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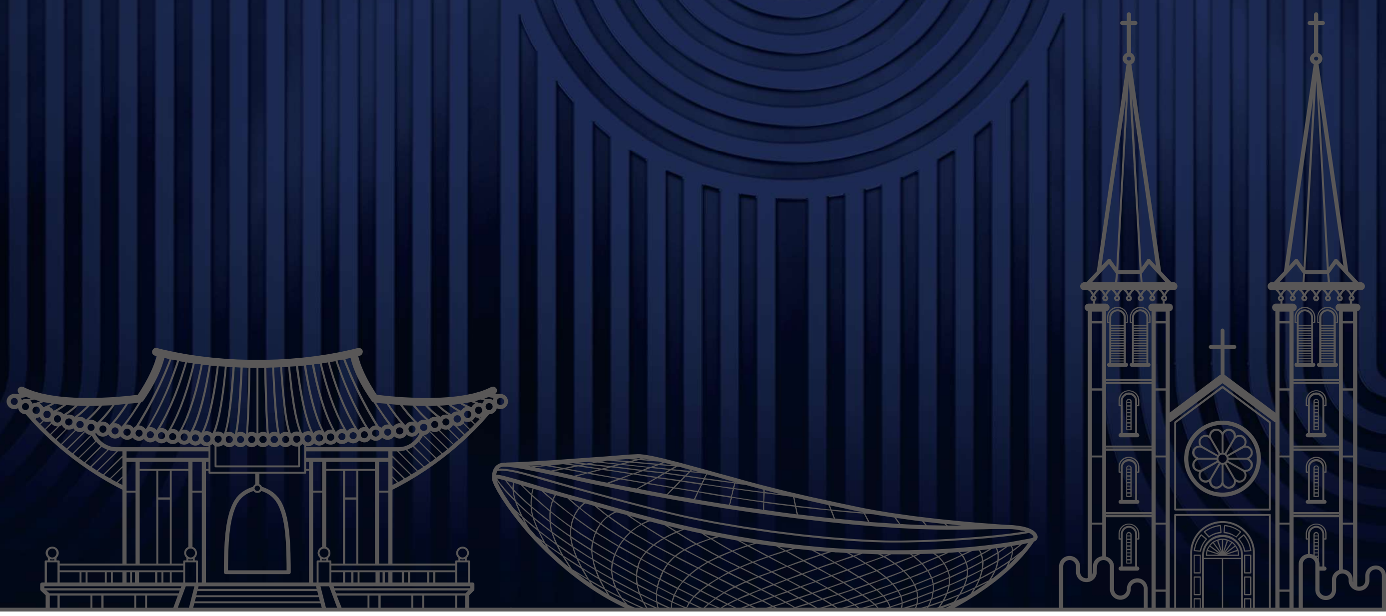
Abstract Book



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INTERACTION CLUSTER MODEL WITH THE MODIFIED TANGENT LINEARIZATION FOR ELASTIC-PLASTIC TWO-PHASE MATERIALS

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Summary The extension of the interaction cluster model to elastic-plastic composites is performed. For this purpose the tangent linearization employing the second moment of stress is used. The proposed mean-field approach allows us to properly quantify the influence of the spatial distribution of inclusions on the anisotropic overall response of heterogeneous materials. The proposed framework is verified using the results of full-field numerical calculations for cubic and random arrangements of spherical particles within the unit cell. The attention is focused on two-phase metal matrix composites and porous metals. It is shown that incorporation of the second moment of stress enables a better assessment of both overall material response and average response per phase compared to other available approaches.

INTRODUCTION

In terms of the current state of computer technology, numerical homogenisation, which allows the microstructure of a material to be directly subjected to mechanical analysis, seems an obvious choice for the assessment of the overall properties of heterogeneous materials. However, this 'brute force' method is still relatively inefficient in terms of resources and time, which incentivizes further advancement of analytical homogenization. The use of swift computational, analytical tools enables the testing of various situations (like adjusting parameters of phases, altering inclusions' spatial distribution or their volume fraction), and finding the optimal solutions based on the intended use of a specific composite. The short calculation times and possibility of automation allow efficient testing of several different scenarios. The goal of this paper is to formulate and verify the extension of the cluster model which properly describes the effective elastic-plastic response of two-phase materials accounting for the spatial distribution of second phase inclusions (reinforcements or voids).

CLUSTER MODEL FOR ELASTIC-PLASTIC COMPOSITES

An interaction cluster model was originally proposed for linear elastic composites in [1] with the aim to account for the spatial distribution of inclusions within the material volume and improve the predictions of classical mean-field schemes for high concentrations. It was later extended to cover thermoelastic properties [2], viscoplasticity and elastic-viscoplasticity [3]. In the present work an extension to elasto-plasticity is discussed.

The scheme assumes identical inclusions of ellipsoidal shape embedded in a uniform matrix with different properties. A representative unit cell with a particular arrangement of heterogeneities is defined, which is next reproduced periodically to fill the whole space. Heterogeneities are grouped into families of symmetrically equivalent ones and the problem reduces to finding the solution which provides the mean strain (or mean stress) experienced by such a family. The resultant strain localization equation providing the mean strain in the inclusion $\boldsymbol{\varepsilon}^{\mathrm{i}}$ in terms of the overall strain $\bar{\boldsymbol{\varepsilon}}$, for the case of a thermoelastic matrix material 'm' is

$$\boldsymbol{\varepsilon}^{\mathrm{i}} = \mathbb{A}^{\mathrm{i}} \cdot \bar{\boldsymbol{\varepsilon}} + \mathbf{e}^{\mathrm{i}} \text{ where } \mathbb{A}^{\mathrm{i}} = \left(\mathbb{I} + \left((1-f_{\mathrm{i}})\mathbb{P}_0 - \boldsymbol{\Gamma}^{\mathrm{i}}\right)\left(\mathbb{C}^{\mathrm{i}} - \mathbb{C}^{\mathrm{m}}\right)\right)^{-1}, \ \mathbf{e}^{\mathrm{i}} = \mathbb{A}^{\mathrm{i}}\left((1-f_{\mathrm{i}})\mathbb{P}_0 - \boldsymbol{\Gamma}^{\mathrm{i}}\right) \cdot \left(\boldsymbol{\beta}^{\mathrm{i}} - \boldsymbol{\beta}^{\mathrm{m}}\right) \quad (1)$$

where $\mathbb{C}^{\mathrm{i/m}}$ are elastic stiffnesses, $\boldsymbol{\beta}^{\mathrm{i/m}}$ are thermal stresses of the two phases and f_{i} is the inclusion volume fraction. The fourth order tensor $\boldsymbol{\Gamma}^{\mathrm{i}}$ includes information about the spatial distribution of inhomogeneities and depends solely on the matrix properties. Relatively simple, closed form formulas for $\boldsymbol{\Gamma}^{\mathrm{i}}$ exist when $\mathbb{C}^{\mathrm{m}}$ is isotropic (see [1] and [4]). More information about the model's development can be found in [3].

The cluster model presented above was derived in the framework of uniform linear thermoelasticity of the matrix material ($\mathbb{C}^{\mathrm{m}}$ and $\boldsymbol{\beta}^{\mathrm{m}}$ are spatially constant). The possible extension of the cluster model for elastic-plastic composites has not been formulated, yet. In the analysis we assume that the metal matrix is described by the Huber-von Mises yield function with the associated flow rule (the so-called isotropic J2 plasticity) and the power-law isotropic hardening. Thus, due to the non-linearity of the local model, the extension requires additional assumptions to be introduced. First, the constitutive law of the matrix has to be linearized and next the obtained stiffness tensor and thermal-like stress, which are in general spatially non-uniform within the matrix, need to be approximated by some 'uniformized' values. Among studied proposals the tangent linearization employing the second stress moment has been selected. According to this method, in elastic-plastic regime the incremental constitutive relation is locally approximated as

$$\dot{\boldsymbol{\sigma}} = \mathbb{C}^{\mathrm{m}}_{\mathrm{ep,tg,iso}}(\bar{\bar{\sigma}}_{eq}) \cdot \dot{\boldsymbol{\varepsilon}} - \boldsymbol{\beta}^{\mathrm{m}}_{\mathrm{ep,tg}}(\boldsymbol{\sigma}^{\mathrm{m}}, \bar{\bar{\sigma}}_{eq}) \quad \text{where} \quad \bar{\bar{\sigma}}_{eq} = \sqrt{\left\langle \sigma^2_{\mathrm{eq}} \right\rangle_m}, \quad (2)$$

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and $< . >_{\rm m}$ denotes averaging over the matrix phase while σ_{eq} is equivalent von Mises stress. Moreover,

$$\mathbb{C}^{\rm m}_{\rm ep,tg,iso}(\bar{\bar{\sigma}}_{eq}) = 3K^{\rm m}\mathbb{I}^{\rm P} + 2G^{\rm m}_{\rm ep,tg}(\bar{\bar{\sigma}}_{eq})\mathbb{I}^{\rm D}\,, \ \boldsymbol{\beta}^{\rm m}_{\rm ep,tg}(\boldsymbol{\sigma}^{\rm m}, \bar{\bar{\sigma}}_{eq}) = (\mathbb{C}^{\rm m}_{\rm ep,tg,iso}(\bar{\bar{\sigma}}_{eq}) - \mathbb{C}^{\rm m}_{\rm ep,tg}(\boldsymbol{\sigma}^{\rm m}, \bar{\bar{\sigma}}_{eq})) \cdot \dot{\boldsymbol{\varepsilon}}^{\rm m}, \quad (3)$$

where $G^{\rm m}_{\rm ep,tg}$ is the elastic-plastic modulus evolving with the accumulated plastic strain [5], and

$$\mathbb{C}^{\rm m}_{\rm ep,tg}(\boldsymbol{\sigma}^{\rm m}, \bar{\bar{\sigma}}_{eq}) \ = \ 3K^{\rm m}\mathbb{I}^{\rm P} + 2G^{\rm m}_{\rm ep,tg}(\bar{\bar{\sigma}}_{eq})\mathbf{N}^{\rm m} \otimes \mathbf{N}^{\rm m} + 2G^{\rm m}(\mathbb{I}^{\rm D} - \mathbf{N}^{\rm m} \otimes \mathbf{N}^{\rm m})\,, \ \mathbf{N}^{\rm m} = \boldsymbol{s}^{\rm m}/\sqrt{\boldsymbol{s}^{\rm m}.\boldsymbol{s}^{\rm m}}\,, \quad (4)$$

$K^{\rm m}$ and $G^{\rm m}$ are the elastic bulk and shear moduli of the matrix, while $\boldsymbol{s}^{\rm m}$ is the deviator of the mean stress $\boldsymbol{\sigma}^{\rm m}$ in the matrix. With such an approximation of the local law all the relations derived for the interaction cluster model with an isotropic thermoleastic matrix material [2] remain valid as long as strain (and stress) tensors are replaced with their rates. Thus for a given loading process calculations are conducted in the incremental way. It should be mentioned that the use of $\bar{\bar{\sigma}}_{eq}$ in the specification of the tangent stiffness instead of the classically used $\bar{\sigma}_{eq} = \sqrt{3/2\boldsymbol{s}^{\rm m}.\boldsymbol{s}^{\rm m}}$ enables us to observe non-linear material response under hydrostatic loading. The evolution of the second moment $\bar{\bar{\sigma}}_{eq}$ is followed using the procedure proposed in [6] for elastic-viscoplastic materials, which adds the Hill lemma to the standard set of equations of the mean-field scheme.

RESULTS

As a means of verification, the results of micromechanical modelling are compared with those of numerical homogenisation performed using the Finite Element Method (FEM). Unit cells with the regular arrangements possessing cubic symmetry: regular cubic (RC), body centred cubic (BCC) or face centred cubic (FCC), composed of spherical inclusions, were generated and subjected to periodic boundary conditions. Results for isochoric extension along the edge and diagonal of a unit cell are demonstrated in Fig. 1. Satisfactory qualitative and quantitative agreement between the proposed modified tangent cluster model and the FE calculations can be seen.

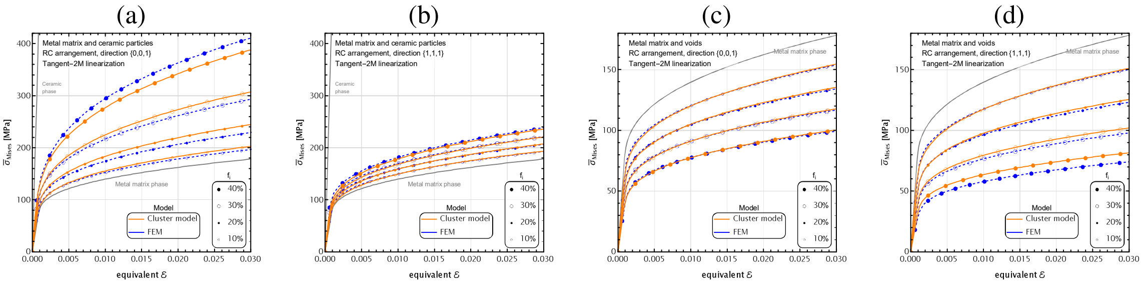

Figure 1. The proposed cluster model and numerical estimations (FEM) of the elastic-plastic response of the metal matrix reinforced with ceramic inclusions (a,b) (material parameters as in [5]) or with pores (c,d). Macroscopic equivalent stress $\bar{\sigma}_{\rm Mises}$ vs. equivalent strain $\bar{\varepsilon}$, for the macroscopic isochoric extension test in two extension directions: (a,c) $(0,0,1)$ and (b,d) $(1,1,1)$. f_i equals $10\%, 20\%, 30\%, 40\%$ and inclusion arrangements are of the RC type.

CONCLUSIONS

In this paper we extended the cluster model to elastic-plastic composites using modified tangent linearization combined with isotropisation of the elastic-plastic tangent stiffness. The obtained formulation is computationally robust. Thus the model is a good candidate for implementation with a view to large-scale finite element calculations. The qualitative and quantitative predictive capabilities of the proposed approach are validated using the more accurate results of numerical homogenization. It is found that the predicted anisotropy of the heterogeneous material response, resulting from periodic arrangements of particles or voids, is quite accurate. To our best knowledge such an extension and verification have not been available in the existing literature.

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References

[1] Molinari A., El Mouden M.: The problem of elastic inclusions at finite concentration. *Int J Solids Struct* **33**:3131–3150, 1996.
[2] El Mouden M., Molinari A.: Thermoelastic properties of composites containing ellipsoidal inhomogeneities. *J Th Stresses* **23**:233–255, 2000.
[3] Kowalczyk-Gajewska K., Majewski M., Mercier S., Molinari A.: Mean field interaction model accounting for the spatial distribution of inclusions in elastic-viscoplastic composites. *Int J Solids Struct*, **224**:111040, 2021.
[4] Kowalczyk-Gajewska K., Maj M., Bieniek K., Majewski M., Opiela K.C., Zieliński T.G.:Cubic elasticity of porous materials produced by additive manufacturing: experimental analyses, numerical and mean-field modelling. *Arch Civ Mech Eng*, **24**:34, 2024.
[5] Majewski M., Holobut P., Kursa M., Kowalczyk-Gajewska K.:Packing and size effects in elastic-plastic particulate composites: Micromechanical modelling and numerical verification. *Int J Eng Sci*, **151**:103271, 2020.
[6] Berbenni S.:A time-incremental homogenization method for elasto-viscoplastic particulate composites based on a modified secant formulation. *Int J Solids Struct*, **229**:111136, 2021.