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Computational framework for a family of methods based on stress-constrained topology optimization

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ABSTRACT

This study presents a general computational framework for topology optimization under constraints related to various engineering design problems, including: reliability analysis, low-cycle fatigue assessment, and stress limited analysis. Such a framework aims to facilitate comprehensive engineering design considerations by incorporating traditional constraints such as displacement and stress alongside probabilistic assessments of fatigue failure and the complex behaviors exhibited by structures made of elastoplastic material. The framework's amalgamation of diverse analytical components offers engineers a versatile toolkit to address intricate design challenges. Notably, the inclusion of reliability analysis introduces a probabilistic perspective, transforming conventional design constraints into random parameters, thereby enhancing the robustness of the design process. Key to the framework's efficacy is its implementation using MATLAB mathematical computing software, leveraging the platform's efficient code execution and object-oriented programming paradigm. This choice ensures an intuitive and potent environment for designers and researchers, facilitating seamless adaptation across various engineering applications. Additionally, the proposed previously by the Authors algorithm for the topology optimization is extended by adaptive strategy allowing for efficient adjustment of an amount of material removed at individual optimization step.

The presented framework is offering a comprehensive and integrated approach to address multifaceted design challenges while enhancing design robustness and efficiency.

1. Introduction

Topology optimization is a pivotal technique in structural design, as it optimizes material distribution within a designated space to meet specific performance objectives. Recent research has extensively explored enhancing the fatigue resistance of structural components through topology optimization methodologies. This literature review aims to delineate the latest advancements in this field however the significant and almost forgotten achievements also presented.

In the realm of engineering optimization, numerous methodologies have been devised to tackle intricate design challenges. This review delves into recent progressions in optimization techniques and their applications in engineering design, particularly in addressing plasticity based fatigue-constrained topology optimization to refine the design process for structural components. The computational framework is very important when a research field has reached the stage of application in industrial software.

Reviewing the historical milestones of topology optimization, which includes discussions on the non-uniqueness of optimal topologies and analytical solutions dating back to the 1950-s, Lógó and Ismail provide valuable insights [26]. Wu et al. [60] furnish a comprehensive review of topology optimization for designing multi-scale structures. Zhou and Rozvany [69] applied the Continuum-based Optimality Criteria (COC) algorithm to simultaneously optimize topology and geometry. Arora et al. [43] introduced a new class of structural design problems focused on ensuring the safety of structures under damage conditions. In 1995, Patnaik et al. [38] explored the merits and limitations of the Optimality Criteria (OC) method for structural optimization. Topping provided a review of mathematical programming methods used in the design of skeletal elastic structures, which includes considerations for altering the shape, position, or layout of members [51]. Rozvany offered a critical review of general solution techniques for topology optimization in 2009 [41], with statements still relevant today, offering guidance for future research directions. Furthermore, a new category of methods for im-

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plementing and solving structural optimization problems, referred to as feature-mapping methods, has emerged over the last 20 years, as outlined in the review paper by Wein et al. [59].

Nguyen and Lee presented several papers [32],[31],[30] recently in the field of stress, stability, and vibration constrained multi-scale, multi-material topology optimization. The compliance minimization objective function of the multi-scale structures constrained by the global stress constraints, critical buckling load factor, and natural frequencies. The algorithm is based on the modification of the modified solid isotropic material with penalization algorithm. The maximum stress is measured using the global p-norm stress aggregation. They extend the dynamic constrained optimization to solve thermoelastic problems [34],[33].

The proposed herein study covers the stress-limited topology optimization, which includes considerations for plasticity-based fatigue and reliability. Therefore, it is necessary to present the fundamental theories underlying stress-limited topology optimization. One of the seminal principles in this field was put forth by Sved [45], later elaborated upon by Sved and Ginos [46]. Their findings state:

"In the case of minimum-weight design of a stress-limited truss, the optimal structure cannot be less stiff in the elastic range or in stationary creep than any other truss using the same amount of material and adhering to the same allowable range of axial stress. It is shown that a truss of minimum weight supporting given point masses from a given rigid foundation and possessing a given fundamental natural frequency has the same layout as the truss designed for minimum-weight under stress limitations, but may have different cross-sectional areas."

The theorem established by Hegemier and Prager [19] provided a generalization for minimum compliance design, showing that stress-constrained minimum weight design and stress-limited compliance minimization can lead to the same optimal topology. These theorems serve as invaluable tools for guiding the optimal layout of truss-like structures under stress constraints.

It is widely acknowledged that topology optimization frequently results in non-unique optimal solutions, a topic thoroughly discussed in [26]. Additionally, the existence of singular topologies in optimum design has been investigated by Kirsch [24]. These singular solutions hold significant importance in verifying the practical applications and efficacy of topology optimization methodologies.

Over the past three decades, significant advancements have been made in the field of stress-limited topology optimization. Bojczuk and Mróz [3] introduced a heuristic algorithm for optimal truss design, using volume generalized cost as the objective function while considering stress and buckling constraints. Their approach treated design variables such as cross-sectional areas, node configurations, and the number of nodes and bars, akin to biological growth models. They observed that the structure evolves based on a characteristic size parameter, with topology bifurcation occurring through the generation of new nodes and bars to minimize the cost function.

Le et al. [25] proposed an algorithm to address stress-constrained topology optimization, employing a combination of density filtering for length scale control, the solid isotropic material with penalization (SIMP) method to generate black-and-white designs, and a SIMP-motivated stress definition to resolve stress singularities. They also incorporated a global/regional stress measure with an adaptive normalization scheme to control local stress levels. The Drucker–Prager failure criterion was utilized to handle materials with different behavior under tension and compression, as demonstrated in the work of Bruggi and Duysinx [4]. They implemented a suitable relaxation of the equivalent stress measure to address singularity-related challenges. Subsequently, these authors, along with Collet [9], extended their optimization problem to include fatigue constraints, overcoming singularity issues through *qp*-relaxation of the equivalent stress measures.

Coelho et al. [8] presented stress-constrained topology optimization of cellular materials with periodic microstructures. They utilized parallel computing to mitigate the computational cost associated with the local nature of stress constraints and finite difference design sensitivities, particularly in cellular materials with periodic microstructures.

Recently, the aspect of manufacturability has gained importance in stress-limited optimization. Mishra et al. [29] proposed a methodology to address the issue of slender intersections in topology optimization. Norato et al. [35] developed a method for stress-constrained topology optimization, where the stress constraint is represented by a differentiable approximation of the maximum element stress violation within the structure. They introduced a differentiable rectifier function to quantify the element stress violation, enabling the generation of designs that satisfy stress limits without the need for constraint renormalization.

Wang and Wu [54] focused on shell-infill structures in their research. They formulated the topology optimization problem as a Robust Minimum Compliance problem with Volume and Stress constraints (RMCVS), utilizing robust methods to eliminate undesired topological characteristics.

While the aforementioned papers primarily assumed elastic materials with stress limits, there has been consideration for plasticity-based topology optimization as well. Blachowski et al. [2] presented a methodology for the topology optimization of elastoplastic structures under stress constraints, expanding the scope of stress-limited optimization to include materials exhibiting plastic behavior. The Authors of this paper presented an effective computational method to solve plasticity based stress constrained topology optimization problems, however this algorithm is limited for single performance constraint reliability optimization [48]. The paper describing the IT aspects of topology optimization computational environment, including the code architecture, has been published by the authors here [49].

To further advance the understanding of stress-limited topology optimization problems from a probabilistic and reliability perspective, several researchers have conducted investigations. However, fundamental questions, such as those related to convexity, also require thorough examination.

Wang et al. [58] explored the interrelation between classical probabilistic methods and convex modeling approaches. Their study illustrated that concepts of probability and convexity are compatible, shedding light on potential synergies between these two frameworks in addressing stress-limited topology optimization problems. In 2004, Du and Chen [13] introduced the Sequential Optimization and Reliability Assessment (SORA) algorithm for reliability analysis in topological design. Honarmandi et al. [21] proposed a reliability-based design optimization methodology specifically for cantilever beams. Kharmanda et al. [23] introduced reliability-based topology optimization (RBTO). Valdebenito and Schuëller [52] provided a survey of approaches for conducting reliability-based optimization. Da Silva and Beck [10] elaborated on a methodology for solving reliability-based topology optimization problems in continuum domains with stress constraints and uncertainties in the magnitude of applied loads. They addressed the entire set of local stress constraints without resorting to aggregation techniques. Zhang et al. [65] proposed a Moving Morphable Void (MMV)-based approach. Huang and Xie [22] proposed a bi-directional evolutionary topology optimization method with material interpolation. Senhora et al. [42] presented a consistent topology optimization formulation for mass minimization with local stress constraints. Fin et al. [16] explored structural topology optimization under limit analysis. Pastore et al. [37] introduced a risk factor-based formulation of stress constraints to handle non-isoresistant materials, such as concrete, in stress-based topology optimization problems. Xia et al. [62] presented an interval reliability-based topology optimization (IRBTO) framework for interval parametric structures. Wang et al. [55] proposed an uncertainty-oriented cross-scale topology optimization model with global stress reliability constraint, local displacement constraint, and micro-manufacturing control based on evidence theory. Xia and Qiu [61] presented a sequential strategy for non-probabilistic reliability-based topology optimization (NRBTO) of continuum structures with stress constraints. Bo et al. [63] elaborated on a surrogate model-based method for reliability-oriented buckling topology optimization under random field load uncertainty. Freitag et al. [17] addressed aleatory and epistemic uncertainties within topology optimization using polymorphic uncertainty models. Cheng et al. [6] proposed a response-surface-based reliability design approach for solving stress-constrained optimization problems with loading uncertainty. De et al. [11] presented a topology optimization approach that optimizes the structural shape and topology at the macroscale while considering design-independent uncertain microstructures.

A hybrid reliability-based topology optimization method for handling epistemic and aleatory uncertainties was presented by Meng et al. [28] They elaborated a new triple-nested RBTO model based on fuzzy and probabilistic theory for describing the multi-source uncertainties. To make the algorithm more effective, an efficient single-loop optimization method is proposed to degrade the triple-nested optimization problem into a deterministic optimization problem using the Karush–Kuhn–Tucker optimality condition. These derived optimality condition (the hybrid reliability constraint with respect to the random probabilistic variables, fuzzy variables, and deterministic design variables) based on the adjoin variable method.

In the realm of frameworks, Wang et al. [56] introduced a non-probabilistic reliability-based topology optimization (NRBTO) framework for continuum structures under multi-dimensional convex uncertainties. They combined the SIMP model with set-theoretical convex methods to derive mathematical approximations and boundary laws for displacement responses. The non-probabilistic reliability was quantified using the principle of hyper-volume ratio. Ortigosa et al. [36] developed an additional framework for designing flexoelectric energy harvesters at finite strains using topology optimization. Caasenbrood et al. [5] presented a pioneering framework for designing pressure-driven soft robots, employing topology optimization techniques.

Fatigue is a critical phenomenon in engineering design, often categorized into high cycle (elasticity-based) and low cycle (plasticity-based) fatigue. Lemaître and Desmorat [12] offered a comprehensive overview of Continuum Damage Mechanics applied to mechanical and civil engineering, focusing on failures and fatigue. Figel and Kamiński [15] presented a sensitivity based probabilistic approach for fatigue problem. They studied the effects of composite parameters in a fatigue delamination problem of a two-layer composite. This approach combines together a fatigue delamination model, Monte Carlo simulations and the finite element method.

Tauzowski et al. [50] introduced a novel method for stress-constrained topology optimization considering reliability analysis with a performance function based on low-cycle fatigue. Chen et al. [7] proposed a methodology for fatigue-constrained topology optimization by penalizing cumulative fatigue damage. Suresh et al. [44] presented an efficiency-enhanced procedure for addressing high-cycle fatigue constraints in topology optimization. Zhao et al. [68] proposed an efficient fail-safe approach for topology optimization considering fatigue considerations. Verbart et al. [53] proposed a damage-based method for topology optimization with local stress constraints. Desmorat [12] introduced a topology optimization approach to maximize fatigue lifetime. Holmberg et al. [20] proposed a methodology for fatigue-constrained topology optimization. As it was mentioned earlier Collet et al. [9] investigated a simplified approach for optimizing structures under fatigue and compliance constraints. Zhang et al. [64] presented a method for the topology optimization of structures subject to non-proportional cyclic loading with regard to fatigue criteria.

Generally, the topology optimization problem is computed by object oriented programming but Tauzowski et al. [47] proposed a novel approach using functor-based programming for topology optimization of elasto-plastic structures. The Authors in the paper [48] presented also an effective computational method to solve plasticity based stress constrainted topology optimization problems, however this algorithm is limited for single performance constraint reliability optimization. Ma et al. [27] developed a fully automatic computational framework for beam structure design based on continuum structural topology optimization. Rade et al. [40] proposed algorithmically-consistent deep learning frameworks for structural topology optimization. Zhang et al. [66] presented TopADD, a parallel-computing framework for integrated topology optimization. Ortiogosa et al. [36] introduce a computational framework for topology optimization of flexoelectric energy harvesters. Zhao et al. [67] present an application framework for designing optimized self-supporting structures. Aage et al. [1] develop a parallel framework for topology optimization using the method of moving asymptotes. Wang et al. [57] introduced a framework for structural shape and topology optimization using a level-set method. Wang et al. [55] proposed a topology optimization formulation including a model of the layer-by-layer additive manufacturing process. Haverroth et al. [18] developed a topology optimization formulation considering the layer-by-layer additive manufacturing process.

The diverse methodologies and advancements showcased in the reviewed literature under- score the potential of topology optimization in addressing real-world engineering challenges. While existing computational frameworks often address individual aspects of design optimization, such as topology or fatigue analysis, they lack a comprehensive and integrated approach. This study introduces a MATLAB framework that seamlessly integrates topology optimization with various constraints such as displacement, stress, low-cycle fatigue assessment, and elasto-plastic analysis within a single user-friendly environment. Additionally, constraints can be based not only on specific values—such as displacement, stress, or low-cycle fatigue assessment—but also on the probability of exceeding these values. This probability assessment is achieved using various reliability analysis methods, such as FORM, SORM, and Monte Carlo. This unique combination sets our framework apart and empowers engineers to tackle complex design challenges with a more holistic and probabilistic perspective. A significant part of our novelty lies in incorporating the Sequential Optimization and Reliability Assessment (SORA) algorithm for topology optimization within the reliability analysis. This allows for the inclusion of design constraints based on the probability of failure, representing a substantial advancement in achieving robust, reliable designs and relatively fast results.

In this paper a novel framework is presented to solve complex, stress based (fatigue including) topology optimization problems. Section 1 provides an overview of structural topology optimization, aiming to elucidate its fundamental principles, methodological variants, and practical applications. Section 2 delves into the detailed formulation of structural topology optimization, encompassing various algorithms and constraint types, including considerations for reliability, fatigue, and displacement based constraints. A series of illustrative examples is presented in Section 3, demonstrating the application of these methodologies through numerical simulations of classic problems such as the L-shaped structure in both 2D and 3D, as well as the cantilever beam and corbel. Finally, Section 4 offers conclusions drawn from the analysis and suggests avenues for future research in this dynamic field.

2. Methods

2.1. General formulation of structural topology optimization

This study formulates topology optimization problem as a size optimization under stress constraints. The derivation of the method starts with general formulation of structural optimization:

$$\begin{aligned} &\text{Find design vector} && \mathbf{x} = [x_1, x_2, \ldots x_n]^T, \\ &\text{which minimizes} && f(\mathbf{x}) \\ &\text{subject to constraints} && g_i(\mathbf{x}, \boldsymbol{\theta}) \le 0, \quad i = 1, 2, \ldots, m \end{aligned} \tag{1}$$

where $\mathbf{x}$ is an n-dimensional vector called the design vector, $\boldsymbol{\theta}$ is a random variable vector, $f(\mathbf{x})$ is termed the objective (cost) function, $g_i(\mathbf{x}, \boldsymbol{\theta})$ are known as inequality constraints.

In our approach, random variables may include values such as material parameters, loads values, and geometric quantities, such as the

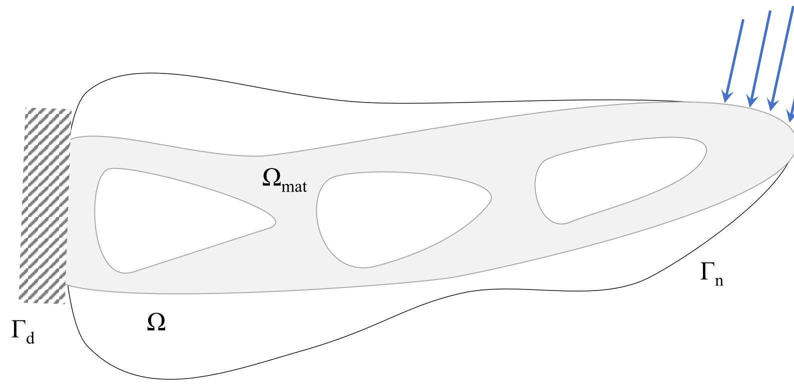

Fig. 1. Design domain Ω optimal topology Ω_{mat}.

location of forces. The nature of this randomness is described by probability distributions implemented in MATLAB. The randomness of model parameters implies the randomness of resulting values from the structural analysis, such as displacements, stresses, and other quantities like the number of fatigue cycles. Given these random results, we can calculate the probability of these results meeting selected conditions, such as the probability of exceeding a certain threshold value (details of such threshold functions are presented later in the paper).

The methodology for determining such probabilities is called reliability analysis. In the simplest Monte Carlo method, to calculate the probability of a result meeting a selected condition, it is sufficient to random N points from the probability distributions, determine the results using FEM, and then check what percentage meets the condition. Although this method is very simple and reliable, it is numerically expensive because it typically requires a large number of realization of random variables (at least 10,000) and an equal number of FEM analyses. Therefore, there are several other methods for estimating such probabilities. Besides the Monte Carlo method, our framework also implements the gradient-based, iterative First Order Reliability Method (FORM) and Hybrid Mean Value (HMV) method, allowing for much more efficient probability estimation. Iterative formulas of these methods are presented in eq. (11) (FORM), eqs (7)-(9) (HMV).

In the proposed approach the objective function $f(\mathbf{x})$ represents overall volume of the structure, $g_i(\mathbf{x})$ are the inequality constraints depending on the specific problem. Graphical representation of the initial and optimal topology is shown in Fig. 1.

Three particularly useful and frequently used types of inequality equations are:

1) $g_i(\mathbf{x}, \boldsymbol{\theta}) = |u(\mathbf{x})| - u_T$, in the case of displacement constrained optimization where $u(\mathbf{x})$ denotes displacement at the optimal solution and u_T is allowable displacement level

2) $g_i(\mathbf{x}, \boldsymbol{\theta}) = \sigma_{HM}(\mathbf{x}) - \sigma_T$, in the case of stress constrained optimization where $\sigma_{HM}(\mathbf{x})$ denotes von Mises stress at the optimal topology and σ_T is a threshold value. The threshold value of stress constraints can be defined locally for each element or as a global measure using p-norms. These norms provide a way to combine stresses from various locations within the structure into a single value. When an elastoplastic material model is chosen, a natural limit on stress exists. This is because the material's yield stress represents the maximum stress it can sustain before undergoing plastic deformation. The optimization process will inherently avoid exceeding this limit.

3) $g_i(\mathbf{x}, \boldsymbol{\theta}) = N_c(\mathbf{x}) - N_T$, in the case of low-cycle fatigue where $N_c(\mathbf{x})$ is number of cycles at optimal design and N_T represents permissible value

The above constraints are related to various cases of deterministic topology optimization. In the case of reliability-based topology optimization the above constraints take the probabilistic form:

$$Pr(\sigma_{HM}(\mathbf{x}) \geq \sigma_T) - P_T \leq 0, \tag{2}$$

where P_T is the probability threshold value.

Next, based on the various constraints mentioned above in the next subsection we will describe algorithms used for proposed topology optimization driven by stress intensity.

2.2. Improved optimality criteria-based algorithm for topology optimization

In this section we will briefly describe our main algorithm for topology optimization and propose its improvements towards reduction of computational time. Our main algorithm for topology optimization is based on elastoplastic analysis and allows to design structures subjected to both displacement and stress constraints. It was described in detail in our previous work [2], presented graphically in Fig. 2 and essentially consists of the following six steps:

- Step 1) Initialize design variables $\rho_e = 1$,
- Step 2) Solve equilibrium equations from the FEM equation

$$\mathbf{K}\left(\rho_e^{(k)}\right)\mathbf{u}\left(\rho_e^{(k)}\right) - \mathbf{f} = \mathbf{0}, \tag{3}$$

- Step 3) Determine element stress intensity $a_e = \frac{\sigma_i^{(k)}}{\sigma_{max}}$,
- Step 4) Apply sensitivity filter,
- Step 5) Update design variables corresponding to the finite elements according to iterative formula:

$$\rho_i^{(k+1)} = \rho_i^{(k)} \left[\frac{\sigma_i^{(k)}}{\sigma_{max}}\right]^p, \tag{4}$$

- Step 6) Remove n_{sel} least stressed finite elements
- Step 7 If end condition meet then STOP
- Step 8 Goto Step 2.

Where $\rho_e^{(k)}$ - is element density – design variable of topology optimization, $\sigma_i^{(k)}$ average Huber-Mises stress at ith element, σ_{max} is maximal stress in the structure, $\mathbf{K}\left(\rho_e^{(k)}\right)$ is global stiffness matrix, $\mathbf{u}\left(\rho_e^{(k)}\right)$ is global displacement vector and $\mathbf{f}$ is global load vector. Besides of standard parameters appearing in other topology optimization methods such as SIMP, in our method another free parameter appears. This parameter is denoted as n_{sel} and represents a number of finite element removed at subsequent iteration of the whole optimization procedure. As it was presented in paper [2] it is always possible to select this sufficiently small to achieve proper result. In general one can say, that the smaller the parameter the better the result. However, such an approach is obviously related to longer time of computations, therefore it is strongly recommended to tune the parameter in such a way that optimal topology is obtained with required accuracy in reasonable time. We have now introduced the capability for the cut threshold to vary as a function of the constraint value (see Fig. 4 for examples of variability functions). This allows for more efficient and more accurate topology optimization, particularly in the initial stages where different cut threshold than in final stages. This improvement not only enhances efficiency but also proves

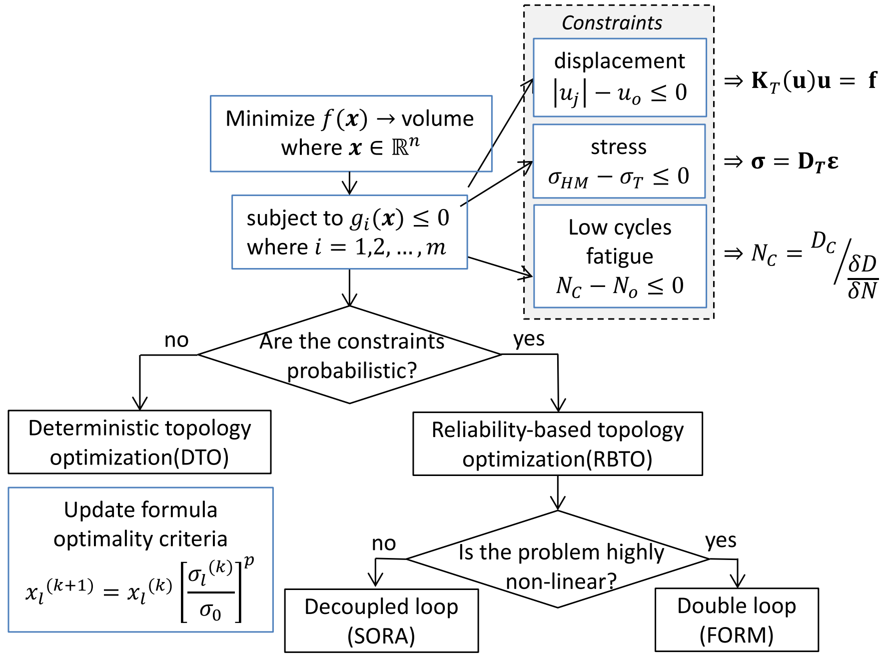

Fig. 2. Computational framework for a family of topology optimization methods.

beneficial when coupled with reliability analysis, where the reliability index can exhibit rapid changes, and a dynamic cut threshold provides better time resolution during these critical iterations.

The proposed herein modification is related to adaptive adjustment of the number of elements selected at subsequent iterations. Optimal topologies of simple cantilever structures are shown in Fig. 3. In contrast Fig. 5 shows the optimal topologies for adjusted number of removed elements.

2.3. Algorithms for constraints evaluation

The main algorithm presented in previous section is targeted on solving reliability-based topology optimization (RBTO) problem under displacement, stress and fatigue constraints. For satisfying constraints in our topology optimization problem the following four algorithms are used, two of which are responsible for mechanical aspects and the remaining two for reliability evaluation. The above mentioned algorithms are:

1) elastoplastic analysis using return mapping approach,
2) fatigue analysis based on continuum damage mechanics,
3) decoupled optimization using sequential optimization and reliability assessment, and
4) nested optimization using first order reliability analysis.

Algorithm 1 *(Elastoplastic analysis).*

Step 1. Initialize loop variable $n = 1$, and assign the total applied load to the residual force vector $\mathbf{r}_n = \mathbf{f}$.

Step 2. Aggregate of tangent stiffness matrix

$$\mathbf{K}_T^n = \int_\Omega \mathbf{B}^T \mathbf{D}^{ep}(\boldsymbol{\sigma}^n, \Delta\gamma)\mathbf{B}\, d\Omega,$$

where $\mathbf{B}$ is matrix of strain derivatives, $\mathbf{D}^{ep}$ is elastoplastic material matrix dependent on stress and $\Delta\gamma$ is stress corrector ratio.

Step 3. Calculate of displacement increment on the current iteration

$$\Delta\mathbf{u}_n = -\left(\mathbf{K}_T^n\right)^{-1}\mathbf{r}_n$$

Step 4. Update strain vector and calculate trial strain

$$\Delta\boldsymbol{\varepsilon}_n = \mathbf{B}\Delta\mathbf{u}_n, \quad \boldsymbol{\varepsilon}_{n+1}^{trial} = \boldsymbol{\varepsilon}_n^{trial} + \Delta\boldsymbol{\varepsilon}_n.$$

Step 5. Update stress vector (return mapping $\boldsymbol{\Psi}$) to ensure that stresses do not exceed the plastic flow surface

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\Psi}(\Delta\boldsymbol{\varepsilon}_{n+1}^{trial}).$$

Step 6. Update residual forces

$$\mathbf{r}_{n+1} = \int_\Omega \mathbf{B}^T \boldsymbol{\sigma}_{n+1}\, d\Omega.$$

Step 7. Check the convergence criterion

$$\text{If } \frac{||\mathbf{r}_{n+1}||}{||\mathbf{f}||} > \epsilon_{tol} \text{ then go to } \textbf{Step 2.}$$

Algorithm 2 *(Number of cycles in low-cycle fatigue analysis).*

Step 1. Computation of plastic deformation increment per cycle $\frac{\delta p}{\delta N}$

$$\frac{\delta p}{\delta N} = \int_{\text{cycle}} \dot{p}\, \text{dt} = 2\Delta\epsilon_{\text{P}},$$

where $\Delta\epsilon_{\text{P}}$ is plastic strain range at each cycle, $\dot{p}$ is plastic strain rate.

Step 2. Number of cycles to damage initiation N_{D}

$$N_{\text{D}} = \frac{p_{\text{D}}}{2\Delta\epsilon_{\text{P}}},$$

where p_{D} is plastic strain at which damage initialization is observed.

Step 3. Increment of damage per cycle $\frac{\delta D}{\delta N}$ computation

$$\frac{\delta D}{\delta N} = \int_{\text{cycle}} \dot{D}\, \text{dt} = \frac{\sigma_{\max}^{2s} + (\Delta\sigma - \sigma_{\max})^{2S}}{2(2ES)^s}\frac{\delta p}{\delta N},$$

where $D \in [0,1]$ is damage factor (0 means undamaged material while 1 is related to fully broken one), $\Delta\sigma$ is stress range at cycle,

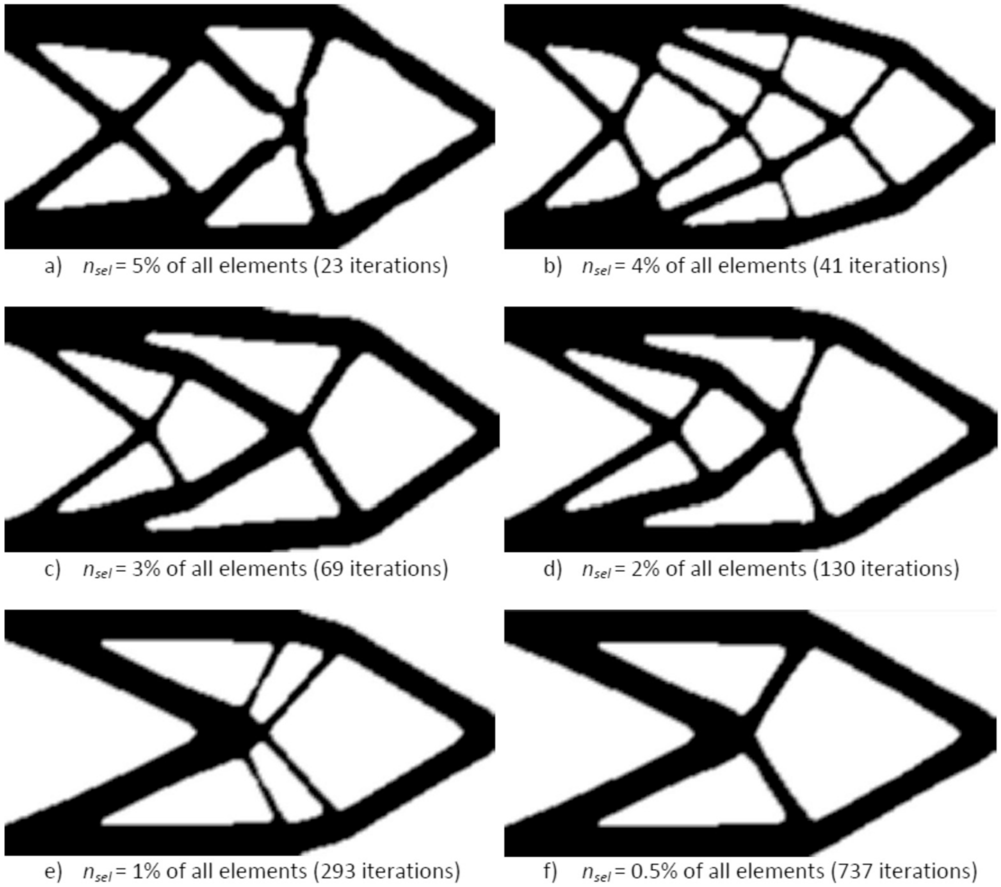

Fig. 3. Optimal topology of cantilever for various numbers of removed elements.

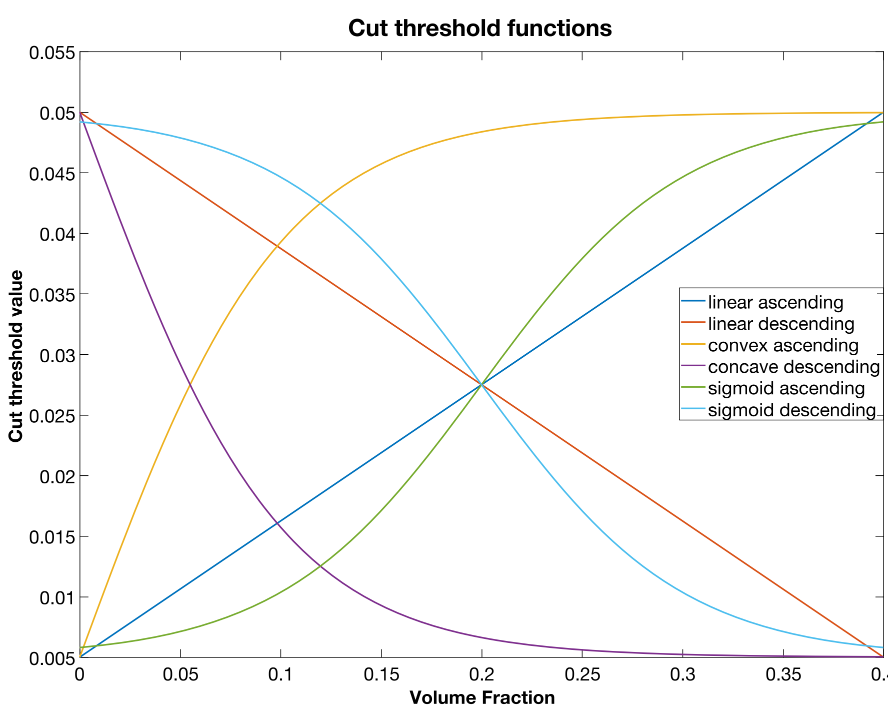

Fig. 4. Functions of the cut threshold parameter variability.

σ_{max} is maximal stress at cycle, S and s are temperature dependent material constants, E is Young modulus.

Step 4. Final calculation of number of cycles to rupture

$$N_R = N_D + \frac{D_C}{\frac{\delta D}{\delta N}}, \tag{5}$$

where D_C is damage factor where rupture occurs.

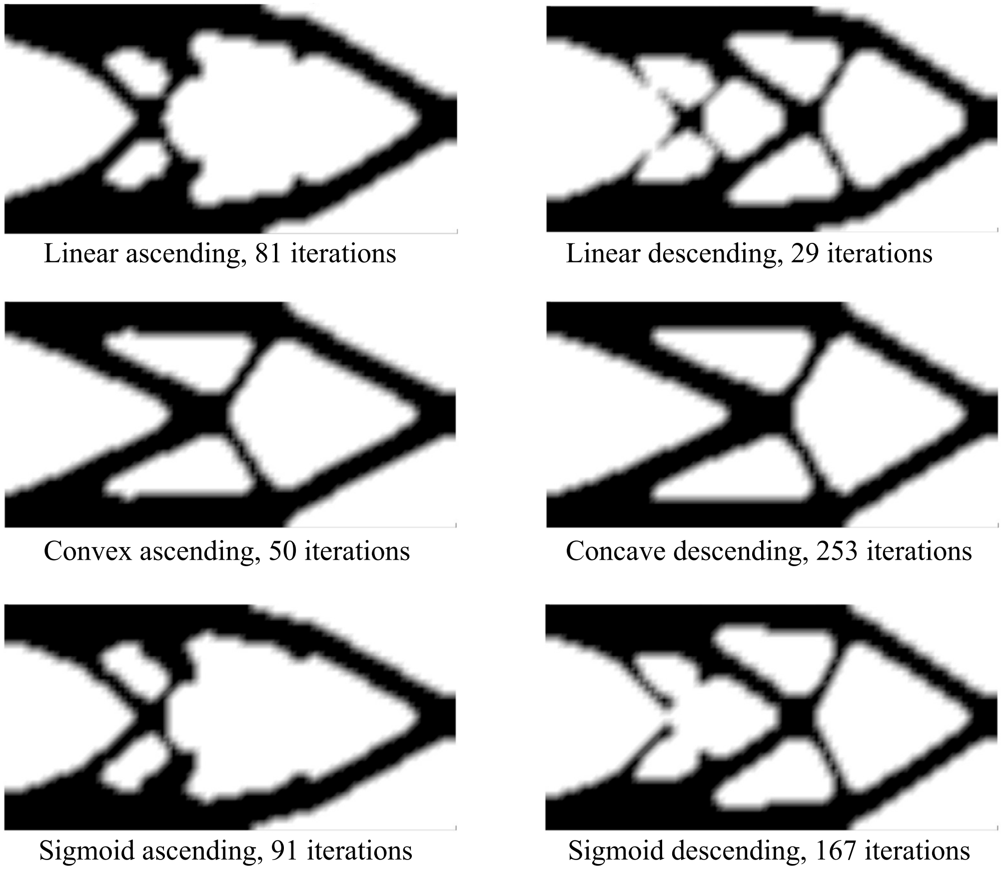

Fig. 5. Adaptive adjustment of the number of elements.

SORA approach which is described below in step-by-step manner:

Algorithm 3 *(Decoupled approach for RBTO).*

Step 1. Initialize the loop variable $n = 1$ and the random variables with their mean values

$$\theta^{(0)} = \bar{\theta}. \tag{6}$$

Step 2. For the values assigned to random variables in step 1 determine optimal topology as in a deterministic case.

Step 3. Find reliability index β_T for topology obtained using deterministic optimization (step 2) and determine the most probable point (MPP) with corresponding failure scenario applying first order approximation in form of Hybrid Mean Value (HMV) where the local loop variable for this procedure is m.

$$\mathbf{n}(\mathbf{y}_{\mathrm{HMV}}^{(0)}) = -\frac{\nabla G(\mathbf{y}_{\mathrm{HMV}}^{(0)})}{||\nabla G(\mathbf{y}_{\mathrm{HMV}}^{(0)})||} \tag{7}$$

$$\mathbf{y}_{\mathrm{HMV}}^{(m+1)} = \beta_T \frac{\mathbf{n}(\mathbf{y}_{\mathrm{HMV}}^{(m)}) + \mathbf{n}(\mathbf{y}_{\mathrm{HMV}}^{(m-1)}) + \mathbf{n}(\mathbf{y}_{\mathrm{HMV}}^{(m-2)})}{||\mathbf{n}(\mathbf{y}_{\mathrm{HMV}}^{(m)}) + \mathbf{n}(\mathbf{y}_{\mathrm{HMV}}^{(m-1)}) + \mathbf{n}(\mathbf{y}_{\mathrm{HMV}}^{(m-2)})||} \tag{8}$$

if $m < 3$ use AMV formula:

$$\mathbf{n}(\mathbf{y}_{\mathrm{HMV}}^{(m+1)}) = \beta_T \mathbf{n}(\mathbf{y}_{\mathrm{HMV}}^{(m)}), \tag{9}$$

where $\mathbf{y}_{\mathrm{HMV}}$ are the random variables transformed into a standard probabilistic space in which family of first order algorithms are usually implemented.

Step 4. Update the random variables by assigning to them values of the most probable point (MPP) determined at Step 3, so

$$\boldsymbol{P}^{(n)} = \mathcal{L}^{-1}(\mathbf{y}_{\mathrm{HMV}}), \tag{10}$$

where $\boldsymbol{P}^{(n)}$ is structural load vector at the nth iteration (in general this vector can represent other random parameters such as material constants). $\mathcal{L} = \Phi^{-1}(F_\theta(\boldsymbol{\theta}))$ is transformation of random variables into standard normal space, where Φ^{-1} is reverse cumulative distribution and F_θ is cumulative density distribution, both functions of random variable θ.

This update of random variable is crucial for adjusting the structural load vector, which is used in next step for deterministic optimization process. Finally, return to Step 2 and repeat the following procedure until the difference between two consecutive update results is smaller than the required accuracy (see Fig. 6).

Algorithm 4 *(Double loop approach for RBTO).*

Step 1. Initialize the loop variable $n = 1$ and the design variables the set the material densities $\mathbf{x}_n = 1$.

Step 2. Determine current reliability index β_T and mth iteration using first order reliability approach according the Rackwitz-Fiessler [39] iterative formula:

$$\mathbf{y}_{\mathrm{FORM}}^{(m+1)} = \frac{1}{||\nabla G(\mathbf{y})||^2}(\nabla G(\mathbf{y})^T \mathbf{y} - G)\nabla G(\mathbf{y}), \tag{11}$$

where $\mathbf{y}$ in this equation means $\mathbf{y}_{\mathrm{FORM}}^{(m)}$ for compactness.

Step 3. Check if current reliability index exceeds prescribed threshold.

$$\beta_n > \beta_{\mathrm{T}},$$

if yes, restore design form previous, safe iteration and finish the algorithm.

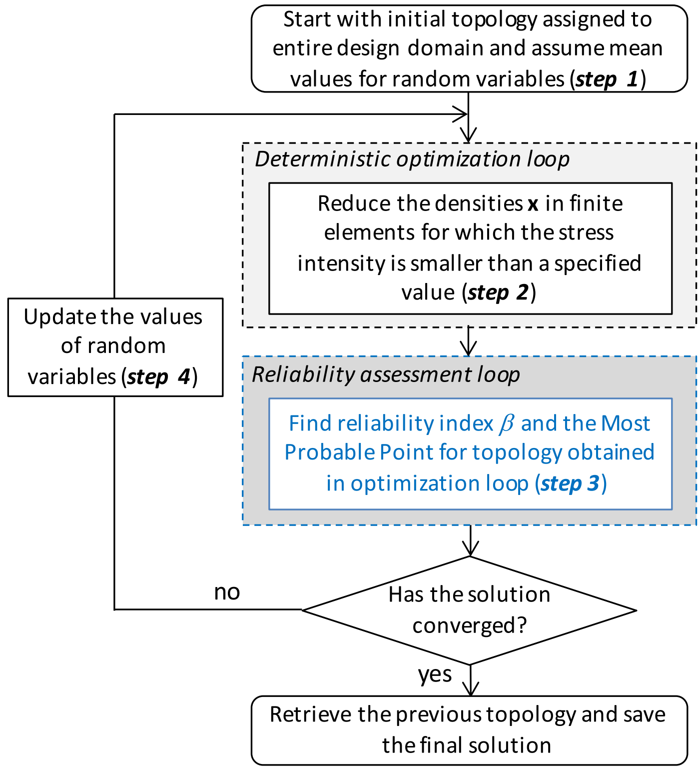

Fig. 6. Flowchart of the decoupled loop RBTO.

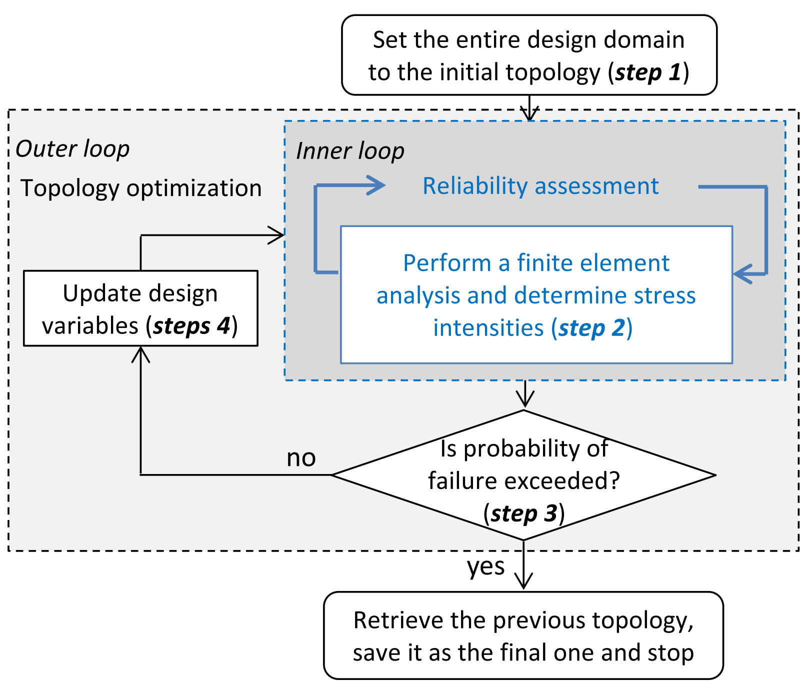

Fig. 7. Flowchart of the double loop RBTO.

Step 4. Update design variables according to particular topology optimization algorithm (see Fig. 7).

Finally, it is worth mentioning a few remarks about the generality of our solution. The numerical implementation of probabilistic problems has been designed to accommodate any probability distribution available in the MATLAB environment. A 'RandomVariable' class was created with an attribute representing a probability distribution and three functions necessary for reliability analyses: 'random' (used in Monte Carlo simulation methods to randomize a set of points from a given distribution), 'toU', and 'fromU' (used to transform variables to and from the standard normal space, used in gradient based methods such as FORM, SORM, HMV). These methods are defined as follows:

```
function u = toU(obj,x)
    u = norminv(cdf(obj.pd, x));
end

function x = fromU(obj,u)
```

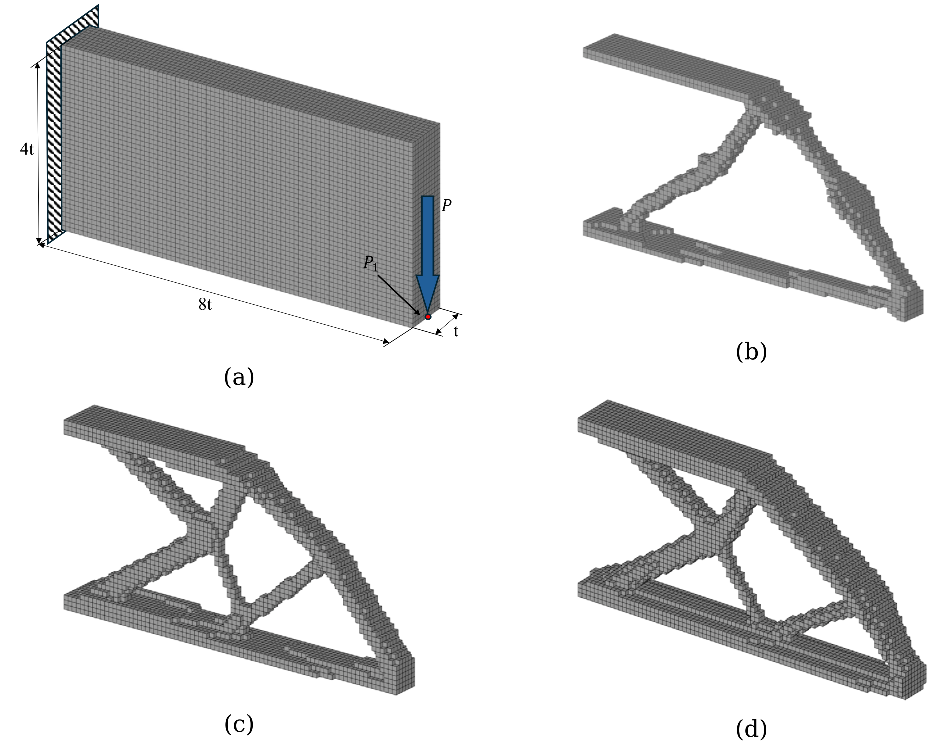

Fig. 8. Cantilever structure with constraint on vertical displacement: a) scheme, b) deterministic solution with $V = 10\%$, $\beta = 0.8$, c) probabilistic solution with $V = 18\%$, $\beta_T = 2.0$ and d) $V = 21\%$, $\beta_T = 3.0$.

```
    x = icdf(obj.pd, normcdf(u));
end
```

In this code: 'obj.pd' is an attribute of the 'RandomVariable' class, which is a MATLAB probability distribution object, '**norminv**' is the normal inverse cumulative distribution function, '**cdf**' is the cumulative distribution function, '**icdf**' is the inverse cumulative distribution function, '**normcdf**' is the normal cumulative distribution function. As demonstrated in the code above, the methods are defined for any probability distribution. A drawback of this approach is the inability to solve problems with correlated random variables, which would require the implementation of more complex variable transformation algorithms, such as the Nataf transformation which is not implemented yet.

Each task in our system is implemented as a separate class, allowing for convenient encapsulation of all task parameters. Such a class can contain methods that enable easy modification of any task parameters, including load parameters, material constants (multi-material model), geometric parameters, or force location parameters. This solution provides significant flexibility in selecting random parameters for reliability tasks. In the numerical methods used in this work, the 'setLoad(P1,P2,…,Pn)' method was implemented to modify the force values, specifically the vector of the right-hand sides of the FEM equation. Similar function is implemented for compute any necessary constraint values (in this example 'computeDisplacements' for obtain constraints values similar function was implemented for computation of 'computePenalizedStress' for stress type constraint or 'computeFatiguneNCycles' for fatigue constraints.

3. Illustrative examples

In the following subsections numerical examples are presented demonstrating effectiveness of the proposed methodology. These examples are two- and three-dimensional cantilever, 2D corbel and 3D L-shape structure. In our numerical examples, we aimed to achieve a target value of the beta reliability coefficient ranging around from 3 to 5. This range is supported by the Eurocode standard [14], which specifies reliability indices from 3.3 to 5.2 (Table B2, page 45, depending on the structure class). This broader range justifies our selection of values for β_T.

3.1. Example 1. Solid cantilever with constraints on displacement

The first numerical example will concern 3D solid cantilever structure (Fig. 8a). It employs a regular mesh of eight-node Lagrange elements, totaling 32,000 finite elements and 109,593 degrees of freedom. In this example, we solve the topological optimization problem with a probabilistic reliability constraint on horizontal displacement. The performance function will be expressed by Eq. (12). The material constants are defined as follows: Young's modulus $E = 210$ GPa, Poisson's ratio $\nu = 0.3$. The probabilistic parameters are: $P_x = \mathcal{N}(0, 10)$, $P_y = \mathcal{N}(0, 0.1)$, $P_z = \mathcal{N}(100, 10)$. In this case, we see slight but noticeable differences in the structure of the truss between the beta2 and beta3 safety models. The safer design has a significantly thicker top band. The position of one node on the bottom node and the adjacent bars have also changed (Figs. 8c, 8d). However, there is a clear difference between a deterministic task and a reliability task where we have a more complex lattice. This is illustrated in Fig. 8a and Figs. 8c, 8d. It is important to highlight that when employing the SORA algorithm, achieving a safer structure does not necessarily entail significantly increasing the volume fraction. Throughout the optimization process, the structure is iteratively reconstructed to achieve the desired level of structural safety. While there exists a correlation between the probability of failure and the volume fraction, the increase in volume fraction is generally modest. This is primarily attributed to the safety improvements resulting from the design adjustments made during the SORA algorithm. This observation is supported by the findings of this study as well as other research [56].

3.2. Example 2. Corbel with constraints on displacement

In the first example, we illustrate a reliability analysis task, as depicted in Fig. 9a. The 2D Corbel model has too many finite elements to

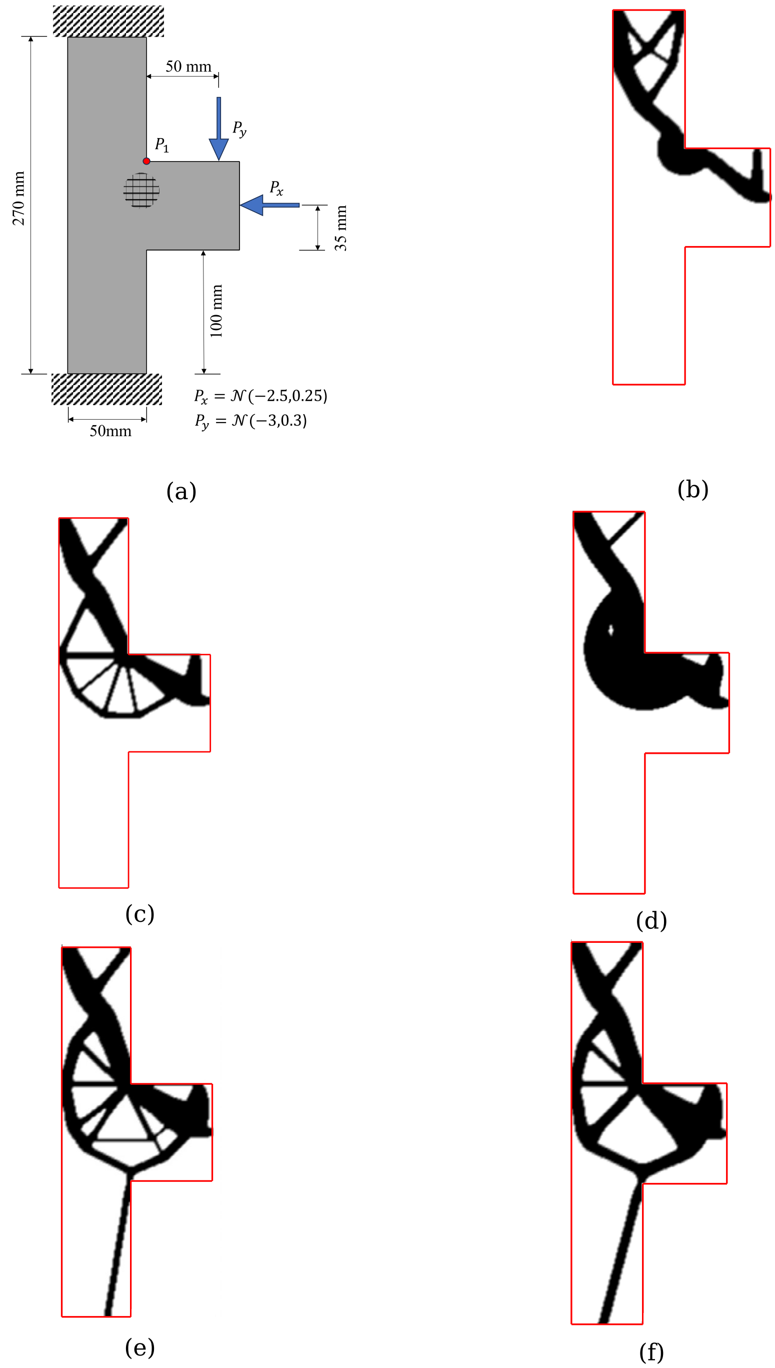

Fig. 9. Optimization of Corbel with constraints on displacement ($u_T = 8$ cm): a) scheme, b) deterministic solution with volume fraction $V = 20\%$ and resulting $\beta = 1.23$; and probabilistic solutions c) $V = 25\%$ for $\beta = 2.0$, $\beta_{MC} = 2.008$, d) $V = 34\%$ for $\beta = 3.0$, $\beta_{MC} = 2.992$, e) $V = 36\%$ for $\beta = 4.0$, $\beta_{MC} = 4.018$ and d) $V = 38\%$ for $\beta = 5.0$, $\beta_{MC} = 5.0012$.

fully display in the diagram due to its complexity. Therefore, a regular grid of four-node Lagrangian elements is symbolically represented on 9a. The model consists of 25,488 finite elements and 51,890 degrees of freedom. The material constants are defined as follows: Young's modulus $E = 210$ GPa and Poisson's ratio $\nu = 0.3$. The performance function represents the condition for the serviceability limit state, specifically, exceeding a specified permissible value of horizontal displacement u at the point P_1 (Fig. 9a). This function is given by the formula:

$$g(\mathbf{x}) = |u_x^{(P_1)}| - u_T, \tag{12}$$

Satisfying the above with a given probability is the topological optimization constraint in this example (see eq. (2)). Four tasks were solved for different values of the threshold probability equal $\beta_T = 2$, $\beta_T = 3$, $\beta_T = 4$ and $\beta_T = 5$. The probabilistic data of the task include two random variables with normal distributions, which are the horizontal and vertical (Fig. 9a). The probabilistic parameters are: $P_x = \mathcal{N}(2.5, 0.25)$, $P_y = \mathcal{N}(3, 0.3)$. These two optimal topologies are presented below (Figs. 9c, 9d, 9e, 9f). Notably, the showcased topologies demonstrate that enhancing the safety of the structure does not necessarily entail an increase in weight. The SORA algorithm m aims to reconfigure the structure in a manner that reduces the probability of failure, which we observe by comparing a deterministic solution (Fig. 9b with reliable ones (Figs. 9c, 9d, 9e, 9f). These figures include also probabilities computed by Monte Carlo method to check corrected of SORA results. Verifying very small

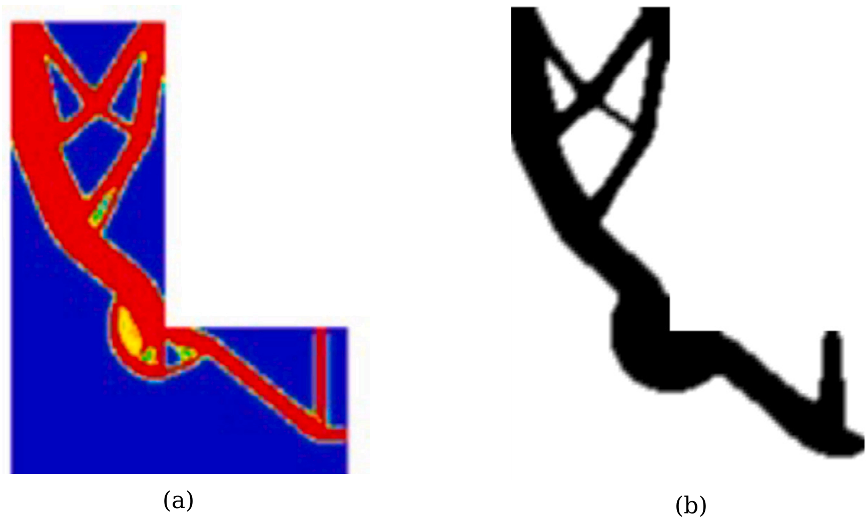

Fig. 10. Comparison of deterministic solution a) paper [56], $V = 19\%$, b) presented solution, $V = 20\%$.

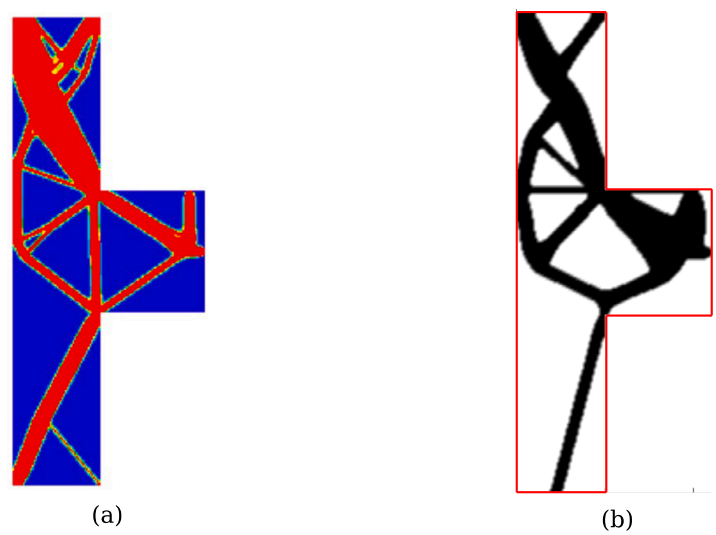

Fig. 11. Comparison of reliability based solution a) paper [56], $V = 36\%$, b) presented solution, $V = 38\%$.

failure probabilities can be challenging. In our Corbel example, we target a reliability index of beta = 5. Such probabilities require an exceptionally large number of points for the Monte Carlo method. For this reason, we assume a large number $N = 10^9$ (one billion) points. The feasibility of solving this task is due to the linear relationship between the applied load and displacement. Displacement can be determined without expensive FEM calculations, as it is a linear combination of displacement values computed once with unit forces.

Figs. 10 and 11 compare the topologies obtained in this study with results from Wang 2018 [56]. Fig. 10 shows high level of consistency between the deterministic topologies, with very similar volume fractions. This reinforces the validity of our implementation.

Fig. 11 compares the topologies for a case with a reliability constraint. While Wang's work uses a non-probabilistic approach, making quantitative comparisons challenging, a qualitative assessment reveals good agreement between the resulting topologies for the case with a threshold reliability index $\beta_T = 5.0$. Both solutions exhibit similar volume fractions.

3.3. Example 3. L-shape 3D with constraints on displacement

The last numerical example solid L-shape structure is taken into consideration (Fig. 8a). In this example, we solve the topological optimization problem with a probabilistic reliability constraint on horizontal displacement, similarly to previous example. The performance function will be expressed by Eq. (12). The material constants are defined as follows: Young's modulus $E = 210$ GPa, Poisson's ratio $\nu = 0.3$. The probabilistic parameters are: $f_x = \mathcal{N}(0, 10)$, $f_y = \mathcal{N}(0, 0.1)$, $f_z = \mathcal{N}(100, 10)$. The finite element model uses an eight-node Lagrangian element with linear interpolation. The number of finite elements in the model is 23328, which makes up the total number of degrees of freedom of the model to 79059. Structures taking into account the reliability condition have wider support at four points, as opposed to only two for the deterministic analysis (Fig. 12b and Figs. 12c, 12d). The differences between the $\beta_T = 2.0$ and $\beta_T = 3.0$ safety indices (Figs. 12c, 12d) are small but noticeable. The results in this example clearly illustrate the operation of the SORA algorithm. Reliability improvement is achieved not only by increasing the volume fraction but also through significant design changes. The deterministic structure, representing the initial design without reliability constraints, appears somewhat flatter. The structures optimized with reliability constraints exhibit a clear change, characterized by the creation of elements stiffening them in a perpendicular direction. There are no substantial differences between the individual reliability solutions in terms of overall safety. However, the design with the higher threshold safety index $\beta_T = 3.0$ has a more rigid V-shaped back wall, while the solution with the lower threshold ($\beta = 2.0$) exhibits

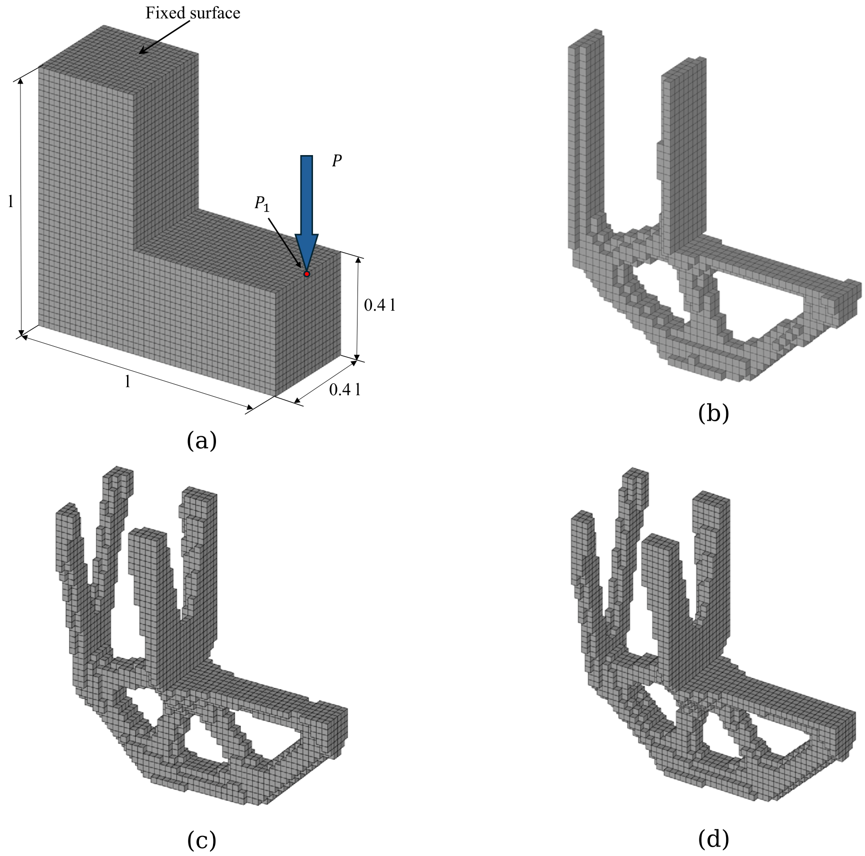

Fig. 12. L-shape structure with constraint on vertical displacement at point P_1: a) scheme, b) deterministic optimal topology $V = 10\%$ and $\beta = 1.05$ determined for comparison purposes, c) probabilistic optimal topology $V = 15\%$ for $\beta_T = 2.0$, and d) $V = 16\%$ for $\beta_T = 3.0$.

a Y-shaped back wall, which can be seen as a transitional shape between the more secure one and the deterministic design.

4. Conclusions

In conclusion, this study introduces a unified computational framework that integrates reliability-based topology optimization with diverse constraints, including considerations for displacements, stresses in both elastic and plastic materials, and the number of cycles in low-cycle fatigue problems. Addressing inherent uncertainties, our MATLAB-based framework utilizes stress intensity at the finite element level and showcases the benefits of the object-oriented programming paradigm. The proposed framework incorporates safety assessment into topology optimization, employing the Sequential Optimization and Reliability Assessment (SORA) method for safety control and the Hybrid Measure Approach (HMA) algorithm to handle design uncertainties. Numerical examples demonstrate the correlation between volume fraction and probability of failure. This comprehensive framework offers a versatile and robust approach for optimizing structures under diverse constraints and uncertainties, showcasing its applicability and effectiveness in practical scenarios.

The variable adjustment of the 'cut threshold' parameter introduced in our algorithm presents a significant advancement in topological optimization methodologies. By allowing for more effective optimization and enhanced resolution over time, particularly in conjunction with reliability analysis, these enhancements pave the way for further advancements in structural engineering and related fields.

The integration of multiple advanced analyses within a transparent architecture, designed according to sound object-oriented programming practices, empowers users to seamlessly assemble even highly intricate tasks. These tasks may encompass a range of analyses, including reliability analysis and low-cycle fatigue analysis. Moreover, the clear architecture facilitates relatively easy extension of the framework with new analyses, offering users flexibility and adaptability for future research and practical applications.

It is worth emphasizing that when using the SORA algorithm, achieving a safer structure does not necessarily equate to a significant increase in volume fraction. The optimization process rebuilds the structure to attain the desired level of structural safety, as measured by the Hasofel-Linde Reliability index. While there is naturally a correlation between the probability of failure and the volume fraction, the increase in volume fraction is often modest due to the significant improvement in safety achieved through the design changes occurring during the SORA algorithm.

CRediT authorship contribution statement

Piotr Tauzowski: Writing – original draft, Software, Conceptualization. **Bartlomiej Blachowski:** Writing – review & editing, Writing – original draft, Methodology. **János Lógó:** Writing – review & editing, Writing – original draft, Supervision, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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