43rd Solid Mechanics Conference – SolMech 2024
Wrocław, Poland, 16–18 September 2024

Book of Abstracts

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Institute of Fundamental Technological Research
Polish Academy of Sciences

43rd Solid Mechanics Conference – SolMech 2024
Wrocław, Poland, 16–18 September 2024

organized by

Institute of Fundamental Technological Research
of the Polish Academy of Sciences (IPPT PAN)

and

Wrocław University of Science and Technology

under the auspices of Committee on Mechanics of the Polish Academy of Sciences
and Polish Association for Computational Mechanics

ISBN 978-83-65550-53-8

Warsaw 2024

Experimental investigation of Euler's elastica: in-situ SEM nanowire post-buckling

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KEYWORDS: *Nanowire, nanomechanics, nonlinear elasticity theory*

ABSTRACT

The Euler Buckling Formula (EBF), which gives the critical value of the compressive load required to buckle a long slender rod, is widely used mainly by structural and mechanical engineers. However, EBF become a subject of wide interest not only on the macroscopic level, but also on the microscopic or nano-scale levels. The increasing demand for nanomaterials with extraordinary properties, e.g. elastic ones, requires mechanical behaviour description. In this research, the post-buckling behaviour of simply supported nanowires is conducted in the framework of the nonlinear elasticity theory. In this theory, the displacement of the end of the wire can be expressed as [1]:

$$u_1(l_0) = 2\,l_0\left(1 - \frac{E(k)}{K(k)}\right), \tag{1}$$

where $K(k)$ and $E(k)$ are the incomplete elliptic integral of the first and second kind, respectively, $k = sin\frac{\alpha}{2}$, $\alpha = \theta(0)$ and $\theta(0)$ is the angle of inclination of the tangent $\mathbf{t}$ to the elastica at the moving end of nanowire. The mid-span deflection can be evaluated as:

$$u_2(l_0/2) = \frac{k\,l_0}{m\,K(k)}, \tag{2}$$

where m is the bifurcation mode and $m = 1,3,5,\ldots$

The experimental method of obtaining a relationship between nanowire end displacement and mid-span deflection is based on SEM images of buckling. Results are presented for three various cobalt nanowires with different length to diameter ratio ($\frac{l_0}{d}$), namely, nanowire *1*: $\frac{l_0}{d} = 12{,}8$; nanowire *2*: $\frac{l_0}{d} = 28{,}4$; nanowire *3*: $\frac{l_0}{d} = 46{,}3$. The elastica with the assumed mth mode and initial angle α can be plotted using *Wolfram Mathematica Software*. In this analysis, the span lengths and diameters of nanowires were the same as in the experiment. According to the book, [1], for cobalt nanowires E = 75 GPa gives Young's modulus mechanical properties used in this simulation. Measured deflection and corresponding nanowire end displacement were compared with elasticaes plotted in *Mathematica*. It is observed that the displacement of the end of the nanowire is aspect-ratio dependent and the mid-span deflection increases by increasing the length-to-diameter ratio. The experiment results indicated a pretty good agreement with those obtained in *Mathematica*. In general, EBF should apply in predicting the shape of elastic, also for nanowires.

ACKNOWLEDGEMENT: The authors acknowledge support and valuable contribution to this work from late Prof. Ryszard B. Pęcherski.

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