\mathrm{Tr}[\sqrt{(\rho-\omega)^2}])^2 = (\sum_i |\lambda_i|)^2$, where the $\{\lambda_i\}$ are the eigenvalues of $(\rho - \omega)$. This quantity is obviously always larger than or equal to the squared Hilbert-Schmidt distance $\mathrm{Tr}[(\rho-\omega)^2] = \sum_i \lambda_i^2$. Therefore, an impact power gap for quantum correlated states exists also in the case in which we replace the Hilbert-Schmidt distance with the trace distance, and hence similar results can be obtained also in this case. As the latter is monotonic under general stochastic maps, this result is relevant in light of a recent observation [34] that due to the fact that the Hilbert-Schmidt distance is not monotonic under stochastic maps, some reversible operations on unmeasured subsystem B can change the value of the quantum correlations.

In conclusion, we have established that all of the quantum correlated states of bipartite quantum systems exhibit a nonvanishing impact power gap, i.e., a nonvanishing minimal change under the action of *any* nontrivial local Hamiltonian. On the contrary, for every classically correlated state, there exists at least one particular nontrivial local unitary operation that leaves the state unchanged. Starting from this observation, we have quantified this global change via the Hilbert-Schmidt distance and showed that the minimal distance achieved along the local time evolution is proportional to the amount of quantum correlations quantified via the geometric measure of discord. Moreover, for two-qubit systems at fixed global purity, we have verified explicitly that Werner states maximize the impact power gap and thus the amount of quantum correlations. We have mainly used as the measure of the effect of the local unitary operations the Hilbert-Schmidt metrics; however, we have shown that similar results can be obtained also using the trace distance. On the other hand, it is expected that the detailed structure of the quantification of nonclassicality and the characterization of maximally quantum-correlated states using the formalism of least-perturbing local unitary operations will depend to some extent on the choice of the metric inducing the distance between quantum states. In this respect, the choice of the Bures metric, which is at the same time monotonic and Riemannian, seems to be the most appropriate one, also in light of the fundamental operational meaning that stems from its intimate relation with the Uhlmann fidelity. The general structure of distance-based measures of quantumness associated to least-perturbing local unitary operations defined via different norms (Bures, trace, and Hilbert-Schmidt) and their detailed comparison are the subject of ongoing investigations and we hope to report on them in the near future [35].

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