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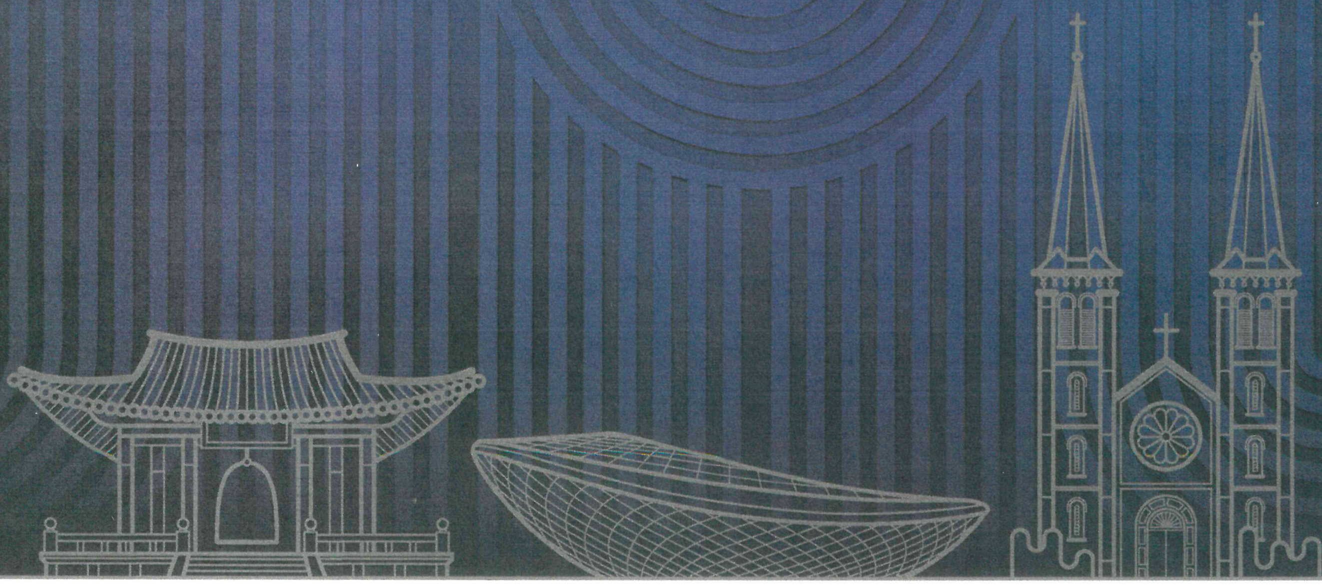
# Abstract Book



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# VERY FLEXIBLE FIBERS IN A SHEAR FLOW

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*Summary* The dynamics of very flexible and relatively short fibers in a simple shear flow have been studied numerically and experimentally. It has been shown that for a sufficiently small ratio of the bending to hydrodynamic forces, fibers tend to form elongated and thin double helices, oriented not far from the vorticity direction. Once formed, the double helices keep their shape for a very long time, until the end of the observations, with almost periodic small deformations. The characteristic time scale of the shape deformations is correlated with the spinning period of the helix. In addition to spinning around their main axes, double-helices perform effective Jeffery orbits.

## THE SYSTEM

We consider a single highly elastic fiber in a shear flow $\boldsymbol{v}_0 = \dot{\gamma} z \hat{\boldsymbol{e}}_x$ of a fluid with a large dynamic viscosity $\eta$. Here $\hat{\boldsymbol{e}}_x$ is the unit vector along $x$-axis. The Reynolds number is much smaller than unity, Re << 1. The fiber is relatively short, with the aspect ratio around 40-60. The bending forces are of the same order of magnitude as the hydrodynamic forces caused by the shear flow, with the bending stiffness ratio $A = E/(64\eta\dot{\gamma})$ of the order of one, where $E$ is Young's modulus. The long-time fiber dynamics are investigated. The time unit is $1/\dot{\gamma}$.

## NUMERICAL SIMULATIONS

### Theoretical method

The fiber is modeled as a chain of $N$ identical spherical beads of diameter $d$. Centers of the consecutive beads are linked by springs, with the fiber stretching potential energy $E_s = \frac{\hat{k}}{2}\sum_{i=2}^{N}(\ell_i - \ell_0)^2$, where $\ell_0 = 1.02d$, $\ell_i = |\boldsymbol{r}_i - \boldsymbol{r}_{i-1}|$, $\boldsymbol{r}_i$ is the position of the center of bead $i = 1, ..., N$, and $\hat{k}/(\pi\eta d\dot{\gamma}) = 1000$. At the elastic equilibrium, the fiber is straight. Its bending costs the energy $E_b = \frac{\hat{A}}{2\ell_0}\sum_{i=2}^{N-1}\beta_i^2$, where $\cos\beta_i = (\boldsymbol{r}_i - \boldsymbol{r}_{i-1}) \cdot (\boldsymbol{r}_{i+1} - \boldsymbol{r}_i)/(\ell_i \ell_{i+1})$ and $\hat{A} = E\pi d^4/64$. For Re<< 1, the fluid flow satisfies the Stokes equations. The dynamics of the fiber beads are $\dot{\boldsymbol{r}}_i - \boldsymbol{v}_0(\boldsymbol{r}_i) = \sum_{j=1}^{N}\left(\boldsymbol{\mu}_{ij}^{tt} \cdot \boldsymbol{F}_j + \boldsymbol{\mu}_{ij}^{td} : \boldsymbol{E}_\infty\right)$, where $\boldsymbol{F}_i = -\frac{\partial}{\partial \boldsymbol{r}_i}(E_s + E_b)$, the mobility tensors $\boldsymbol{\mu}_{ij}^{tt}$ & $\boldsymbol{\mu}_{ij}^{td}$ are evaluated by the multipole method [1] and the Hydromultipole numerical codes [2], and $\boldsymbol{E}_\infty$ is the rate-of-strain tensor.

### Results

Two examples of a double helix spontaneously formed on a highly elastic fiber are shown in Fig. 1, with rotation

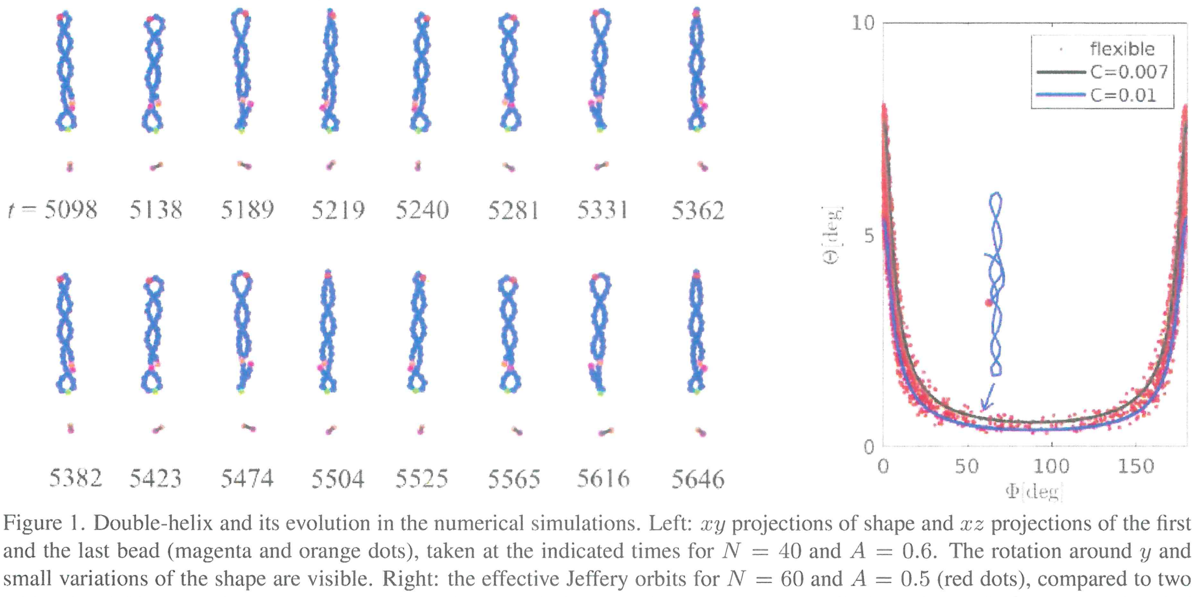


Figure 1. Double-helix and its evolution in the numerical simulations. Left: $xy$ projections of shape and $xz$ projections of the first and the last bead (magenta and orange dots), taken at the indicated times for $N = 40$ and $A = 0.6$. The rotation around $y$ and small variations of the shape are visible. Right: the effective Jeffery orbits for $N = 60$ and $A = 0.5$ (red dots), compared to two orbits of a rigid spheroid with the aspect ratio 13.5 and the indicated values of the constant $C$. The polar angle $\Theta$ and the azimuthal angle $\Phi$ determine the orientation of the main eigenvector of the fiber inertia tensor with respect to the polar axis $y$.

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around the main axis of the double helix (left) and Jeffery orbits close to the vorticity direction $\Theta = 0$ (right). The results correspond to the fiber initially almost straight at $(\Theta_0, \Phi_0) = (80°, 175°)$ (left) and $(20°, 90°)$ (right), with a small perturbation. Similar results (typically with less regular double-helix shapes) are obtained for other initial orientations not very close to the vorticity direction, other initial shapes, and also for a curved elastic equilibrium.

## EXPERIMENTS

### Experimental setup

The fiber fabrication method aligns with that in Ref. [3, 4, 5], but the fibers are one order of magnitude more flexible than in Ref. [4, 5], with Young's modulus $E = O(10^4)$ Pa. In the elastic equilibrium, they are slightly curved. The fiber diameter, length and aspect ratio are $d = 34.8 \pm 3.3\ \mu$m, $l = 1.31 \pm 0.07$ mm, and $r = 38$, respectively. A dilute suspension of those fibers in PEG-DA (fluid viscosity $\eta = 55$ mPa s, volume fraction approximately 0.5%) is placed in the rheometer with transparent glass parallel plates, described in Ref. [4, 5], A constant shear flow is applied between the plates, with a shear rate of $\dot{\gamma} = 523\ \mathrm{s}^{-1}$ near the field of view. The dynamics of the fibers are recorded in the flow-vorticity plane, as described in Ref. [4, 5].

### Results

A snapshot from the experiments, typical for large times, is shown in Fig. 2. Regular or irregular spinning double helices dominate. Their orientation is correlated with the vorticity direction and changes with time almost periodically.

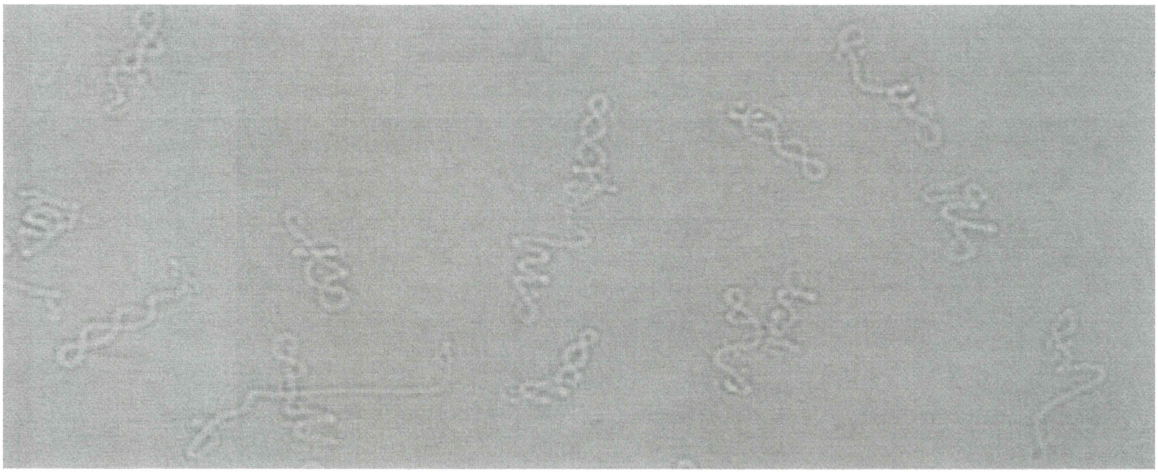

Figure 2. Projection of the fiber shapes on the flow-vorticity plane $xy$ in the experiments with $\dot{\gamma} = 523 s^{-1}$ at time $t = 1453$.

## CONCLUSIONS

Both in experiments and numerical simulations with highly elastic fibers, at large times we observed a spontaneous formation of double helices for a sufficiently small value of the bending stiffness ratio $A$. The double helices tend to orient relatively close to the vorticity direction while performing Jeffery orbits and spinning around their main axes. Such a behavior has not been reported so far. It differs from the dynamics of moderately elastic fibers [6, 7], and occurs at much larger timescales than the buckling [5, 8, 9].

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