Behavior of Quantum Correlations under Local Noise

Alexander Streltsov,* Hermann Kampermann, and Dagmar Bruß
Heinrich-Heine-Universität Düsseldorf, Institut für Theoretische Physik III, D-40225 Düsseldorf, Germany
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We characterize the behavior of quantum correlations under the influence of local noisy channels. Intuition suggests that such noise should be detrimental for quantumness. When considering qubit systems, we show for which channels this is indeed the case: The amount of quantum correlations can only decrease under the action of unital channels. However, nonunital channels (e.g., such as dissipation) can create quantum correlations for some initially classical states. Furthermore, for higher-dimensional systems even unital channels may increase the amount of quantum correlations. Thus, counterintuitively, local decoherence can generate quantum correlations.

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Composite quantum states often reveal puzzling features of nature. Recently, much interest [1] has been devoted to the study of quantum correlations that may arise without entanglement: Here, the quantumness of a composite system manifests itself even in a separable state. The fact that such quantum correlations are present [2] in an algorithm for mixed state quantum computing [3] has stimulated intensive investigations into measures for quantum correlations [4–13] and their properties and interpretations [14–28]. Experimental detection of quantum correlations beyond entanglement is also receiving more and more attention [29]. Some studies of the dynamics of quantum correlations have been presented in Refs. [30–34]. The importance of quantum correlations beyond entanglement is also highlighted by the task of efficiently locking classical correlations in quantum states [35]. There, two parties can arbitrarily increase their classical correlations by sending only one classical bit. The fact that no entanglement is needed in this process leads to the conclusion that other types of correlations are responsible for this phenomenon. Understanding fundamental properties of such correlations is the aim of this Letter.

An appeal of mixed state quantum computation lies in the possibility to be run in a noisy environment: Pure entangled states are typically fragile, and the resource of entanglement is easily destroyed by noise. For an open system the transition from entangled to separable states is only a matter of time—as the volume of the set of separable states is nonzero [36], typically it takes a finite time for entanglement to disappear under noise such as dissipation or decoherence [37].

Mixed state quantum computation as suggested in Ref. [3] already uses separable states, so it is natural to assume that it can be run also in a noisy environment. However, in order to verify or falsify this conjecture, one has to study the behavior of quantum correlations under noisy channels (described by trace-preserving completely positive maps). Here we consider only local noisy channels—as correlated channels may also preserve entanglement (with or even without some degradation, depending on the amount of correlation); see, e.g., [38]. The goal of this Letter is to answer such questions as the following: Which types of noisy channels decrease the amount of quantum correlations? Are there any noisy channels that might even increase the amount of quantum correlations? How does dissipation influence quantum correlations, and how are they affected by decoherence? We point out that our answers to these questions also apply to the situation where one actively performs local operations on a composite quantum system, e.g., with the aim of creating or preserving quantum correlations.

In general, a bipartite quantum state is called fully classically correlated [39] if it can be written in the form [6,7]

$$\rho_{cc} = \sum_{i,j} p_{ij} |i^A\rangle\langle i^A| \otimes |j^B\rangle\langle j^B|, \quad (1)$$

where $\{|i^A\rangle\}$ and $\{|j^B\rangle\}$ are sets of orthogonal states of party A and B, respectively, with non-negative probabilities p_{ij} that add up to 1. If a state cannot be written as in Eq. (1), it is called quantum correlated. These definitions can be extended to any number of parties [13].

As a simple example, consider the classically correlated state of two qubits $\rho_{cc} = \frac{1}{2}|0^A\rangle\langle 0^A| \otimes |0^B\rangle\langle 0^B| + \frac{1}{2}|1^A\rangle \times \langle 1^A| \otimes |1^B\rangle\langle 1^B|$. By using a local channel on qubit A only, it is possible to create the quantum correlated state

$$\rho = \tfrac{1}{2}|0^A\rangle\langle 0^A| \otimes |0^B\rangle\langle 0^B| + \tfrac{1}{2}|+^A\rangle\langle +^A| \otimes |1^B\rangle\langle 1^B| \quad (2)$$

with $|+^A\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle)$. The quantum channel that achieves this transformation can be formally written as the completely positive trace-preserving map

$$\rho = \Lambda_A(\rho_{cc}) = E_1 \rho E_1^\dagger + E_2 \rho E_2^\dagger \quad (3)$$

with local Kraus operators $E_1 = |0^A\rangle\langle 0^A|$ and $E_2 = |+^A\rangle\langle 1^A|$ acting only on qubit A. The state in Eq. (2) is not of the form (1); i.e., it is quantum correlated.

As will become clear below in this Letter, one reason why the local quantum channel in Eq. (3) is able to create quantum correlations lies in its action on the maximally mixed state $\frac{1}{2}\mathbb{1}_A$. Observe that $\Lambda_A(\frac{1}{2}\mathbb{1}_A) = \frac{1}{2}|0^A\rangle\langle 0^A| + \frac{1}{2}|+^A\rangle\langle +^A| \neq \frac{1}{2}\mathbb{1}_A$. This property is also known as nonunitality. A single-qubit quantum channel Λ is called unital if and only if it maps the maximally mixed state onto itself: $\Lambda(\frac{1}{2}\mathbb{1}) = \frac{1}{2}\mathbb{1}$; see also Fig. 1. We will turn this observation into Theorem 1 by showing that nonunitality is one property which enables a local channel to create quantum correlations in a multiqubit system. In Theorem 2, we will show that, on the other hand, local unital quantum channels cannot increase quantum correlations in a multiqubit system. However, this statement does not hold for higher dimensions.

Before presenting the main result of this Letter, we introduce the semiclassical channel Λ_{sc}. It maps all input states ρ onto states $\Lambda_{sc}(\rho)$ which are diagonal in the same basis: $\Lambda_{sc}(\rho) = \sum_k p_k(\rho)|k\rangle\langle k|$. The non-negative probabilities $p_k(\rho)$ can, in general, depend on the input state ρ, while the orthogonal states $|k\rangle$ are independent of ρ. Such a channel is, e.g., realized by complete decoherence, after which only the diagonal elements of a density matrix may be nonzero. Channels of this form were also considered in Ref. [40], where they were called measurement maps. We are now in the position to prove the following theorem.

Theorem 1.—A local quantum channel acting on a single qubit can create quantum correlations in a multiqubit system if and only if it is neither semiclassical nor unital.

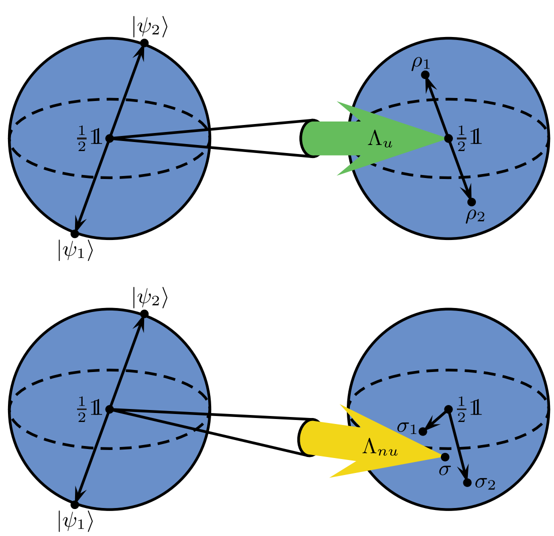

FIG. 1 (color online). Quantum channels on a single qubit: The upper figure shows a unital quantum channel Λ_u (green arrow) which maps the maximally mixed state $\frac{1}{2}\mathbb{1}$ onto itself: $\Lambda_u(\frac{1}{2}\mathbb{1}) = \frac{1}{2}\mathbb{1}$. Two orthogonal states $|\psi_1\rangle$ and $|\psi_2\rangle$ with collinear Bloch vectors are mapped onto the states $\rho_1 = \Lambda_u(|\psi_1\rangle\langle\psi_1|)$ and $\rho_2 = \Lambda_u(|\psi_2\rangle\langle\psi_2|)$ with collinear Bloch vectors. The lower figure shows a nonunital quantum channel Λ_{nu} (yellow arrow) which maps the maximally mixed state onto the state $\sigma = \Lambda_{nu}(\frac{1}{2}\mathbb{1}) \neq \frac{1}{2}\mathbb{1}$. The Bloch vectors of $\sigma_1 = \Lambda_{nu}(|\psi_1\rangle\langle\psi_1|)$ and $\sigma_2 = \Lambda_{nu}(|\psi_2\rangle\langle\psi_2|)$ add up to twice the nonzero Bloch vector of σ; see the main text.

Proof.—For simplicity, we restrict ourselves to two qubits only. A generalization to an arbitrary number of qubits is straightforward. The action of a local semiclassical channel Λ_{sc}^A on the classically correlated state (1) is, due to linearity, $\Lambda_{sc}^A(\rho_{cc}) = \sum_{i,j} p_{ij}\Lambda_{sc}^A(|i^A\rangle\langle i^A|) \otimes |j^B\rangle\langle j^B|$. The definition of a semiclassical channel directly implies that $\Lambda_{sc}^A(\rho_{cc})$ is classically correlated.

Now we will show that a local unital channel never creates quantum correlations in a multiqubit system. A local unital channel Λ_u^A on the qubit A takes a classically correlated state to the state $\Lambda_u^A(\rho_{cc}) = \sum_{i,j} p_{ij}\Lambda_u^A(|i^A\rangle\langle i^A|) \otimes |j^B\rangle\langle j^B|$. The action of the unital channel on the pure state $|i^A\rangle\langle i^A|$ can be studied by using the Bloch representation: $|0^A\rangle\langle 0^A| = \frac{1}{2}(\mathbb{1}_A + \sum_i r_i\sigma_i^A)$, where σ_i^A are the Pauli operators with $i \in \{x, y, z\}$ and $|1^A\rangle\langle 1^A| = \frac{1}{2}(\mathbb{1}_A - \sum_i r_i\sigma_i^A)$. Using linearity and unitality of Λ_u^A, we see that the state $|0^A\rangle\langle 0^A|$ is mapped onto the state $\rho_0^A = \Lambda_u^A(|0^A\rangle\langle 0^A|) = \frac{1}{2}[\mathbb{1}_A + \sum_i r_i\Lambda_u^A(\sigma_i^A)]$. The same procedure for $|1^A\rangle\langle 1^A|$ results in $\rho_1^A = \Lambda_u^A(|1^A\rangle\langle 1^A|) = \frac{1}{2}[\mathbb{1}_A - \sum_i r_i\Lambda_u^A(\sigma_i^A)]$. Note that the Bloch vectors of the states ρ_0^A and ρ_1^A point into opposite directions; see Fig. 1 for illustration. States with this property can be diagonalized in the same basis. This implies that it is possible to write the state $\Lambda_u^A(\rho_{cc})$ in the form (1). Thus we proved that local unital quantum channels cannot create quantum correlations in a classically correlated multiqubit state.

In the following, we will complete the proof of Theorem 1 by showing that any local quantum channel Λ_{nu}^A that is neither unital nor semiclassical can create quantum correlations. By definition, Λ_{nu}^A maps the maximally mixed state $\frac{1}{2}\mathbb{1}_A$ onto some state that is not maximally mixed: $\Lambda_{nu}^A(\frac{1}{2}\mathbb{1}_A) = \frac{1}{2}(\mathbb{1}_A + \sum_i s_i\sigma_i^A)$, with $\sum_i s_i^2 \neq 0$. Since we demand that the quantum channel is not semiclassical, there exists a state ρ^A such that $\Lambda_{nu}^A(\rho^A)$ is not diagonal in the eigenbasis of $\Lambda_{nu}^A(\frac{1}{2}\mathbb{1}_A)$. Again we consider the Bloch representation $\Lambda_{nu}^A(\rho^A) = \frac{1}{2}(\mathbb{1}_A + \sum_j r_j\sigma_j^A)$ and note that the two Bloch vectors $\mathbf{r}$ and $\mathbf{s}$ are linearly independent. Otherwise, the states $\Lambda_{nu}^A(\rho^A)$ and $\Lambda_{nu}^A(\frac{1}{2}\mathbb{1}_A)$ could be diagonalized in the same basis, which is in contradiction to the definition of ρ^A. We can write the state as $\rho^A = \frac{1}{2}(\mathbb{1}_A + \sum_i v_i\sigma_i^A)$. Consider now the classically correlated state $\rho_{cc} = \frac{1}{2}\rho^A \otimes |0^B\rangle\langle 0^B| + \frac{1}{2}\tau^A \otimes |1^B\rangle\langle 1^B|$ with $\tau^A = \frac{1}{2}(\mathbb{1}_A - \sum_i v_i\sigma_i^A)$. We define the vector $\mathbf{w}$ such that the equality $\Lambda_{nu}(\sum_i v_i\sigma_i^A) = \sum_i w_i\sigma_i^A$ with $\sum_i w_i^2 \neq 0$ is satisfied. This is always possible, since Λ_{nu}^A is trace-preserving. The action of the channel onto the two states ρ^A and τ^A is as follows: $\Lambda_{nu}^A(\rho^A) = \frac{1}{2}[\mathbb{1}_A + \sum_i(s_i + w_i)\sigma_i^A]$ and $\Lambda_{nu}^A(\tau^A) = \frac{1}{2}[\mathbb{1}_A + \sum_i(s_i - w_i)\sigma_i^A]$. As noted above, the two Bloch vectors $\mathbf{s}$ and $\mathbf{r} = \mathbf{s} + \mathbf{w}$ are linearly independent. The same must hold for the vectors $\mathbf{s} + \mathbf{w}$ and $\mathbf{s} - \mathbf{w}$. This implies that the two states $\Lambda_{nu}^A(\rho^A)$ and $\Lambda_{nu}^A(\tau^A)$ are not diagonal in the same basis. This completes the proof. ∎

So far, we have seen that local unital and local semiclassical channels acting on a single qubit cannot create quantum correlations from a classically correlated multiqubit state. These results hold independently of the chosen measure for quantum correlations. In the following, we will go one step further by showing that these local channels never increase a very general class of measures for quantum correlations in multiqubit systems. We consider distance-based measures of quantum correlations Q_D, which are defined via the minimal distance D to the set of the classically correlated states CC [8,9]: $Q_D = \min_{\sigma \in CC} D(\rho, \sigma)$, where D does not necessarily have to be a distance in the mathematical sense. The statement mentioned above will be shown to hold for all distance measures D with the property of being nonincreasing under any quantum channel Λ, i.e., $D(\Lambda(\rho), \Lambda(\sigma)) \leq D(\rho, \sigma)$. This property is also frequently used for defining entanglement measures [41,42].

Theorem 2.—Quantum correlations in multiqubit systems, quantified by a distance-based measure Q_D, do not increase under local unital channels Λ_{lu} and local semiclassical channels Λ_{lsc}:

$$Q_D(\Lambda_{\text{lu}}(\rho)) \leq Q_D(\rho), \tag{4}$$

$$Q_D(\Lambda_{\text{lsc}}(\rho)) \leq Q_D(\rho). \tag{5}$$

Proof.—Let ξ be the classically correlated state which minimizes the distance, i.e., $Q_D(\rho) = D(\rho, \xi)$. Using the property of the distance to be nonincreasing under quantum channels, we obtain $Q_D(\rho) = D(\rho, \xi) \geq D(\Lambda_{\text{lu}}(\rho), \Lambda_{\text{lu}}(\xi))$ and $Q_D(\rho) = D(\rho, \xi) \geq D(\Lambda_{\text{lsc}}(\rho), \Lambda_{\text{lsc}}(\xi))$. Now we use Theorem 1 noting that local unital channels Λ_{lu} and local semiclassical channels Λ_{lsc} map the classically correlated multiqubit state ξ onto another classically correlated state $\Lambda(\xi)$ which is not necessarily the one that minimizes the distance to $\Lambda(\rho)$. This observation finishes the proof. ■

One example for a measure that satisfies the properties (4) and (5)—and thus Theorem 2 holds—is the geometric measure of quantumness, which we define as

$$Q_G(\rho) = \min_{\sigma \in CC} [1 - F(\rho, \sigma)] \tag{6}$$

with the fidelity $F(\rho, \sigma) = (\text{Tr}[\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}])^2$. Using the fact that the fidelity is nondecreasing on quantum channels together with Theorem 2, we see that the geometric measure of quantumness does not increase under local unital channels and local semiclassical channels in multiqubit systems. Alternatively, we can use the quantum relative entropy $S(\rho \| \sigma) = -\text{Tr}[\rho \log_2 \sigma] + \text{Tr}[\rho \log_2 \rho]$, which is also nonincreasing on quantum channels [41,42]. From Theorem 2 follows that the resulting measure of quantum correlations $Q_S = \min_{\sigma \in CC} S(\rho \| \sigma)$ does not increase under local unital and local semiclassical channels in multiqubit systems. Q_S was also studied in Ref. [13], where it was called relative entropy of quantumness.

So far, we have considered states consisting of an arbitrary number of qubits. We have shown that local unital and local semiclassical channels acting on a single qubit never increase quantum correlations as defined by a distance-based measure Q_D, where the minimization is done over all classically correlated multiqubit states. On the other hand, any local channel which is nonunital and not semiclassical can, in principle, create quantum correlations, independently of the considered measure, out of a classically correlated state. An example for such a channel is the amplitude damping channel as a model for dissipation. Thus, dissipation can increase quantum correlations.

At the present stage, it is natural to ask the question, for what kind of input states this behavior can or cannot be observed in general. The following theorem shows that pure states are special.

Theorem 3.—The geometric measure of quantumness of multipartite systems with arbitrary dimension cannot increase under any local quantum channel, if the initial state is pure:

$$Q_G(\Lambda_l(|\psi\rangle\langle\psi|)) \leq Q_G(|\psi\rangle\langle\psi|), \tag{7}$$

where Λ_l is an arbitrary local quantum channel.

Proof.—Let $\xi \in CC$ be defined such that $Q_G(|\psi\rangle \times \langle\psi|) = 1 - F(|\psi\rangle\langle\psi|, \xi)$. Using the properties of the fidelity F, we see that ξ can be chosen to be a pure product state $\xi = |\phi\rangle\langle\phi|$. Moreover, $1 - F$ does not increase under the action of any quantum channel, i.e., $1 - F(|\psi\rangle \times \langle\psi|, |\phi\rangle\langle\phi|) \geq 1 - F(\Lambda_l(|\psi\rangle\langle\psi|), \Lambda_l(|\phi\rangle\langle\phi|))$. Since $|\phi\rangle$ is a product state, $\Lambda_l(|\phi\rangle\langle\phi|)$ is also a product state. This observation completes the proof. ■

So far, we have shown that quantum correlations in multiqubit systems cannot increase under local unital quantum channels. A prominent example for a unital channel is the phase-damping channel, which is a model for decoherence in a quantum system. Under decoherence the quantum state $\rho = \sum_{i,j} \rho_{ij} |i\rangle\langle j|$ is transformed to the state

$$\Lambda(\rho) = \sum_i \rho_{ii} |i\rangle\langle i| + (1 - p) \sum_{i \neq j} \rho_{ij} |i\rangle\langle j| \tag{8}$$

with the damping parameter $0 \leq p \leq 1$. Since Λ is unital, it is not possible to create quantum correlations with local phase damping in a multiqubit system. Surprisingly, this is not true if the local systems are not qubits: Qubits are special. This can be demonstrated via the classically correlated state as input: $\rho_{cc} = \frac{1}{2} |\psi^A\rangle\langle\psi^A| \otimes |0^B\rangle \times \langle 0^B| + \frac{1}{2} |\phi^A\rangle\langle\phi^A| \otimes |1^B\rangle\langle 1^B|$ with the orthogonal single-qutrit states $|\psi^A\rangle = (1/\sqrt{3})(-|0^A\rangle + |1^A\rangle + |2^A\rangle)$ and $|\phi^A\rangle = \frac{1}{\sqrt{2}}(|0^A\rangle + |1^A\rangle)$. We will show that a local phase-damping channel Λ_A acting on subsystem A generates quantum correlations. We consider the action of the channel (8) with the damping parameter $p = \frac{1}{2}$ on the state ρ_{cc}: $\Lambda_A(\rho_{cc}) = \frac{1}{2} \sum_{i=1}^3 \lambda_i |\psi_i^A\rangle\langle\psi_i^A| \otimes |0^B\rangle\langle 0^B| + \frac{1}{2} \sum_{j=1}^3 \mu_j |\phi_j^A\rangle\langle\phi_j^A| \otimes |1^B\rangle\langle 1^B|$, where the states $\{|\psi_i^A\rangle\}$ are the eigenstates of $\Lambda_A(|\psi^A\rangle\langle\psi^A|)$ with the corresponding eigenvalues λ_i. Similarly, the states $\{|\phi_j^A\rangle\}$ are eigenstates of $\Lambda_A(|\phi^A\rangle\langle\phi^A|)$ with the eigenvalues μ_j. One can see as follows that the state $\Lambda_A(\rho_{cc})$ is quantum correlated: The eigenvalues of $\Lambda_A(|\psi^A\rangle\langle\psi^A|)$ are given by $\lambda_1 = \frac{2}{3}$ and $\lambda_2 = \lambda_3 = \frac{1}{6}$. The eigenstate to the largest eigenvalue λ_1 is given by $|\psi_1^A\rangle = |\psi^A\rangle$. It is easy to check that $|\psi_1^A\rangle$ is not an eigenstate of $\Lambda_A(|\phi^A\rangle\langle\phi^A|)$, and therefore the state $\Lambda_A(\rho_{cc})$ is not classically correlated. Thus we proved that it is possible to create quantum correlations with a local phase-damping channel, i.e., via local decoherence.

In conclusion, we have investigated the effect of local noisy channels (i.e., trace-preserving completely positive maps) on quantum correlations. While entanglement can never increase under such local channels, quantum correlations without entanglement may or may not increase, depending on the type of channel and the type of input state. For multiqubit systems, we fully answer the question which local channels can increase quantum correlations: Unital and semiclassical local channels cannot enhance quantum correlations, while nonunital and nonsemiclassical local channels (e.g., dissipation, corresponding to amplitude damping) can increase quantum correlations. Surprisingly, for higher-dimensional systems, even unital channels such as decoherence, corresponding to phase damping, can generate quantum correlations from an initially classically correlated state. However, quantum correlations as quantified by the geometric measure of quantumness can become larger under local channels only when the initial state is mixed. Thus, we have shed some light on the behavior of quantum correlated states in a noisy environment.

We also mention the connection of our approach to the quantum discord; see [4] for a definition. A quantum state has zero quantum discord if it can be written in the classical-quantum form $\rho_{cq} = \sum_i p_i |i^A\rangle\langle i^A| \otimes \rho_i^B$. Note that Theorem 1 also holds in this case, if the subsystem A is a qubit. Moreover, Theorems 2 and 3 also hold if the corresponding measure is defined via the minimal distance to the set of classical-quantum states. The proofs follow the same lines as above.

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Note added.—While finishing this Letter, we became aware of two related works. In Ref. [43], the authors show that the quantum discord can increase under a local amplitude damping channel. The dynamics of quantum correlations in a spin chain under the action of local noise is studied in Ref. [44].

*streltsov@thphy.uni-duesseldorf.de

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